

Reconciling General Relativity with Quantum Theory and Obtaining a Unified Theory of Quantum Gravity

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Abstract :- The general theory of relativity (GFE) is subjected to the quantum principle in the framework of a new concept (Duality of Space) that leads to the unification of forces in nature on a base of a general principle, including the two theories, and contains a universal constant C.

Keywords:- Quantum Gravity, Quantum Cosmology, Unification of Forces, Spacetime Structure, Universal Constant.

I. INTRODUCTION

The general theory of relativity is a classical theory of gravitation, which does not include the uncertainty principle, as does the quantum theory that describes all other fields. The lack of such consistency is not important in most space-time since the scale at which space-time curvature is very small, and the scale at which quantum effects are important is a very small scale. Gravitational quantity is an important function, so our current region of space-time is constrained to the past by boundaries in which quantum gravitation is important. To answer the important question: How did our universe begin? We will need a quantum theory of gravity. But "at present there is no complete and convincing quantum theory of gravity" [1]

In this research, it has been possible to subject the generalized field equation to the quantum principle within the framework of a unified geometric structure based on a new concept "space duality", which leads to the completion of the unification of forces in nature on the basis of a general principle that combines the two theories, general relativity and quantum theory.

This unified geometric structure opens a wide door for the development of theories in general and gravitational theory in particular, as it is possible by it to explain many phenomena in physics and cosmology, whether those related to microscopic universe or macroscopic universe.

One of the most important things we studied in this research is the concept of acceleration, which is closely related to quantum gravity, since acceleration is exactly equivalent to gravity, and Einstein replaced the geometric concept, which is curvature instead of gravity; accordingly, the curvature is equivalent to acceleration. We have obtained in this research the maximal universal acceleration discussed in the research [2 – 8] as a critical curvature, and its value was calculated using the principle that was extracted in this paper. This principle includes a universal constant C to which the curvature is related according to a definite relation. The maximal universal acceleration in the previous works is on the order of magnitude ($\sim 10^{54} \text{ cm.s}^{-2}$), while its exact value obtained in this paper is $5.7 \times 10^{53} \text{ cm.s}^{-2}$. This agreement with the results of previous research reinforces our principle and emphasizes the importance of the constant C.

II. GFE IN THE PRESENCE OF A SOURCE

Lanczos obtains - first - the generalized field equation GFE [9], and it is derived in another way by Ali El-Tahir [11,10]. The equation of the field in Ali Al-Tahir's model was derived from the principle of least action by taking the metric tensor $g_{\mu\nu}$ to perform the task of field variables, where the integral of the action is invariant in his theory and depends on a function f representing the Lagrangian density $\mathcal{L}$ depending on the scalar curvature R [10,11]

$$f = f(x^\lambda, g_{\mu\nu}, \partial_\lambda g_{\mu\nu}, \partial_\gamma g_{\mu\nu}) = \sqrt{g} \mathcal{L}(R) \quad (2.1)$$

Ali Al-Tahir [12] deduced the following equation:

$$\mathcal{L}'''(g_{\mu\nu} g^{\lambda\gamma} R_{;\lambda} R_{;\gamma} - R_{;\mu} R_{;\mu}) + \mathcal{L}''(g_{\mu\nu} \square^2 R - R_{;\mu;\nu}) - \mathcal{L}' R_{\mu\nu} + \frac{1}{2} g_{\mu\nu} \mathcal{L} - \frac{1}{2} (T_{\mu\nu(m)} - g_{\mu\nu} \gamma_\nu) = 0 \quad (2.2)$$

where [13]:

$$\begin{aligned} \square^2 &\equiv;_{\sigma;\rho} g^{\rho\sigma} \\ R_{;\mu;\nu} &= \partial_\nu R_{;\mu} - \Gamma_{\mu\nu}^\lambda R_{;\lambda} \\ \Gamma_{\mu\nu}^\lambda &= \frac{1}{2} g^{\lambda\gamma} (\partial_\nu g_{\gamma\mu} + \partial_\mu g_{\gamma\nu} - \partial_\gamma g_{\mu\nu}); g_{\rho\sigma} = (g^{\rho\sigma})^{-1} \\ R_{;\mu} &= \partial_\mu R = \frac{\partial R}{\partial x^\mu}; R = g^{\mu\nu} R_{\mu\nu} \\ R_{\mu\nu} &= \partial_\nu \Gamma_{\mu\lambda}^\lambda - \partial_\lambda \Gamma_{\mu\nu}^\lambda + \Gamma_{\mu\lambda}^\eta \Gamma_{\nu\eta}^\lambda - \Gamma_{\mu\nu}^\eta \Gamma_{\lambda\eta}^\lambda \end{aligned}$$

Equation (2.2) is the familiar generalized field equation and includes the term for the source.

When we consider the linear Lagrangian $\mathcal{L} = \beta R$, where

$$\beta = \frac{1}{16\pi G}, \gamma_v = 0$$

Equation (2.2) returns to Einstein's field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi GT_{\mu\nu(m)} \quad (2.3)$$

Thus, equation (2.2) shares Einstein's equation with all its successes [14] in the domain of the weak field.

III. SOLUTION OF A SHORT-RANGE GRAVITATIONAL FIELD EQUATION

Let us see what the field of a star looks like to us within the framework of the gravitational field equation [12]. By reducing the field equation (2.2) by $g_{\mu\nu}$, the result is

$$\square^2 R = \frac{\mathcal{L}'R - 2\mathcal{L} - \mathcal{L}''R;_{\rho}R;_{\rho}}{3\mathcal{L}''} + \frac{1}{6\mathcal{L}''}T_{\mu(m)}^{\mu} - \frac{2}{3\mathcal{L}''} \quad (3.1)$$

A second-order term only in addition to the linear term, that

$$\mathcal{L} = -\alpha R^2 + \beta R \quad (3.2)$$

$$\square^2 R = \frac{\beta}{6\alpha}R + \frac{\rho - 3P}{12\alpha} + \frac{\gamma_v}{3\alpha} \quad (3.3)$$

The field of any isolated star can be described by a static isotropic metric[13]

Using the proper time interval, we write

$$ds^2 = -g_{\mu\nu}dx^{\mu}dx^{\nu} \quad (3.4)$$

In static spherically symmetric coordinates

$$ds^2 = -[A(r)dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2 - B(r)dt^2] \quad (3.5)$$

All components of metric tensor ($g_{\mu\nu} = 0; \mu \neq \nu$) vanish away, leaving only the following

$$g_{rr} = A(r), g_{\theta\theta} = r^2, g_{\varphi\varphi} = r^2\sin^2\theta, g_{tt} = -B(r) \quad (3.6)$$

Thus, by using the relations (3.6) and ignoring the pressure (P), relation (3.3) becomes in the formula

$$\square^2 R = \frac{1}{A}[\ddot{R} - \dot{R}(\frac{\dot{A}}{2A} - \frac{\dot{B}}{2B} - \frac{2}{r})].$$

$$\square^2 R = \frac{\beta}{6\alpha}R + \frac{\rho}{12\alpha} + \frac{\gamma_v}{3\alpha} \quad (3.7)$$

At $R = R_o$, equation (3.7) becomes

$$R_o = \frac{-\rho}{2\beta} - \frac{2\gamma_v}{\beta} \quad (3.8)$$

This means that R is affected by the presence of both matter and space, as it is affected by the presence of matter because it is a function of (ρ), while it is affected by space by the space energy density function γ_v . Matter and space can be considered here as a “frozen” gravitational field, or in other words, the constant gravitational background field manifests itself in the form of matter and space [12].

Equation (3.7) is very complex and highly nonlinear, but it can be simplified by assuming a flat structure of the spacetime coordinates [13], that is,

$$A \rightarrow 1, B \rightarrow 1$$

Therefore, in the off-source range, equation (3.7) becomes

$$\ddot{R} + \frac{2}{r} \dot{R} = \frac{\beta}{6\alpha} R + \frac{\gamma_v}{3\alpha} \quad (3.9)$$

We consider the following solution to this equation:

$$R = \frac{c_1}{r} \exp c_2 r + R_o \quad (3.10)$$

Putting (3.10) into (3.9), we obtain

$$\frac{c_1 c_2^2}{r} \exp c_2 r = R_o + \frac{\beta c_1}{6\alpha r} \exp c_2 r + \frac{\gamma_v}{3\alpha}$$

$$c_2 = \pm \left(\frac{\beta}{6\alpha}\right)^{\frac{1}{2}}; R_o = \frac{-\gamma_v}{3\alpha}$$

$$R = \frac{c_1}{r} \exp -\left(\frac{\beta}{6\alpha}\right)^{\frac{1}{2}} r - \frac{\gamma_v}{3\alpha} \quad (3.11)$$

where c_2 with a positive sign can be excluded because $R \rightarrow 0$, when $r \rightarrow \infty$.

The "quasi-Minkovsky approximation" [15] can be used to express R in terms of potential function (φ), where it becomes

$$\begin{aligned} g^{ii} &\rightarrow \eta^{ii} \\ g^{00} &= \eta^{00} \eta^{00} g_{00} = -(1 + 2\varphi) \\ R &= g^{00} g^{ii} R_{ioio} = g^{oo} g^{ii} \frac{1}{2} \nabla^2 g_{oo} \end{aligned}$$

$$R = 8\pi G\rho\varphi + 4\pi G\rho \quad (3.12)$$

Looking at relations (3.11) and (3.12), it is clear that the gravitational potential is in the formula:

$$\varphi = \frac{c_1}{8\pi G\rho r} \exp -\left(\frac{\beta}{6\alpha}\right)^{\frac{1}{2}} r \quad (3.13)$$

This potential is similar to the Yukawa potential, so it shows the existence of a short-range gravitational field. This means that there is a possibility of linking the two strong and gravitational forces [12].

IV. GRAVITATIONAL WAVE PROPAGATION

Let us now search for the possibility of propagation of gravitational waves by a certain source in empty space [12]

When considering the free space, equation (3.3) becomes

$$\square^2 R = \frac{\beta}{6\alpha} R \quad (4.1)$$

Taking into account the case of semi-Euclidean coordinates, equation (4.1) can be written in the following form:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial \tau^2}\right) R = \frac{\beta}{6\alpha} R \quad (4.2)$$

One of the possible solutions to this equation is written in the following form [12]

$$R = R_c \sin(\omega t - kr) \quad (4.3)$$

where R_c is the maximal curvature. ω is equal to [12]

$$\omega = c \left(\frac{\beta}{12\alpha} \right)^{\frac{1}{2}} \quad (4.4)$$

V. DISCRETE SPACETIME - PERMANENT REST POSITIONS

Because $r \rightarrow \infty$, we will be satisfied with the special form of relation (4.3). which we write

$$R = R_c \sin \omega t \quad (5.1)$$

But: $R \rightarrow 0$, when: $r \rightarrow \infty$

Thus, we write

$$R_c \sin \omega t = 0$$

Since $R_c \neq 0$, then $\sin \omega t = 0$. That is,

$$\omega t = n\pi; n = 0, \pm 1, \pm 2, \pm 3, \dots \quad (5.2)$$

From relation (4.4), we write

$$\omega t = ct \left(\frac{\beta}{12\alpha} \right)^{\frac{1}{2}}$$

Substituting in (5.2), we obtain

$$r = ct = n\pi \left(\frac{12\alpha}{\beta} \right)^{\frac{1}{2}} \quad (5.3)$$

The case ($n = 0$) can be excluded, and only positive values are satisfied to achieve continuity r for all possible n values since $0 < r_n \leq r < \infty$. We will also choose - for the same reason - the positive sign of the root product.

The continuity of r allows us to avoid "singularity" automatically because $r > 0$. Therefore, we write

$$r_n = n\pi \left(\frac{12\alpha}{\beta} \right)^{\frac{1}{2}}, n = 1, 2, 3, \dots \quad (5.4)$$

➤ Discreteness of Space

Far from the source $r \rightarrow \infty$, then $n \rightarrow \infty$, so the space separation is not apparent due to the large number n . However, it appears clearly when r is of the order of magnitude r_1

We have

$$R = \frac{c_1}{r} \exp c_2 r - \frac{\gamma_v}{3\alpha}$$

Putting $\gamma_v = 0$, we write

$$(5.5) R = \frac{c_1}{r} \exp c_2 r$$

We also have

$$c_2 = - \left(\frac{\beta}{6\alpha} \right)^{\frac{1}{2}}$$

Referring to relation (5.4), it becomes clear that

$$c_2 = \frac{-\pi\sqrt{2}}{r_1}$$

Or we write

$$c_2 = -\frac{1}{r_a}$$

Then,

$$r_a = \left(\frac{6\alpha}{\beta}\right)^{\frac{1}{2}} \quad (5.6)$$

Becomes as (5.5) and relation

$$R = \frac{c_1}{r} \exp\left(-\frac{r}{r_a}\right) \quad (5.7)$$

(r_a) is the permanent rest in this space, where the field quantifies, i.e., it becomes a particle with an inertial mass m_a . Therefore, the stability of a particle is its position in a reference frame at which the change in its decreasing velocity approaches zero. In other words, the accelerating reference frame eventually results in an inertial reference frame that travels at a speed close to zero.

When $r = r_a$, therefore

$$R = \frac{c_1 e^{-1}}{r_a}$$

Or

$$R = \frac{b}{r_a} \equiv R_a \quad ; \quad b = c_1 e^{-1} \quad (5.8)$$

where R_a is the curvature at the permanent rest position r_a

VI. THE DUAL PROPERTIES OF SPACETIME

The apparent contradiction between the discrete and continuous pictures of spacetime can be explained on the basis of a dual system in geometry, in which these two different pictures are combined to form a unified dual properties spacetime. That is, it is possible to reconcile these two seemingly contradictory pictures and to hold them together as dual properties of spacetime.

Now, we return to relation (5.7) and write it as follows:

$$R = \frac{b}{r} \exp\left(1 - \frac{r}{r_a}\right); 0 < r_a \leq r < \infty \quad (6.1)$$

This relation describes a continuous space: $0 < r_a \leq r < \infty$; $R = R(r)$, where r_a is a permanent rest position in this space. Considering the discreteness of space, there is not one permanent rest position but an infinite number of ones $(r_{a_1}, r_{a_2}, r_{a_3}, \dots, r_{a_n})$, where energy is transferred between them in the form of quanta.

Therefore, we can generally write relation (6.1) as

$$R = \frac{b_n}{r} \exp\left(1 - \frac{r}{r_{a_n}}\right); 0 < r_{a_n} \leq r < \infty; n = 1, 2, 3, \dots \quad (6.2)$$

Considering permanent rest positions, the space is discontinuous, that is, r takes discrete values $(r_{a_1}, r_{a_2}, r_{a_3}, \dots, r_{a_n})$. The domains are then quantized, that is, they form particles, where their quantum properties depend on the selected permanent rest positions, and the exponential factor goes to unity. Thus, relation (6.2) becomes at the permanent rest positions $r = r_{a_n}$ as follows:

$$R = \frac{b_n}{r_{a_n}} \equiv R_{a_n}; n = 1, 2, 3, \dots \quad (6.3)$$

By analogy with (5.4), relation (5.6) can be generalized to be written as follows:

$$r_{a_n} = n \left(\frac{6\alpha}{\beta}\right)^{\frac{1}{2}} \quad (6.4)$$

Or

$$r_{a_n} = nr_{a_1}; n = 1, 2, 3, \dots \quad (6.4a)$$

A similar formula can be found for relation (6.4) by making α negative and by putting $\gamma_v = 0$ in relationship (3.11), so it is

$$R = \frac{c_1}{r} \exp - i \left(\frac{\beta}{6|\alpha|} \right)^{\frac{1}{2}} r$$

or we write

$$R = \frac{c_1}{r} \exp - ikr; k = \left(\frac{\beta}{6|\alpha|} \right)^{\frac{1}{2}}$$

This relation is also expressed as

$$R = \frac{c_1}{r} (\cos kr + i \sin kr)$$

By taking the scalar curvature R to be a real number, the imaginary part of this relation must be equal to zero, and therefore, $\sin kr = 0$: that is, $kr = n\pi$.

Or we write

$$r = \frac{n\pi}{k} = n\pi \left(\frac{6|\alpha|}{\beta} \right)^{\frac{1}{2}}$$

Therefore, relation (6.3) with a substitution of (6.4a) can be written as follows:

$$R_{a_n} = \frac{1}{r_{a_1}} \left(\frac{b_n}{n} \right); n = 1, 2, 3, \dots \quad (6.5)$$

This means that the scalar curvature has discrete values due to the discreteness of spacetime

Relation (6.2) expresses what we might call “space duality” or “discrete -continuous of space”. That is, this relation combines the two properties of space and removes the apparent contradiction between them, and then there are dual properties of space that combine the two theories of general relativity and quantum theory.

VII. PERMANENT RESTS AND QUANTUM PROPERTIES

Mass m_a , permanent rest position r_a and permanent rest time t_a are fundamental properties that are not reduced to other properties but are expressed by others by their significance. Energy, momentum and other quantum properties are derived from them.

We write the Compton wavelength of a particle of mass m as

$$\lambda = \frac{h}{mc}$$

Based on Bohr's hypothesis and its generalization, we can consider that the mass of a particle at its stationary state m_a is a special that arises from the bending of a wave of length λ_a and folding on itself in the form of a circle, where the particle can be located in a stable state at a distance r_a from the center of this circle. Therefore, based on Bohr's hypothesis, we can consider that the mass of a particle at its permanent rest position m_a is a special that arises from the bending of a wave of length λ_a and folding on itself in the form of a circle, where the particle can be located in a permanent rest position r_a from the center of this circle. Therefore,

$$\lambda_a = 2\pi r_a \quad (7.1)$$

Therefore, we write the mass of the particle at its permanent rest r_a as follows:

$$m_a = \frac{\hbar}{r_a c}$$

Or we write - in general - the masses of all particles at their permanent rest positions as follows:

$$m_{a_n} = \frac{\hbar}{r_{a_n} c}; n = 1, 2, 3, \dots \quad (7.2)$$

Relation (6.3) indicates that the existence of a constant b_n depends on the permanent rest position, but we can find from it a constant C that has the same value at all permanent rest positions. To obtain C , we have relation (3.12) as

$$R = 8\pi G \rho \varphi + 4\pi G \rho$$

It is noted from the previous relations that φ and ρ , R refer to very small values when r turn to a very large value, so the value $\rho \varphi$ is very small as we can neglect the term it contains. to write

$$R = 4\pi G \rho \quad (7.3)$$

By excluding $r = 0$ and considering the constant r_a as the first position after it, the particle becomes a hollow object (a hollow sphere); that is, its matter is distributed to form a spherical surface. Therefore, the surface density σ - not the volume density ρ - should be considered in this case.

Therefore, instead of relation (7.3), we write R at the permanent rest position as follows:

$$R_a = 4\pi G \sigma_a \quad (7.4)$$

Where

$$A_a = 4\pi r_a^2, \quad m_a = \frac{\hbar}{r_a c}, \quad \sigma_a = m_a / A_a$$

Or

$$R_a = \frac{G \hbar}{c r_a^3}$$

That is,

$$r_a \cdot R_a = \frac{G \hbar}{c r_a^2} = b$$

Then, we write

$$b r_a^2 = \frac{G \hbar}{c} \quad (7.5)$$

By comparing the dimensions in relation (7.5), we find that: $b = v_a^2$

Where v_a is the particle's momentum per unit mass at its permanent rest position r_a .

So we write the relation (7.5) in the following form:

$$v_a \cdot r_a = \left(\frac{G \hbar}{c} \right)^{\frac{1}{2}} = C \quad (7.6)$$

$$C = 4.84 \times 10^{-23} \text{ cm}^2 \cdot \text{s}^{-1}$$

This relation describes an essential principle in nature that includes the two theories, GR and quantum theory, and is written in the following form:

"The product of a permanent rest position r_a of a particle and its momentum per unit mass v_a at that position is equal to a constant C ," that is, "the moment of inertia of a unit mass per its rest radius ($v_a \cdot r_a$) is invariant under transforms from one rest position to another".

Thus, the most profound progress in physics occurs when the principles of various theories are organized into one framework. Now, we have come to know the principle behind the unification of the two theories, general relativity and quantum theory, that govern the fundamental theory.

Based on this principle, the unit of mass m_c can be written as

$$m_c = \frac{\hbar}{c} = 2.2 \times 10^{-5} g \quad (7.7)$$

This mass is the mass of our universe at the beginning of time and is equivalent to the Planck's mass

Now we have

$$b = v_a^2 = \left(\frac{p}{m_c}\right)^2$$

Or we write

$$v_a = \frac{mc}{m_c}; p = mc$$

We can take

$$a \equiv \frac{m}{m_c}.$$

Then, we write

$$v_a = ac.$$

If $m = m_c$, then $a = 1$. Therefore, we write in this case $v_a = c$

If the particle has "no" mass, then $a \rightarrow 0$, that is, $v_a \rightarrow 0$

Therefore, v_a changes between zero and the maximal value c at permanent rest positions in space

$$0 < v_a \leq c \quad (7.8)$$

This result confirms that the speed of light c is constant at all permanent rest positions r_{a_n} in space, while its momentum per unit mass ac is variable because $0 < a \leq 1$.

A particle permanent rest position r_a , if given enough energy, can be shifted to a higher rest position in space near $r = 0$, but there is a boundary position r_c that cannot be exceeded, where the curvature is maximal $R = R_c$.

To find r_c , we consider the maximum speed c that the particle can be accelerated to

o. Therefore, we write relation (7.6) as follows:

$$r_c = \frac{c}{c^2} = 1.6 \times 10^{-33} cm \quad (7.9)$$

The r_c is the same position where the energy was released at the beginning of time when the universe was created. The time of our universe began at $t = t_c$, as

$$t_c = \frac{c}{c^2} = 5.4 \times 10^{-44} s \quad (7.10)$$

It is clear from relations (7.10) and (7.9) that the time during which the "unified process" takes place is $5.4 \times 10^{-44} s$, which is the closest time to zero, and the range of the "unified force" is $1.6 \times 10^{-33} cm$, which is the lowest position in our universe from zero.

The creation of our universe began at (r_c, t_c) . At that time, the general theory of relativity does not collapse, but rather, there can be an understanding of how creation began.

The mass m_c in relation (7.7) is the mass of the "first particle", and we will call it "al-amr"(means: the command), which was launched at the beginning of time t_c , descending from the position r_c , carrying the unified force, transforming phase by phase, where its mass "vanishes" at the end of its path $r \rightarrow \infty$ and "all" turns into small stable particles and energy. Therefore, the mass unit can be taken to be equal to the mass of the first particle in our macroscopic universe (al-amr).

Principle (7.6) can be formulated in another form as follows:

$$m_a \cdot r_a = m_c \cdot r_c = D ; \quad D = 3.5 \times 10^{-38} g \cdot cm \quad (7.11)$$

That is, "the moment of inertia for a particle per its rest radius ($m_a \cdot r_a$) is an invariant when transforms from one permanent rest position to another".

Based on principle (7.11), the quantization masses can be expressed similarly to relation (6.3). where we write

$$m = \frac{D}{r} \exp\left(1 - \frac{r}{r_{a_n}}\right); 0 < r_{a_n} \leq r < \infty \quad (7.12)$$

This relation becomes in the permanent rest positions $r = r_{a_n}$ as follows:

$$m = \frac{D}{r_{a_n}} \equiv m_{a_n} ; n = 1, 2, 3, .. \quad (7.13)$$

It is completely equivalent to the relation (7.2)

At $n = 1$, in the case of the unified field, then $r_{a_1} = r_c$, that is, $m_{a_1} = m_c$

At $n \rightarrow \infty$, then $r_{a_\infty} \rightarrow \infty$, that is, $m_{a_\infty} \rightarrow 0$

The mass m_{a_∞} is the mass of the first particle (al-amr)- it is also the mass of photon - at present time, and it is not equal to zero, but rather approaches it infinitely close to it, so this smallest mass can be calculated, as we write

$$m_{a_\infty} \cdot r_{a_\infty} = m_c \cdot r_c = 3.5 \times 10^{-38} g \cdot cm$$

However, $r_{a_\infty} = 1.6 \times 10^{28} cm$, radius of the universe. So then

$$m_{a_\infty} = 2.2 \times 10^{-66} g \quad (7.13a)$$

Now, we return to relation (7.5) and write it as follows: $br_a^2 = C^2$

Dividing both sides by r_a^3 , we obtain

$$\frac{b}{r_a} = \frac{C^2}{r_a^3} = R_a$$

Therefore, we write the maximal universal curvature at r_c as follows:

$$R_c = \frac{C^2}{r_c^3}$$

Referring to relation (7.9), it becomes

$$R_c = \frac{c^3}{c} = 5.7 \times 10^{53} cm \cdot s^{-2} \quad (7.14)$$

Obtaining a limited value for the maximal universal curvature negates the existence of the "singularity" within our universe, and matching the curvature dimensions with the acceleration dimensions is consistent with the principles of general relativity.

Relation (7.14) indicates the existence of a maximal universal acceleration, which depends on the constants c and C and is mainly related to the existence of a microscopic scale for space-time (r_c, t_c). This result is consistent with previous research [2-8].

The presence of the maximal universal acceleration on this scale indicates “the occurrence of an accelerated expansion (decreasing acceleration) of our universe at the beginning of time” [16], as the force of universal attraction arises from this decreasing acceleration.

The maximal universal curvature per square of the velocity unit $\left(\frac{R_c}{c^2}\right)$ can be expressed by the following formula:

$$R_c = \frac{1}{r_c} \quad (7.15)$$

Where we have taken here $c = 1$.

We write the curvatures at all permanent rest positions

$$R_{a_n} = \frac{C^2}{r_{a_n}^3}$$

$R = R(r)$ for the dual space has the following form:

$$R = \frac{C^2}{r^3} \exp\left(1 - \frac{r}{r_{a_n}}\right); 0 < r_{a_n} \leq r < \infty; n = 1, 2, 3, \dots \quad (7.16)$$

We have

$$r_{a_n} = nr_{a_1} \quad ; \quad n = 1, 2, 3, \dots$$

Therefore, we write relation (7.16) at permanent rest positions $r = r_{a_n}$ as follows:

$$R_{a_n} = \frac{C^2}{r_{a_1}^3} \left(\frac{1}{n^3}\right); n = 1, 2, 3, \dots \quad (7.17)$$

r_{a_1} an arbitrary constant that specifies the range of the field.

For the unified field, it is $r_{a_1} = r_c = \frac{c}{\sigma}$, so we write

$$R_{a_n} = \frac{c^3}{C} \left(\frac{1}{n^3}\right); n = 1, 2, 3, \dots \quad (7.17a)$$

at $n = 1$, then $R_{a_1} = R_c$. At $n \rightarrow \infty$, then $R_{a_\infty} \rightarrow 0$

Thus, the generalized field equation can be subjected to the quantum principle within the framework of a geometrical structure that accommodates the two theories – general relativity and quantum theory - and leads to the completion of the process of unification of forces in nature.

The critical density σ_c of the matter at r_c can be calculated from the following relation:

$$\sigma = \frac{m_c}{4\pi r_c^2} = 6.7 \times 10^{59} g.cm^{-2} \quad (7.18)$$

Or we write

$$\sigma_c = \frac{m_c}{r_c^2} \quad (7.18a)$$

where $\sigma_c = 4\pi\sigma$ is the critical density of the matter.

Therefore, when we return to the starting position r_c at the beginning of time t_c , we find that our universe matter is not of infinite density, but its density is limited, and it is very large. There are no "singularities" in the universe, i.e. positions where the density of matter is infinite

Relation (7.18) can be written in terms of C as follows:

$$\sigma_c = \frac{\kappa}{C} \quad (7.18b)$$

Where κ represents the rate of creation of matter in our macroscopic universe and is also equal to the mass of the universe M per its age T

$$\kappa = \frac{M}{T} \quad (7.19)$$

Therefore,

$$M = 10^{61} m_c \quad (7.19a)$$

Note that

$$m_c = 10^{61} m_{a_\infty}$$

Or we write

$$m_c = N m_{a_\infty}$$

We will call : "Ibrahim's number", which represents the "maximal universal number", or the "cosmic quantum number".

$$N = 10^{61} \quad (7.19b)$$

Therefore,

$$M = \frac{m_c^2}{m_{a_\infty}} \quad (7.19c)$$

The different forces combine at a very high energy ($\sim 10^{19} \text{ GeV}$), where they form one unified force, and then the particles and forces are the same. Thus, the structure of the unified theory is simple and consistent, and this confirms the belief that nature must be like that in its deepest structure.

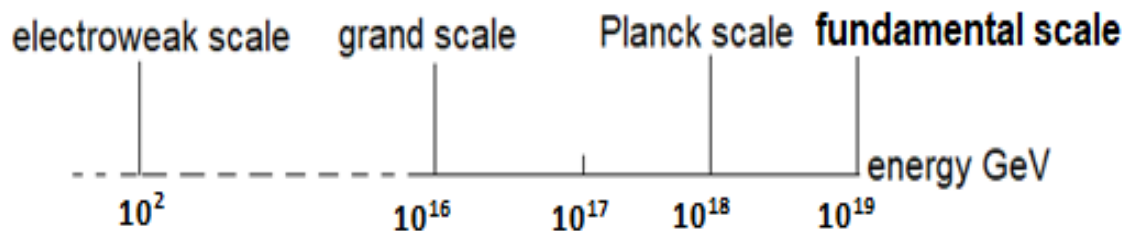


Fig 1 Fundamental Unification Energy and Other Unification Energies

This theory is basically tested on the extent of its success in interpreting the values of the universal constants and their agreement with the measured values. The theory can be tested by experimental investigation of its principle at low energies. By understanding the behavior of particles and forces at energy (10^{19} GeV), we reach the beginning of time when the beginning of time when Allah created our universe by His command (Be) and it has been.

The discovery of the unified theory allows us to answer the deepest questions in physics and cosmology: How did our universe begin? what is the beginning of spacetime of our universe? How does our universe evolve, and how will it end? why do constants like $\hbar$ and c have their known values?

VIII. INTERPRETATION OF UNIVERSAL CONSTANTS

A system of units is chosen once the fundamental units of mass M , length L and time T have been assigned. MKS system of units is often used in practical and applied physics

$$[M] = kg, [L] = m, [T] = s \quad (8.1)$$

In theoretical physics, the Gaussian system of units CGS is used

$$[M] = g, [L] = cm, [T] = s \quad (8.2)$$

in particle physics, Planck's constant $\hbar$ is used as the unit of action, and the speed of light c as the natural unit of velocity

$$[c] = LT^{-1}, [\hbar] = ML^2T^{-1} \quad (8.3)$$

This is because all natural quantities that have action dimensions such as angular momentum and spin are measured in $\hbar$, and all velocities are measured in units of the speed of light" [18].

$$[M] = am, [L] = ad, [T] = aq \quad (8.4)$$

We will symbolize the system of unit of unified force: adnaa .amr .aqrab - abbreviated - in letters(AAA). So this system of units can be used in theoretical physics

➤ The fundamental units are illustrated in the following table.

Table 1 Fundamental Universal Units

Unit	Value (CGS)	Value(AAA)	Dimensions
mass unit (m_c)	$2.2 \times 10^{-5}g$	1 amr	M
length unit (r_c)	$1.6 \times 10^{-33}cm$	1 adnaa	L
time unit (t_c)	$5.4 \times 10^{-44}s$	1 aqrab	T

➤ The universal constants can be derived from the fundamental units included in the following table:

Table 2 Derived universal constants

Constant	Definition	Value(AAA)	Dimensions
Hubble const.	$H = \frac{1}{t_c}$	$1aq^{-1}$	T^{-1}
The cosmological const.	$\Lambda = \frac{1}{r_c^2}$	$1ad^{-2}$	L^{-2}
Moment of inertia/radius	$D = m_c r_c$	$1am \cdot ad$	ML
Rate of matter creation	$\kappa = \frac{m_c}{t_c}$	$1am \cdot aq^{-1}$	MT^{-1}
Critical surface density	$\sigma_c = \frac{m_c}{r_c^2}$	$1am \cdot ad^{-2}$	ML^{-2}
length density	$\lambda = \frac{m_c}{r_c}$	$1am \cdot ad^{-1}$	ML^{-1}
Speed of light	$c = \frac{r_c}{t_c}$	$1ad \cdot aq^{-1}$	LT^{-1}
Critical curvature	$R_c = \frac{r_c}{t_c^2}$	$1ad \cdot aq^{-2}$	LT^{-2}
Al-amr const.(Ibrahim's const.)	$C = \frac{r_c^2}{t_c}$	$1ad^2 \cdot aq^{-1}$	L^2T^{-1}
Planck's const.	$\hbar = \frac{m_c r_c^2}{t_c}$	$1am \cdot ad^2 \cdot aq^{-1}$	ML^2T^{-1}
Critical charge(sq)	$q_c^2 = \frac{e^2}{\alpha} = \frac{m_c r_c^3}{t_c^2}$	$1am \cdot ad^3 \cdot aq^{-2}$	ML^3T^{-2}

Newton's const.	$G = \frac{r_c^3}{m_c t_c^2}$	$1 am^{-1}. ad^3. aq^{-2}$	$M^{-1} L^3 T^{-2}$
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This is how the values of universal constants such as $\hbar$, c and G are interpreted in terms of the fundamental units of Al-amr Theory(Ibrahim's quantum gravity theory). The constant α is the fine structure constant. At the beginning of time" unified field"

$$\alpha = 1, \text{ i.e. } q \equiv e \quad (8.4)$$

where we will call q "the critical charge".

If $r, t = 0$, then $\hbar$, c and the other constants become undefined values, i.e., "break" then the laws of nature. This condition does not belong to our universe but is outside and separate from it. It is unique, and it is the highest position in existence(absolute height).This space-time is not befitting except with the Creator of the universe, his Lord and the One in charge of it, Allah - Glory be to Him -.

➤ The universal constants of physics can be expressed in terms of Ibrahim's constant C as follows:

Table 3 Relations with the Al-Amr Constant

Universal const.	Relation with C
maximal universal acc.	$a_c = c^3 C^{-1}$
Cosmological const.	$\Lambda = c^2 C^{-2}$
Hubble's const.	$H = \Lambda C$
the unified energy	$E_c = \kappa C$
Planck's const.	$\hbar = m_c C$
Newton's const.	$G = c m^{-1} C$
critical charge(sq)	$q^2 = c m_c C$
critical magnetic moment(Ibrahim magneton)	$\mu_c = q_c C$

The table shows the importance of Ibrahim's constant C and its functional task in formulating the main relations of physics and cosmology.

The following relation is fulfilled for our macroscopic universe:

$$\frac{m_c}{r_c} = \frac{M}{R} \quad (8.5)$$

Relation (8.5) expresses the equilibrium condition for our macroscopic universe

Generally, we can write this condition as

$$\frac{r}{m} = \frac{1}{\lambda} \quad (8.6)$$

$$; \quad r_c, m_c \leq r, m \leq R, M$$

where λ , is the length density of a matter in our macroscopic universe.

We can write the equilibrium condition for our microscopic universe as

$$r. m = D \quad (8.7)$$

$$; \quad r_{a_\infty} \geq r \geq r_c \quad ; \quad m_{a_\infty} \leq m \leq m_c$$

where D is the moment of inertia of a mass m_{a_n} per its rest radius r_{a_n}

The constant C combines $1/\lambda$ (macroscopic) and D (microscopic) as follows:

$$C = c \left(\frac{D}{\lambda} \right)^{\frac{1}{2}} \quad (8.8)$$

IX. CONCLUSION

➤ *The Main Results in this Paper can be Summarized in the Following Points:*

- The general theory of relativity (the generalized field equation) is subjected to the quantum principle according to the relationship (7.17a) within the framework of a unified geometric structure based on the concept of "space duality" expressed in formula (7.16).
- This dual structure of space-time combined the two theories, GR and quantum theory, and then revealed a geometric symmetry between our macroscopic and microscopic universe
- Relation (7.6) in this research describes a general principle in nature (al-amr principle) that includes the two theories, GR and quantum theory, and includes a universal constant C that has the same value at all permanent rest positions in space-time.
- There is a universal unit of mass, which is determined by the following relation:

$$m_c = \frac{\hbar}{C}$$

➤ *This Mass Unit Is A Special Characteristic That Distinguishes The First Particle (Al-Amr), The Carrier Of The Unifying Force.*

- Relation (7.8) indicates that the constant v_a depends on the permanent rest positions, where it changes between zero and the largest value c . This result confirms the constancy of the speed of light c at all rest positions in spacetime.
- Relation (7.9) indicates the existence of a critical position r_c in spacetime. It cannot be exceeded by any accelerating particle, where the curvature is maximal R_c , it is the same position where the creation of our universe began
- The beginning of the time t_c - when the universe was created - is determined according to the following relation:

$$t_c = \frac{C}{c^2}$$

- The previous two results lead directly to determining the range of the unified force and the time of the unified process.
- Relations (7.14) and (7.18b) resolve the existence of "singularities" within our universe and agree with the principles of general relativity.
- Relation (8.6) expresses the equilibrium condition for our macroscopic universe, while relation (8.7) expresses the equilibrium condition for our microscopic universe. The two conditions are combined together by relation (8.8).
- The maximal universal number N (Ibrahim's number), or the cosmic quantum number, is discovered within the

results of this paper, which is the product of dividing the maximum value of any universal quantity by the minimum value of the same universal quantity.

- The "singularity" at $r, t = 0$ does not belong to our universe but is outside and separate from it. It is unique, and it is the highest position in existence (absolute Height), and this space-time is not befitting except with the Creator of the universe, his Lord and the One in charge of it, Allah - Glory be to Him -.
- An elementary particle is something hollow, not holding. All previous results were based on the "particle model" concluded in this paper.
- Data availability
- The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

REFERENCES

- [1]. S. Weinberg (1972); Gravitation and Cosmology. Wiley and Sons.
- [2]. R.G. Torrome (2007); On the Maximal Universal Acceleration in Deterministic Finslerian Models, Germany.
- [3]. R. Gallego Torrome (2006); A Finslerian version of 't Hooft Deterministic Quantum Models, Jour. Math. Phys. 47, 072101.
- [4]. G. Lambiase, G. Papini, R. Punzi, G. Scarpetta (2006); Lower Neutrino Mass bound from SN1987A data and quantum geometry, Class.Quant.Grav. 23,1347
- [5]. W.G. Unruh (1974); Notes on Black Hole Evaporation, Phys. Rev. D 14,870.
- [6]. H.E. Brandt, (1996); Finslerian Space-Time, Contemporary Mathematics 196, 273.
- [7]. E. R. Caianiello (1992); Quantum and Other Physics as System Theory, La Rivista del Nuovo Cimento, Vol 15, Nr. 4.
- [8]. C. Castro (2005); The Extended Relativity Theory in Clifford PHASE spaces with a lower and upper scale Foundations of Physics, 35, no. 6971
- [9]. C. Lanczos (1932); Phys. Rev. 31,716.
- [10]. Ali El-Tahir (1982); PhD thesis (City University, London).
- [11]. Ali El-Tahir (1987); "A Generalized Variational Principle of Gravitation" ICTP Preprint .IC/87/243.
- [12]. M. Dirar. Ali El-Tahir (1996); "A New Derivation of the Generalized Field Equation with a Source Term" ICTP Preprint.IC/96/158.
- [13]. G. 't Hooft (2002); "Introduction to General Relativity" Princetonplein 5.3584CC trecht, the Netherlands.
- [14]. Chongming Xu, G.F.R. ; Ellis, Xuejun Wu and Ali El-Tahir (1992); S.Afr.J. Phys.15,5.
- [15]. Derk, F. Lawden (1982); An Introduction to Tensor Calculus and Relativity. Wiley & Sons, New York.
- [16]. Diego Pavon (2006); "Holographic dark energy and late cosmic acceleration" 08193 Bellaterra. Spain.

- [17]. A. Seidan (2004); Particle Physics : A Comprehensive Introduction. Addison & Wesley.