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Why Poynting's Theorem P = Ex H is Quite Valid for DC Circuits

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Abstract:- Poynting's vector theorem P=ExH is one of the universal laws of physics that applies to electromagnetic fields in AC and DC circuits. A rigorous analysis of two arbitrary cases of DC and AC circuit electromagnetic fields shows that Poynting's law P=ExH applies to both station ary and time-varying electromagnetic fields.

Keeping the generality, we analyze two simple cases of time-varying and stationary fields of a regular cylindrical wire carrying direct or alternating current where in both cases the electromagnetic energy flow calculations validate the hypothesis that the theorem of Poynting is absolute. Moreover, the interpretation of the results also suggests that the photons of light beams or electromagnetic field bundles cannot live forever, they do not have an infinite but a finite lifespan because they can be created or annihilated during their interaction with free or bound charges such as electrons.

I. INTRODUCTION

There are at least two incompatible theories, classical EM theory based on Maxwell's equations, where wave energy is continuous and quantum theory QM, where the energy of EM waves is essentially discrete or quantized in photons.

A. Soit depends on the theory you use to analyze the problem.

In this article, we follow classical EM theory combined with Einstein's special theory of relativity similar to a previous article explained in ref. [1,2] to arrive at a rigid conclusion.

First, the Poynting vector P is mathematically and physically correct and represents the energy flux per unit area per unit time for electromagnetic fields in 4D space which is the subject of this article.

However, in the present study, we discuss the case where the time variation of electromagnetic fields is small enough, which means that we limit our study to the Poynting vector in non-radiative electrical circuits.

B. Some might simply expect that Poynting's vector formula for energy flow in EM fields would only apply to time-varying fields, but in fact it is also valid for electromagnetic fields of DC circuits.

The classical theory of electromagnetic fields is mathematically and physically based on Maxwell's equations which are a set of four equations, described in integral or differential form. They form the theoretical basis for describing classical E&H fields in electromagnetism.

Moreover, it should be noted that Maxwell's equations are valid in all inertial frames and are the first equations in physics compatible with the laws of special relativity [1,2,3].

The set of four Maxwell's equations in differential form is composed of two divergences and two curls as follows:

- Div E= Rho/∈
- Div B= 0
- Curl E = (partial) d (B/dt)
- Curl B= μ J + (partial) d (D/dt)

In normal conventions [2,3].

Where J is the conduction current density due to motion of free charges, Rho is the volumetric electric charge density and D = Eps E is the displacement vector while its partial time derivative is the displacement electric current density as proposed by Maxwell.

μ is free space permeability

μ =4 Pi E-7 Kg m C^-2

[μ] = [M L T^0 Q^-2]

∈ is Permittivity of free space

∈=(E-9/36 Pi) Kg^-1m^-3s^2C^2

[∈] = [M ^-1L^-3 T^2 Q^2]

C. It should be mentioned that equation 4 by itself implies that E and H must be mutually perpendicular when E and H emanate from the same electric charge.

This property is particularly useful for calculating the pointing vector P = ExH for DC and AC circuits, as discussed later in Section II.

Considering, for example, vacuum or any medium with no free charge (Rho = 0), then equation 4 reduces to,

μ. Curl H = ∈. (partial) d/dt E . . . . (5)

Since the LHS of Eq 5 is a vector orthogonal to H and the RHS of Eq 5 is a vector in the direction of E since t is a scalar. then E is perpendicular to H for the dependent fields coming from the same Rho source.

Assuming that Maxwell's four equations are universal imperial laws and a valid relativistic invariant for time-dependent and stationary fields, then we should expect the Poynting vector,

P=ExH . . . .(6)

Is also an imperial law that is relativistic invariant for time-dependent and stationary fields.

The important question here is how it represents both the stationary and time-dependent flux of electromagnetic power in amplitude and direction that is the subject of this article.

II. THEORY

In order to show that the Poynting vector P is mathematically and physically correct and represents the energy flux per unit area per unit time for electromagnetic fields in 3D configuration space, we validate it in both DC and AC circuits arbitrarily non-radiating without loss of generality.

More precisely, we consider the conservation of electromagnetic energy and momentum at the surface of a cylindrical conductor carrying current in both cases, direct and alternating.

In order to simplify the calculations and not to worry about the details of the electric circuit, we limit our analysis in the two different cases A and B to a long cylindrical wire traversed by a current.

A. Case IIA

a) DC current carrying conductor
Consider the simplest case of pure resistive DC circuits, a uniform cylindrical wire carrying DC hence the length of the wire L and the radius a [4].

Here we arbitrarily define the electro dynamic quantities in SI units as,
Voltage across wire = V volts
Current in the wire = I amps
And the electromagnetic field strengths due to the current in the wire,
$H = I / (2 \pi r)$ ampere m^-1 . . . . (A1)

For r equal to or greater than a.
π =3.1416 …… dimensionless.

And,

$E = V / L$ volt m^-1 . . . . . (A2)

For r equal to or less than a.

while E=0 outside the wire, that is to say for r greater than a.

The two vectors E and H are perpendicular to each other as shown by equation 4.

And, the Poynting vector P is the vector multiplication of two vectors, E & H,
$P= IV / (2 \pi a L)$ watts/m^2 . . . . . (A3)

At any point on the surface of the wire r = a with its direction of power flow normal to the E&H plane, i.e. parallel to the axis of the wire.

The classical Gaussian divergence theorem and Stokes' curl theorem are fundamental theorems of calculus that have an essential place in the theory of electromagnetic fields.
If we apply the first theorem keeping in mind the orthogonality property of the E&H fields, we conclude that the total electromagnetic power flux P(tot) through the entire surface of the wire is given by,

P (tot) = P x total wire passage area A
In other words,
$P (tot) = E \times H \times 2 \pi a L$ . . . . . (A4)
Obviously, the traverse area $A = 2\pi. a.L$
When we substitute the values of E&Hon the wire surface given by equations A1 and A2 in the expression A4 we get,
$P(tot) = IV / (2\pi a L) * 2$ Pi a L watts. . . . . . (A5)
In other words,
P (tot) = IV watts. . . . . . (A5)

This is exactly the total Joule heating through the wire in Joules/s i.e. (Watts).

b) Which means that the heat losses in the wire are provided by the electromagnetic field in the outer space of the wire.
Further, the E/H ratio at the wire surface r=a is given by,
$E/H = (V/I) * (2\pi a /L)$ ohms

Since,V/I is the total resistance of the wire R in ohms which varies with the physical properties of the conductor, so we have,
$E/H= R * 2\pi a /L$ . . . . (A6)

and not to be confused with the impedance to propagate electromagnetic waves which is 120 π or 376 ohms in a vacuum.

The wave impedance of an electromagnetic wave is the ratio of the transverse components of the electric and magnetic fields E/H denoted by Z0
Z0 = Sqrt $(\mu/\in)$ where $\mu = 4\pi\times 10^{-7}$ H/m (Henries per meter) is the magnetic permeability and $\in = (1/36\pi) \times 10^{-7}$ F/m is the electrical permittivity.

Z0 Approximation at 120π or 376 ohms.
Furthermore, we have shown in a previous article [2] when Maxwell's equations are combined with the relativistic mass-energy equivalence relation E=mc^2, that the total energy of the magnetic field in entire space corresponds to the kinetic energy of a single charged particle producing this field, which is also true in the case of a current-carrying conductor.

Assuming that u is the drift velocity of electrons in the wire which is directly proportional to the electric field E,
u=KE,
Where K is the material-electron mobility,
And assuming that e, Me and Ne are respectively the charge of the electron, its rest mass and the total

number of electrons in the wire, simple calculations can show that the total magnetic energy in the whole space Ut(H) is ,

Ut(H)=Ne Me u^2/2 . . . . . . A(7)

This means that the total energy density of the magnetic field Ut(H) corresponds to the drift kinetic energy of the electrons Ne.

B. Case IIB

a) AC current carrying conductor

In case IIB we consider a uniform cylindrical wire carrying an alternating current, where the length of the wire L and the radius a [4,5].

Case B is similar to case A except that the voltage V and the current I and therefore E and V vary slowly over time, which does not affect the calculations.

Consider an inductive-resistive R-L alternating current electrical circuit where the reactance X is the opposition presented to the alternating current by inductance. Here we assume that both I&V are time dependent sine waves.

Following similar procedure to that of section A we have,

Voltage across the wire V(t) = Vmax sin wt volts

Current in the wire I(t)= Imax sin (wt-Phi) amperes

Where w is the angular frequency and Phi is the offset or phase angle called the phaseshift measured in radians.

Phi=jwL / R in the complex phasor diagram.

Where the imaginary unit j, defined by its property j^2 = −1, is introduced to simplify the calculations.

Imaginary time is a mathematical representation of time that appears in many approaches such as special relativity and quantum mechanics. Mathematically, imaginary time is real time that has been rotated counter-clockwise so that its coordinates are multiplied by the imaginary unit j.

It is obvious that w=2 Pi f where f is the temporal frequency of I and v.

I(t)=V(t) /Sqrt (R^2+w^2.L^2) . . . . (B1)

Obviously,

H(t) = I(t) / (2 πa) ampere / m

And,

E(t) = V(t) / L volts / m

On the surface of the wire.

Again, the two vectors E(t) and H(t) are perpendicular to each other by Eq5.

When we apply P(t) = E(t) x H(t), (vector multiplication of two vectors, E and H, we have,

P(t) = Im .Vm / (2.πa L) sin(wt) sin (wt-Phi). . watts/m^2 . . .(B2)

At any point on the surface of the wire r=a.

Applying the Gaussian divergence theorem again with the orthogonality property of the E & H fields, we get,

P(t) = ImVm . sin(wt) sin(wt-Phi)/2 π a L. . watt/m^2 . . .(B2)

And the total Poynting vector power traversing the entire wire P(tot) is given by,

P(tot) = ImVm . sin(wt) sin(wt-Phi) watts. . .(B3)

What exactly is the instantaneous heating per joule in the wire.

In other words,

b) Heat losses in the wire are ensured instantaneously by the electromagnetic field in the outer space of the wire.

The ratio E(t)/H(t) = Sqrt (R^2+X^2)=Z ohms varies with circuit impedance Z but their pointing vector multiplication ExH remains matched to the electromagnetic power flow through the wire.

If we need to calculate the time average of the Poynting vector Pav, then,

Pav = 1/2 Vmax.Imax. cos (Phi) /2.π.a L watts/m^2. . . . (B4)

In equation B3 we have used the property of sin functions, the time average of sin wt .sin (wt-Phi) from 0 to 2 Pi/w , for the Phielement of [0, π/2] is equal to ½ cos Phi.

Similar to case IIA, the time average of the total power flow through the entire wire area P(tot)av is obtained by multiplying P(t) by the wire area, i.e. 2 π a L, hence,

P (tot)av = P x total passage area A

P (tot)av = ½ Imax Vmax watts. . . . . . (B5)

III. INTERPRETATION OF THE RESULTS

Poynting's vector theorem P = ExH is one of the universal laws of physics that applies to electromagnetic fields in AC and DC circuits. The analysis of two arbitrary cases II-A & II-B of electromagnetic fields of DC and AC circuits proves that Poynting's law P = ExH applies equally well to AC circuits as to DC circuits generating electromagnetic fields that they are time-dependent or stationary.

The assumption that a bundle of electromagnetic energy called a photon in quantum mechanics has an infinite lifespan relative to n' any reference, greater than 10^18 yearsis beyond our imagination, whereas an alternative understandable can be explained in the following[6,7],

- Although the above hypothesis or mathematical model has had reasonable success in solving modern quantum mechanical problems, it is inadequate in electromagnetic field problems.
  In electromagnetic field problems, it is preferable to apply Maxwell's equations and Poynting's universal theorem.
- The photon cannot live forever and an alternative to the assumption that the photon has an infinite lifespan is that the photon, defined as a bundle of electrical and magnetic energy in EM theory, is annihilated or created by its interaction with electromagnetic field of bound or free charges such as electrons in case IIA&IIB.

- The photon (EM field bundle)-electron interaction is possible in several ways, such as DC and AC circuits generating stationary or time-varying EM fields where the energy of the magnetic field matches the kinetic energy of the electron of mass Me.
- There is an obvious analogy between the EM energy bundle in EM wave propagation called photon and the EM energy bundle of AC&DC circuit fields. The latter has two components of two time-varying electric and magnetic fields, both of frequency f and normal to each other and having a phase difference Phi. The E/H ratio differs and is equal to the impedance Z of the electrical circuit while their product ExH still represents the EM power flow.

Photons are expected to decay or completely transform into kinetic energy of interacting electric charge like what happens in the physical phenomena of Compton scattering and the creation of electron-positron pairs. The lifetime of the photon, which seems like an eternity, is in fact limited by the chances of interaction with free or bound charges.

In the context of special relativity, taking into account the relativistic dilation of time, that is to say that the photon crosses a relativistic micro instant with respect to its own reference frame.

However, 4D EMW bundles bounded by obstacles of electrical charges in empty space can be assumed to have a mean free path or mean lifetime regardless of duration, but not infinite.

IV. CONCLUSION

A rigorous analysis of EM fields from AC and DC circuits can show that Poynting's theorem P=ExH is a universal law of physics that applies to both stationaryand time-varying electromagnetic fields.

The extensive study of EM power flow in the simple case of DC and AC circuits, a uniform cylindrical wire carrying current validates this proposition.

In addition, the present study suggests that photons from electromagnetic wave radiation and electromagnetic energy bundles cannot live forever. An alternative to the assumption that the photon has an infinite lifetime is that the photon, by analogy to a bundle of electrical and magnetic energy in EM theory, is annihilated or created by its interaction with the electromagnetic field of charges free or bound.

The EM field-electron interaction is possible in several ways, examples are DC and AC circuits generating stationary or time-varying EM fields where the energy of the magnetic field matches the kinetic energy of the free electron.

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