ISSN No:-2456-2165

Modeling the Anisotropy of the Fatigue Limits of Azobe (Lophira Alata Banks) and Bilinga (Nauclea Diderrichii Merr.) in the Orthotropic Plane (L, R)

Kalameu Nouboum Alain Brice, Djeumako Bonaventure, Samon Jean Bosco
Department of Mechanical Engineering, National School of Agro-industrial Sciences
ENSAI University of Ngaoundere
Ngaoundere, Cameroon

Kenmeugne Bienvenu
Mechanical and Material Laboratory
University of Yaounde,
Yaounde, Cameroon

Abstract:- Our study focuses on modeling the macroscopic anisotropy of fatigue limit in fully reversed loading of two tropical wood species, Azobe (Lophira alata Banks) and Bilinga (Nauclea diderrichii Merr.). The proposed formula for modeling anisotropy from the orthotropic directions of wood is a rewrite of a symmetry plane formula (R, T) to the plane of symmetry (L, R). This formula requires the knowledge of three parameters: the fatigue limit in fully reversed loading at 0°, the fatigue limit in fully reversed loading at 90° and the shear modulus in the plane (L, R). A discussion of the acuteness of the formula is made in relation to the literature through its graphical representation.

Keywords:- *Fatigue Limit. Symmetrical Alternating Traction. Anisotropy. Bois.*

I. INTRODUCTION

Anisotropy is the variation of the physical response of the material according to the orientation, with respect to the material, of the imposed stresses. The term "anisotropic material" is very ambiguous and confusing. This expression can mean both the anisotropy of the physical properties of the material (induced anisotropy), and the anisotropy of its internal structures (initial anisotropy). These two notions are intimately linked, the induced anisotropy being the result of the initial anisotropy. Several studies have focused on the study of this phenomenon (Boehler, 1984), (Guitard, 1987), (Charron et al., 2003), (Gachet and Guitard, 2006) and (Brémaud et al., 2010).

The initial anisotropy results from the processes of formation of natural materials (such as wood or rocks) or manufacturing processes of artificial materials (such as metals or composite materials) and the induced anisotropy is related to the nature and the material evolution of material damage (Rajhi, 2014).

It is also known as heterotropy, the anisotropy of the wood material and the natural variability of its mechanical properties make its characterization difficult compared to isotropic materials. It is possible to model wood by assuming the orthotropic material when it is cut in its preferred directions (Figure 1). From the knowledge of the mechanical properties along the growth axes, we can deduce the mechanical behavior of the material solicited outside its axes (Grazide, 2014).

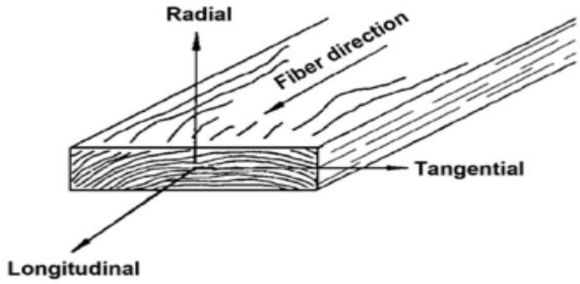

Fig. 1. The three main axes of wood in relation to grain direction and growth rings

II. SOME WORK ON WOOD ANISOTROPY

Reference (Soh Fotsing et al., 2003) propose to take into account the anisotropy of the fatigue limits by the Ankinson formula. We have in any plane (L, R) and for any angle γ contained in this plane (Figure 2) the following fatigue limit:

$$\sigma_{-1}(\gamma)=\frac{\sigma_{-1}(0)\sigma_{-1}(90)}{\sigma_{-1}(90)\cos^2(\gamma)+\sigma_{-1}(0)\sin^2(\gamma)} \quad (1)$$

$\sigma_{-1}(0)$ and $\sigma_{-1}(90)$ are respectively the fatigue limits in symmetrical alternating traction at 0° and 90°

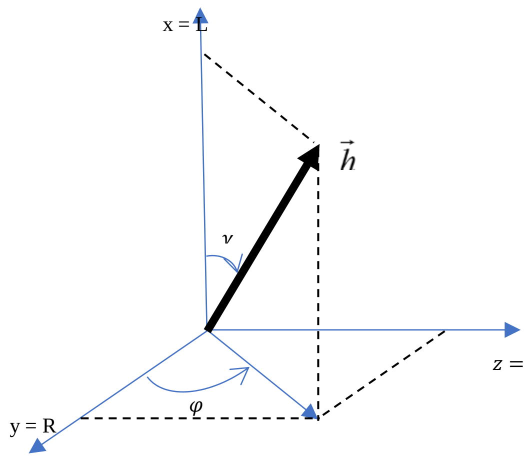

Fig. 2. Repérage d'une direction par rapport à la direction des fibres (L)

ISSN No:-2456-2165

We have already proposed (Kalameu Nouboum et al., 2017) an equation called anisotropy ellipse given in (2) to model the fatigue limit in fully reversed loading similar to (1).

$$\sigma_{-1}(\gamma)=\frac{\sigma_{-1}(0)\sigma_{-1}(90)}{\sqrt{\sigma_{-1}^2(90)\cos^2(\gamma)+\sigma_{-1}^2(0)\sin^2(\gamma)}} \qquad (2)$$

When we talk about anisotropy we refer to a different property in at least two directions as in the case of equations 1 and 2 fatigue limit in fully reversed loading. These two formulas require the knowledge of 2 parameters in the preferred directions of the material for their use; the fatigue limit in fully reversed loading at 0° (longitudinal direction) and the fatigue limit in fully reversed loading at 90° (radial direction). The addition of a third parameter that does not depend on the direction of loading would allow an adjustment of the curve representative of the anisotropy functions proposed to the experimental curve. Below, we propose a formula that allows it's adjustment.

III. PROPOSAL OF THE NEW FORMULA

It was a question of identifying those that could help us to take into account the phenomenon of anisotropy that is to say to be able to find an equation which would easily represent the evolution of the limits of fatigue.

Most of the methods mentioned above are coefficients that allow us to appreciate the degree of anisotropy. It would be expensive and tedious to determine them in all directions. The proposal of (Soh Fotsing et al., 2003) allows, for the given plan, to capture all the values. It is on a similar model that we make our proposal of anisotropy formula. This function with three parameters (a discussion on the importance of this parameter is made in the following), also allows a scan of the values contained in the plane (L, R).

A. Starting point

Simon's modeling (3) (Simon, 2009) calculates the stiffness of the wood in all the directions contained in the plane (R, T). The equation is adapted at the macroscopic level considering θ as the angle between the tangential direction T and the direction of loading R (Figure 3).

$$E(\theta)=\left(\frac{\cos^4\theta}{E_T}+\frac{\cos^2\theta\sin^2\theta}{G_{RT}}+\frac{\sin^4\theta}{E_R}\right)^{-1} \qquad (3)$$

$E(\theta)$ is the elastic modulus of the wood according to the angle of loading.

E_R, E_T and G_{RT} radial, tangential and shear modules

θ orientation of the rings relative to the loading (tangential if $\theta=0°$

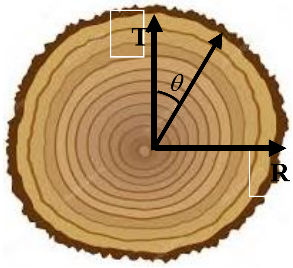

Fig. 3. Spotting angle θ in the transverse plane RT

The two wood species are considered as orthotropic materials and having a symmetry of revolution around the L axis. We will therefore get rid of the angle φ in Figure 2 in the modeling of anisotropy.

We are interested in our work with macroscopic solicitations in the direction of L. Although the deformations have a microscopic origin (Guitard, 1987), the wood is solicited at the macroscopic level. To use (3) in modeling the fatigue limits in fully reversed loading, we will make the following considerations:

- We make a rotation of 90° of the mark RT so that the direction T becomes the direction L without changing the direction R and therefore φ becomes γ;
- Young 's moduli E_T and E_R become, for fatigue limits in fully reversed loading, σ_{-1L} and σ_{-1R}. This mean that in terms of angle the fatigue limits become, respectively at 0° and at 90°, $\sigma_{-1}(0)$ and $\sigma_{-1}(90)$;
- The shear modulus G_{RT} in the RT plane becomes G_{LR} in the LR plane.

Equation (3) becomes (4) for the calculation of fatigue limits in fully reversed loading in all directions contained in the plane (L, R). The formula is rewritten in this plan and gives:

$$\sigma_{-1}(\gamma)=\left(\frac{\cos^4\gamma}{\sigma_{-1}(0)}+\frac{\cos^2\gamma\sin^2\gamma}{G_{LR}}+\frac{\sin^4\gamma}{\sigma_{-1}(90)}\right)^{-1} \qquad (4)$$

$\sigma_{-1}(\gamma)$: fatigue limit in fully reversed loading in all directions of the plane (L, R)

$\sigma_{-1}(0)$: fatigue limit in fully reversed loading in the longitudinal direction

$\sigma_{-1}(90)$: fatigue limit in fully reversed loading in the radial direction

G_{LR}: shear modulus

The main unknown in this equation is the parameter G_{LR}. The fatigue limits at 0° and 90° are determined in the literature by (Soh Fotsing et al., 2003). Indeed, it determines the fatigue limits of two tropical wood species Azobe and Bilinga in the longitudinal and radial directions.

B. Determination of the parameter G_{LR}

The dimensioning of structures made up of structural sawn timber (Sawed pieces of wood used in the construction of a structure and having as main function the resistance to the different loads that this structure must bear over time) is carried out in accordance with Eurocode 5 (EC5) and the characteristic constraints to be used are those defined by standard NF EN 338.

According to this standard, the shear modulus is 1/16 of the parallel modulus of elasticity of softwood and hardwood (Hannouz, 2014). We didn't get the standard. The Young and Coulomb modules found in the literature (Guitard, 1987) give reports contained in the interval [9; 13]. Since the Azobe shear module is already available in this literature, we will vary the Bilinga shear module in this interval. Our formula is now ready to be used to describe the anisotropy of the fatigue limits in fully reversed loading. The table below gives the average static characteristics of the species used for a moisture content of 12%.

Table 1. Static characteristics of selected species (Soh Fotsing, 2000)

Species	Azobe	Bilinga
Longitudinal Young Module EL (MPa)	19200	13400
Calculated shear modulus G (MPa)	2000	820,75

C. Representation of the formula

The representative curve of our formula and that of the experimental results is given in figure 4 for Azobe and in figure 5 for Bilinga. We note that our curves (Azobe and Bilinga) are outside the experimental curve. But from a mathematical point of view and for certain values of the shear modulus G_{LR}, determined in paragraph 4, the curve of our anisotropy function approaches the experimental curve both in the case of Azobe and in the case of Bilinga.

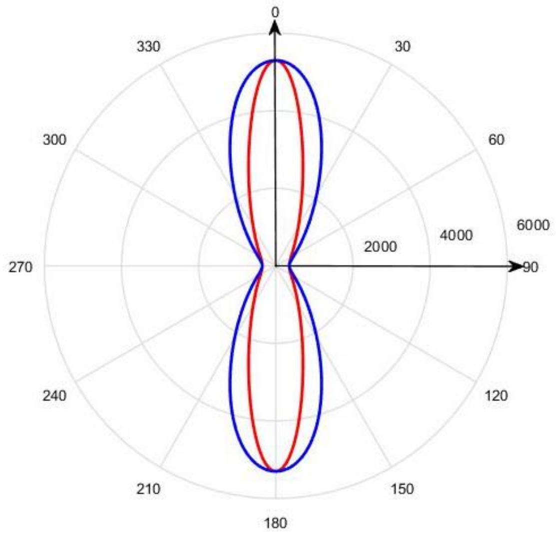

Fig. 4. Anisotropy curve (blue) and experimental curve (red) of Azobe

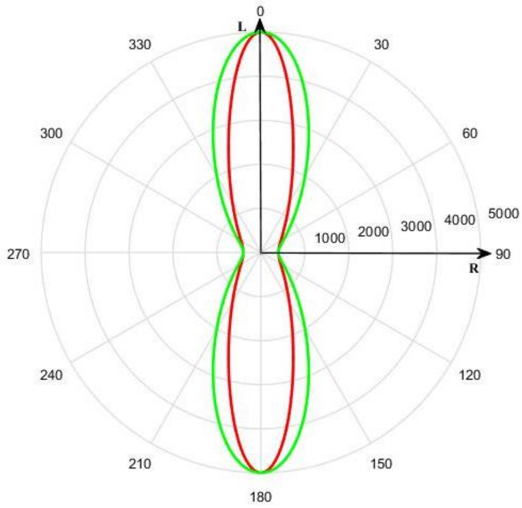

Fig. 5. Curve of anisotropy (green) and experimental curve (red) of Bilinga

IV. RESULTS AND DISCUSSION

Our formula has the same mathematical form as the equation of (Soh Fotsing et al., 2003). This equation is interesting although it requires knowledge of three initial parameters. Outside the fatigue limits in fully reversed loading at 0° and 90°, the formula requires knowledge of the shear modulus in the plane (L, R).

The curves obtained from this formula for the two species of wood are outside the curve obtained from the experimental data. They are therefore non-conservative. Shear modulus is the key to our modeling of anisotropy. This term allows an adjustment of the curve. Indeed, for a value of G_{LR} gives at (5) one can catch the curve of the experimental tests.

$$G_{LR}=\frac{\sigma_{-1}(0°)\sigma_{-1}(90°)\sin^2\gamma\cos^2\gamma}{\sqrt{\left(\left(\sigma_{-1}^2(90°)-\sigma_{-1}^2(0°)\right)\cos^2\gamma+\sigma_{-1}^2(0°)\right)-\sigma_{-1}(90°)\cos^4\gamma-\sigma_{-1}(0°)\sin^4\gamma}} \quad (5)$$

Isotropic or supposedly isotropic materials have equal fatigue limits in all directions; that would mean that $\sigma_{-1}(0)=\sigma_{-1}(90)=\sigma_{-1}$. Equations (1), (2) and (4) becomes (6):

$$\sigma_{-1}(\gamma)=\sigma_{-1} \quad (6)$$

The fatigue limit is no longer a function of γ. The set of points of the plane (L, R) satisfying this equality is the circle of center O and radius σ_{-1} shown in Figure 6 below.

We obtain a similar curve (with the same radius σ_{-1}) with the anisotropy formula of (Soh Fotsing et al., 2003).

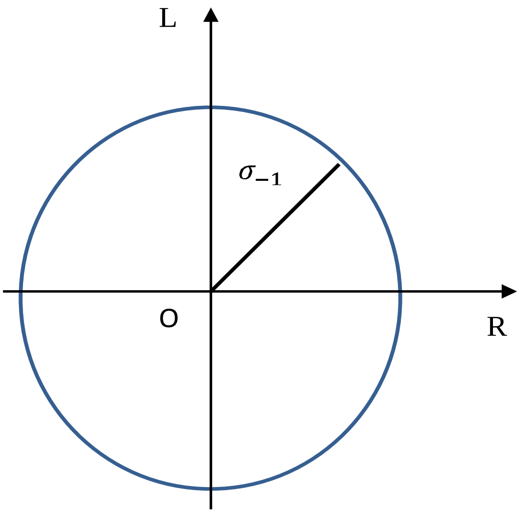

Fig. 6. Isotropy Curve

V. CONCLUSION

At the end of this investigation on the macroscopic anisotropy of two tropical wood species, we proposed a formula for the modeling of fatigue limits in fully reversed anisotropy. The formula is promising. The third parameter of this formula, which can be obtained from the Young's modulus, allows an adjustment of its curve. It could be applied to other stresses such as bending or twisting. This formula can be used to take into account the anisotropy in the fatigue criteria by replacing the fatigue limit in fully reversed that appears in certain criteria.

REFERENCES

[1]. Boehler, J.P., 1984. Anisotropie et comportement rhéologique des matériaux, in: Comptes rendus du 19ème colloque national annuel. Presented at the Rhéologie des matériaux anisotropes, Cépadues-Editions, Paris, pp. 19–50.

[2]. Brémaud, I., Cabrolier, P., Gril, J., Clair, B., Gérard, J., Minato, K., Thibaut, B., 2010. Identification of anisotropic vibrational properties of Padauk wood with interlocked grain. Wood Sci Technol 44, 355–367. https://doi.org/10.1007/s00226-010-0348-0

[3]. Charron, S., Jourez, B., Marchal, M., Hébert, J., 2003. Étude comparative des caractéristiques physiques et mécaniques du bois des mélèzes d'Europe (Larix decidua Mill.), du Japon (Larix kaempferi (Lambert) Carr.) et de leur hybride (Larix x eurolepis Henry). Biotechnol. Agron. Soc. Environ. 7, 5–16.

[4]. Gachet, C., Guitard, D., 2006. Influence relative de la morphologie cellulaire et de l'angle des microfibrilles sur l'anisotropie élastique tissulaire Longitudinale/Tangentielle du bois sans défaut des résineux. Ann. For. Sci., INRA/EDP Sciences 63, 275–283. https://doi.org/10.1051/forest:2006006

[5]. Grazide, C., 2014. Une modélisation de la résistance en flexion du pin maritime utilisé en construction. Université de Bordeaux, Bordeaux.

[6]. Guitard, D., 1987. Mécanique du matériau bois et composites. Cépadues-Editions, Toulouse.

[7]. Hannouz, S., 2014. Développement d'indicateurs pour la caractérisation mécanique et la durabilité des bois traites thermiquement. Ecole Nationale Supérieure d'Arts et Métiers, Paris.

[8]. Kalameu Nouboum, A.B., Kenmeugne, B., Djeumako, B., Soh Fotsing, B.D., Tchotang, T., Robert, J.L., 2017. Elliptic model for the consideration of the,anisotropy by the fatigue criteria: application to wood material. IJSER 8, 2484–2488.

[9]. Rajhi, W., 2014. Anisotropie induite par l'endommagement ductile : mécanismes physiques, modélisation et simulation numérique. Université de Technologie de Troyes.

[10]. Simon, P., 2009. Approche multiéchelle du comportement mécanique du bois dans le plan transverse. Institut National des Sciences Appliquées de Lyon, France.

[11]. Soh Fotsing, B.D., Anago, G., Fogué, M., Robert, J.L., 2003. Etude expérimentale de l'anisotropie des matériaux en fatigue, in: 16ème Congrès Français de Mécanique. Nice.