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Bayesian Analysis of Gross Domestic Product in Nigeria

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Abstract:- The Gross Domestic Product (GDP) measures all commodities and services generated in the country, regardless of whether they are produced by domestic or foreign firms. It examines the country's economic growth, pattern, and rate. As a result, the focus of this research is on Nigeria's GDP rate.

The data was subjected to exploratory data analysis (GDP rate).The data's minimum and maximum values were established. Easy fit was used to match the optimal distribution for the data set after the histogram and box plot were shown. In order to determine their respective posterior distributions, the Bayes theorem was also applied to both the conjugate and non-informative priors.

The two priors' posterior means and standard deviations, as well as their credible intervals, were calculated.The results showed that the mean and standard deviation for the data were 1411.6 and 928.8775, with the minimum and maximum values to be 383 and 3567 respectively. The histogram showed that the data is positively skewed to the right, the box plot indicated the lower and the upper quartiles were 620 and 2187. The application of Bayes theorem to the data set, assumed a beta distribution for conjugate prior, while a uniform distribution was assumed for the Non-informative prior. The parameter values for the conjugate prior, the likelihood and the posterior distributions were Beta(4.21,7.39), Beta(5.07,10.83) and Beta(8.28 and 17.01) respectively. The posterior Mean (Bayes estimate) and Standard deviation as well as 95% credible interval for Beta prior were 0.32373, 0.0917 and [0.31, 0.34]. Also the Posterior mean, standard deviation as well as the 95% credible interval for the uniform prior were 0.3229, 0.114543 and [0.20836, 0.4374].

The posterior mean for the uniform prior was higher than the posterior mean for the conjugate prior, the credible interval for the conjugate prior was closer than the credible interval for the uniform prior, and the posterior standard deviation for the conjugate prior was higher than the posterior standard deviation for the uniform prior.

As a result, the conjugate prior outperforms the uniform prior. Beta likelihood is recommended for fitting the rate of GDP in Nigeria.

Keywords:- *Beta distribution, Uniform Prior, Credible Interval, Bayes estimates, Posterior distribution.*

I. INTRODUCTION

The idea that demand is the motor of the economy is one of the primary theories introduced by economist **John Maynard Keynes**. We can estimate the level of production in our economy if we can assess the economy in terms of what everyone spends. GDP can be used to assess the economy.

GDP is the abbreviation for **Gross Domestic Product**. It is a measure of an economy's overall output of goods and services. The following is the definition of GDP: It is the total market value of all final goods and services produced within a country's internal borders over a certain time period. The term 'domestic' ('in Gross domestic product') signifies that we're only counting goods made within our domestic borders, whether they're made by Americans or foreigners; nothing made beyond our domestic borders is counted in the GDP..

Changes in a country's Gross Domestic Product have been the most widely acknowledged metric of economic success for more than half a century (GDP). GDP is a measure of market throughput that adds up the value of all final goods and services produced and sold for money in a specific time period. It is usually calculated by adding a country's personal consumption expenditures (payments by households for goods and services), net exports (the value of a country's exports minus the value of imports), and net capital formation (the increase in the value of a country's total stock of monetized capital goods) together.

GDP is a measure of economic activity, not economic well-being, according to economists who are familiar with the GDP and SNA methodologies. The principal inventor of the United States national accounting system, Simon Kuznets, warned against associating GDP growth with economic or social well-being in 1934. GDP is used to answer questions like "how fast is the economy growing," "what is the pattern of spending on goods and services," "what percent of the increase in production is due to inflation," and "how much of the income produced is used for consumption as opposed to investment or savings," according to the US Bureau of Economic Analysis (McCulla and Smith 2007).

Gross Domestic Product (GDP) = Consumption + Investment + Government Spending + (Exports - Imports). Or GDP = C + I + G + (X-M).

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II. REVIEW OF LITERATURE

The Bretton Woods Conference further enhanced the use of GDP as a global indicator of economic growth. Economic instability in a number of countries, induced by unstable currency exchange rates and discriminatory trade regulations that impeded international trade, was a major factor in the start of WWII. To avert a repeat of such volatility, leaders of the 44 allied nations met in Bretton Woods, New Hampshire, in 1944 to establish a method for international commerce and currency exchange coordination.

The meeting's goal was to "accelerate economic progress worldwide, facilitate political stability, and promote peace" (IBIBLIO/AIS 1946). Jobs would be created in all countries as a result of international commerce. Those employment would provide enough revenue for people all around the world to be able to afford appropriate food, housing, medical care, and other necessities. As a result, improving economic well-being was critical to achieving long-term global peace. The way to economic prosperity was considered as growing the economy.

As a result, the IMF and the World Bank adopted GDP as the major indicator of economic success during the next 60 years. The US has had a less prominent position within the World Bank and the IMF since their restructuring in the 1970s; yet, GDP remains the most generally cited metric of economic success. Since its inception, economists have warned that GDP is a specialized instrument, and that using it as a gauge of general well-being is both erroneous and harmful.

Changes in GDP are frequently used by governments as a measure of the success of their economic and budgetary policies. GDP is one of the most comprehensive and closely watched economic statistics in the United States, as it is used by the White House and Congress to prepare the Federal budget, the Federal Reserve to formulate monetary policy, Wall Street as an indicator of economic activity, and the business community to prepare economic forecasts that serve as the foundation for production, investment, and employment planning (McCulla & Smith, 2007). Both the IMF and the World Bank utilize changes in a country's GDP to steer policy and determine how and which projects are supported around the world.

Today, major economists, politicians, top-level decision-makers, and the media frequently refer to GDP and economic growth in general as though they indicate overall progress. Indeed, according to a recent World Bank assessment, nothing other than long-term high rates of GDP growth (particularly, a doubling of GDP every decade) can tackle the world's poverty problem (Commission on Growth and Development 2008).The association between six economic variables, including debt service ratio, imports/reserves, amortization, income per capital inflows, and country growth, was evaluated using the logit model, and the results suggest that these variables have a substantial relationship (Feder & Just, 1997).

(Adesola, 2009) looked into the impact of Nigeria's external debt service payments on the country's economic growth, using ordinary least square multiple regression.

It has been discovered that debt service payments have a negative impact on GDP and Gross Fixed Capital Formation (GFCF) (GFCF). The dynamic influence of GDP, debt service, capital stock, and labor force on Pakistan economic growth was investigated, and the results suggest that debt service has a negative effect on labor and capital productivity, and so has a negative impact on economic growth (Abid, Hammad & Ali, 2003).

A vector error correction model was developed using multivariate analysis to explore the long term effects of external debt service on Turkey's GNP level, and it was discovered that the two variables have a unidirectional negative relationship (Erdal Karagol, 2003).

Debt service load has been found to stifle growth through crowding out investment, reducing imports, and eventually slowing growth (John & Sanny, 2003). Current macroeconomic models properly forecast real GDP and employment rate decreases, as seen in the current crisis, but they are unable to demonstrate the failure of economies to recover once the financial crisis has passed (Hall, 2010).

Nigeria's economy output is being harmed by rising debt and debt service payments, and the country requires proper debt management to grow (James Akperan, 2005). Most time-series studies included several factors, but the conclusion was that foreign debt servicing had an impact on growth in a variety of ways, including per-capital income, GDP growth rate, trade openness, currency reserves, and capital inflows (E. Kohlscheen, 2004).

By using unrelated regression, the effects of external debt servicing constraints and public expenditure composition in Sub-Saharan Africa were investigated, and it was discovered that debt servicing constraints shift spending away from the social sector, having negative effects on health and education (Augustin Kwasin, 2009).

The causal relationship between Pakistan's exports, economic growth, and debt servicing was investigated using the Toda- Yamamoto Granger Causality Test, which revealed that there is tri-variant causality between these variables, leading to the conclusion that debt service payments cause Pakistan's GDP to rise (M.Afzal, Hafeez, Jamshed, 2008).Furthermore, using the Eagle and Granger method, it was discovered that debt service payments have no detrimental influence in the long or short run in the instance of Sri Lanka (Albert et al, 2008). Because of the easy conditions of borrowing and quick discussions, the conclusion was reached that GDP growth does not appear to have a substantial relationship with Pakistan's debt-service ratios.

Reductions in foreign debt service payments might enhance economic development indirectly by increasing governmental investment (Benedict et al, 2003). Further research reveals that deferring payment of external debt service to external creditors does not help a debtor country

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escape sluggish growth, but rather adds to its external debt load (Micheal and Lars, 1994).

Finance does not have a causal relationship with growth; rather, it is the other way around, with financial development following economic expansion (Robinson, 1952). The impact of the stock market on economic growth has been studied by researchers (Atje & Jovanovic, 1993). They believe active stock market expansion crucial, and conclusions concerning stock market activity were later provided by (Greenwood & Jovanovic, 1990), based on the theoretical premises set by (Greenwood & Jovanovic, 1990). (Levine & Zervos , 1998).

To decide which regressors' growth should be included in linear cross-country growth regressions, Bayesian Averaging of Classical Estimates (BACE) was utilized (Raftery, 1995; Sala-i- Martin et al, 2004). The completely Bayesian Model Averaging (BMA) technique, which was based on cross-sectional data, has the same goal (Fernandez et al; 2001). The list of growth drivers derived using BMA techniques was found to be vulnerable to arguably little variations in the international GDP data utilized in the computations (Ciccone & Jarocinski, 2009).

- **Objectives**

  The goal of this study is to select the prior distribution to be used for the study and to estimate the posterior mean and posterior standard deviation of economic growth in Nigeria utilizing both informative and non-informative priors.

III. METHODOLOGY

A. Bayesian Inference

The unknown parameters of a model are explicitly treated as random variables in Bayesian inference. The prior distribution ('before' to seeing data) can be used to describe the current level of knowledge about the parameters. The model's data is designed to provide information about the parameters. The Bayes theorem is used to blend prior information and data-based information to produce a posterior distribution.

Bayesian inference is a statistical inference method in which the Bayes theorem is used to update a hypothesis' probability when more data or information becomes available. In statistics, and particularly in mathematical statistics, Bayesian inference is a crucial technique. Bayesian inference is used in a variety of fields, including science, engineering, philosophy, medicine, sports, and law. Subjective probability, often known as "Bayesian Probability," is closely related to Bayesian inference.

B. Basic Steps in Bayesian Analysis

The following step explained the essential methods of Bayesian analysis

a) Construction of Likelihood

This is a function of the statistical model's parameters given data. In statistical inference, particularly methods of estimating a parameter from a set of statistics, likelihood functions play an important role. "Likelihood" is frequently used as a synonym for "probability" in informal circumstances. In statistics, a differentiation is formed based on the roles that the results play in relation to the parameters.

After data are available, likelihood is used to characterize the function of a parameter (or parameter vector) for a given result. The accessible information provided by the sample is contained in the likelihood, likelihood function, or$P(yi|\theta)$. The likelihood is the data's joint probability function seen as a function of the parameters, with the observed data treated as constants. The likelihood function is given by assuming that the data values, $y = (y_1, \ldots, y_n)$ are obtained independently, the likelihood function is given by

$$L(\theta|y = P(y_1, \ldots, y_n) = \prod_{i=1}^{n} P(yi|\theta) \qquad (1)$$

The likelihood in the Bayesian model contains all of the information about that comes directly from the data. The parameters whose values correlate to the biggest values of the likelihood are the ones that the data most strongly supports. The probability function is defined as follows:

$$P(\theta|\alpha\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1} \qquad (2)$$

Where $\alpha > 0$ and $\beta > 0$ and they are the parameter. The mean of a Beta distribution of first kind is $\frac{\alpha}{\alpha+\beta}$ and the variance is given$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$ the value $\alpha + \beta$has an interpretation as the amount of information about θviewed as a sample size.

b) Choosing a Prior Distribution

A prior probability distribution of an unknown quantity, also known as the prior in Bayesian statistical inference, is the probability distribution that would describe one's assumptions about the quantity before any evidence is taken into account. In the same way, the prior probability of a random event or an ambiguous proposition allocated before any relevant data is considered.

$P(\theta)$ often known as the prior distribution or simply the prior, expresses the probability distribution forθ. Prior to acquiring any information from the current data set y, the researcher's belief about the parameter of interest is described by the prior distribution. **Informative and non-informativeprior** distribution approaches are the two primary categories of prior distribution selection methods.

c) Informative Prior

The first strategy entails selecting an informative prior distribution. With this method, the statistician constructs a prior distribution that accurately reflects his opinions about unknown parameters based on his knowledge of the substantive problem, sometimes based on other data, and obtained expert opinion if possible.

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d) Non-informative Prior
The second most common method for selecting a prior distribution is to create a non-informative prior distribution that mimics model parameter ignorance. This type of distribution is also referred to as objective, imprecise, and diffuse, as well as a reference prior distribution. By acting as though no prior information of the parameters exists before observing the data, a non-informative prior distribution is chosen in an attempt to achieve impartiality. This is accomplished by assigning the same probability to all of the parameter's values.

e) Posterior Distribution
To obtain the posterior distribution, $P(\theta|y)$, the probability distribution of the parameters once the data have been observed, we apply Bayes theorem:

$$P(\theta|y_i) = \frac{P(y_i|\theta) \times P(\theta)}{P(y_i)} \qquad (3)$$

$P(y_i)$ is the normalizing constant of the posterior distribution. It is also the marginal distribution of y, and it is sometimes called the marginal distribution of the data.
For discrete function:

$$P(y_i) = \sum_{i=1}^{n} P(y_i|\theta) \times P(\theta) \qquad (4)$$

For Continuous Function:

$$P(y_i) = \int P(y_i|\theta) \times P(\theta) d(\theta) \qquad (5)$$

By replacing $p(yi)$ with c, which is short for a 'constant of proportionality', the model-based formulation of Bayes' theorem becomes:

$$P(\theta|y_i) = \frac{P(y_i|\theta) \times P(\theta)}{(c)} \qquad (6)$$

By removing c from the equation, the relationship changes from 'equals' (=) to 'proportional to'($\propto$)

$$P(\theta|yi) \propto P(yi|\theta) \times P(\theta)$$

Posterior $\propto$*likelihood* × *prior*

C. Posterior Summary
Inferential deductions can be summarized with an appropriate analysis once the posterior distribution has been found. The mean or mode of the posterior distribution are widely used to derive point estimates of parameters. It is also possible to compute interval estimates.

a) Posterior Distribution for Beta Prior
Combining beta likelihood and Beta prior

The distribution of the parameter $\theta$is given as:

$P(\theta|\alpha\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1}$ Where α

and β are the likelihood parameter (7)

The prior distribution $P(\theta)$ is

$P(\theta) = \frac{\Gamma(\alpha_0+\beta_0)}{\Gamma(\alpha_0)\Gamma(\beta_0)}\theta^{\alpha_0-1}(1-\theta)^{\beta_0-1}$ Where α_0 and β_0 are the prior parameter (8)

The posterior distribution is derived as

$$P(\theta|y_i) = \frac{P(y_i|\theta)\times P(\theta)}{\int_0^1 P(y_i|\theta)\times P(\theta)d(\theta)}$$

(9)

Therefore:

$$= \frac{\theta^{\alpha-1}(1-\theta)^{\beta-1}/B(\alpha\beta) \times \theta^{\alpha_0-1}(1-\theta)^{\beta_0-1}/B(\alpha_0\beta_0)}{\int_0^1 \theta^{\alpha-1}(1-\theta)^{\beta-1}/B(\alpha\beta) \times \theta^{\alpha_0-1}(1-\theta)^{\beta_0-1}/B(\alpha_0\beta_0)d\theta}$$

$$= \frac{\theta^{\alpha-1}(1-\theta)^{\beta-1} \times \theta^{\alpha_0-1}(1-\theta)^{\beta_0-1}}{\int_0^1 \theta^{\alpha-1}(1-\theta)^{\beta-1} \times \theta^{\alpha_0-1}(1-\theta)^{\beta_0-1}d\theta}$$

$$= \frac{\theta^{\alpha-1+\alpha_0-1}(1-\theta)^{\beta-1-\beta_0-1}}{\int_0^1 \theta^{\alpha-1+\alpha_0-1}(1-\theta)^{\beta-1+\beta_0+1}d(\theta)}$$

$$= \frac{\theta^{\alpha+\alpha_0-2}(1-\theta)^{\beta+\beta_0-2}}{\int_0^1 \theta^{\alpha+\alpha_0-2}(1-\theta)^{\beta+\beta_0-2}d(\theta)}$$

Recall from Beta function that

$$\int_0^1 \theta^{\alpha-1}(1-\theta)^{\beta-1}\, d(\theta) \quad = B(\alpha\beta)$$

$$\frac{\theta^{\alpha+\alpha_0-2}(1-\theta)^{\beta+\beta_0-2}}{B(\alpha+\alpha_0-1,\ \beta+\beta_0-1)} \qquad (10)$$

b) Posterior Mean For Beta Prior
Using the Mean Square Error as risk, the Bayes estimate of the unknown parameter is simply the mean of the Posterior distribution,

$$E(\theta|y_i) = \int_o^1 \theta \times P(\theta|y_i)d(\theta) \qquad (11)$$

$$\int_0^1 \frac{\theta \times \theta^{\alpha+\alpha_0-2}(1-\theta)^{\beta+\beta_0-2}}{B(\alpha+\alpha_0-1,\ \beta+\beta_0-1)}d(\theta)$$

$$\frac{1}{B(\alpha+\alpha_0-1,\ \beta+\beta_0-1)}\int_0^1 \theta^{\alpha+\alpha-2+1}(1-\theta)^{\beta+\beta_0-2}\ d(\theta)$$

$$\frac{1}{B(\alpha+\alpha_0-1,\ \beta+\beta_0-1)}\int_0^1 \theta^{\alpha+\alpha-1}(1-\theta)^{\beta+\beta_0-2}\ d(\theta)$$

$$\frac{1}{B(\alpha+\alpha_0-1,\ \beta+\beta_0-1)}B(\alpha+\alpha_{0,}\beta+\beta_0-1$$

$$\frac{B(\alpha+\alpha_{0,}\beta+\beta_0-1)}{B(\alpha+\alpha_0-1,\ \beta+\beta_0-1)}$$

Recall that $B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$

Also, Recall that: $\Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1)$

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$$E(\theta|y_i) = \frac{\Gamma(\alpha+\alpha_0)\times\Gamma(\beta+\beta_0-1)}{\Gamma(\alpha+\alpha_0+\beta+\beta_0-1)} \div \frac{\Gamma(\alpha+\alpha_0-1)\times\Gamma(\beta+\beta_0-1)}{\Gamma(\alpha+\alpha_0-1+\beta+\beta_0-1)}$$

$$E(\theta|y_i) = \frac{\Gamma(\alpha+\alpha_0)\times\Gamma(\beta+\beta_0-1)}{\Gamma(\alpha+\alpha_0+\beta+\beta_0-1)} \times \frac{\Gamma(\alpha+\alpha_0-1+\beta+\beta_0-1)}{\Gamma(\alpha+\alpha_0-1)\times\Gamma(\beta+\beta_0-1)}$$

$$E(\theta|y_i) = \frac{\Gamma(\alpha+\alpha_0)}{\Gamma(\alpha+\alpha_0+\beta+\beta_0-1)} \times \frac{\Gamma(\alpha+\alpha_0-1+\beta+\beta_0-1)}{\Gamma(\alpha+\alpha_0-1)}$$

$$E(\theta|y_i) = \frac{\Gamma(\alpha+\alpha_0)}{\Gamma(\alpha+\alpha_0+\beta+\beta_0-1)} \times \frac{\Gamma(\alpha+\alpha_0+\beta+\beta_0-2)}{\Gamma(\alpha+\alpha_0-1)}$$

$$\mathrm{E}\ (\theta|y_i) = \frac{\alpha+\alpha_0-1\Gamma(\alpha+\alpha_0-1)}{\alpha+\alpha_0+\beta+\beta_0-1-1\Gamma(\alpha+\alpha_0+\beta+\beta_0-1-1)} \times \frac{\Gamma(\alpha+\alpha_0+\beta+\beta_0-2)}{\Gamma(\alpha+\alpha_0-1)}$$

$$\mathrm{E}\ (\theta|y_i) = \frac{\alpha+\alpha_0-1\Gamma(\alpha+\alpha_0-1)}{\alpha+\alpha_0+\beta+\beta_0-2\Gamma(\alpha+\alpha_0+\beta+\beta_0-2)} \times \frac{\Gamma(\alpha+\alpha_0+\beta+\beta_0-2)}{\Gamma(\alpha+\alpha_0-1)}$$

$$\mathrm{E}\ (\theta|y_i) = \frac{\alpha+\alpha_0-1}{\alpha+\alpha_0+\beta+\beta_0-2} \qquad (12)$$

c) Posterior Mean For Uniform Prior

The uniform prior is used with Beta likelihood. The uniform prior (non-informative) is implemented by giving all parameter values the same probability. That is $P(\theta) = 1$ The Posterior distribution is derived as:

$$P(\theta|y_i) = \frac{P(y_i|\theta)\times P(\theta)}{\int_0^1 P(y_i|\theta)\times P(\theta)d(\theta)} \qquad (13)$$

$$P(\theta|y_i) = \frac{1\times\theta^{\alpha-1}(1-\theta)^{\beta-1}/B(\alpha,\beta)}{\int_0^1 1\times\theta^{\alpha-1}(1-\theta)^{\beta-1}/B(\alpha,\beta)d(\theta)} \qquad (14)$$

Therefore:

$$P(\theta|y_i) = \frac{\theta^{\alpha-1}(1-\theta)^{\beta-1}}{\int_0^1 \theta^{\alpha-1}(1-\theta)^{\beta-1}d(\theta)}$$

Recall from Beta function that $B(\alpha\beta) = \int_0^1 \theta^{\alpha-1}(1-\theta)^{\beta-1}d(\theta)$

Therefore:

$$P(\theta|y_i) = \frac{\theta^{\alpha-1}(1-\theta)^{\beta-1}}{B(\alpha\beta)} \qquad (15)$$

d) Posterior Mean For Uniform Prior

Using the Mean Square Error as risk, the Bayes estimate of the unknown parameter is simply the mean of the Posterior distribution,

$$E(\theta|y_i) = \int_0^1 \theta\times P(\theta|y_i)d(\theta) \qquad (16)$$

$$= \int_0^1 \frac{\theta\times\theta^{\alpha-1}(1-\theta)^{\beta-1}}{B(\alpha,\beta)}$$

$$= \frac{1}{B(\alpha,\beta)}\int_0^1 \theta^{\alpha-1+1}(1-\theta)^{\beta-1}\,d(\theta)$$

$$= \frac{1}{B(\alpha,\beta)}\int_0^1 \theta^{\alpha}(1-\theta)^{\beta-1}\,d(\theta) \qquad (17)$$

Recall from Beta function that $B(\alpha,\beta) = \int_0^1 \theta^{\alpha-1}(1-\theta)^{\beta-1}d(\theta)$

So, $E(\theta|y_i) = \frac{1}{B(\alpha,\beta)}B(\alpha+1,\beta)$

$= \frac{B(\alpha+1,\beta)}{B(\alpha,\beta)}$

Recall that $B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$

Therefore $\frac{B(\alpha+1,\beta)}{B(\alpha,\beta)} = \frac{\Gamma(\alpha+1)\times\Gamma(\beta)}{\Gamma(\alpha+1+\beta)} \div \frac{\Gamma(\alpha)\times\Gamma(\beta)}{\Gamma(\alpha+\beta)}$

$$= \frac{\Gamma(\alpha+1)\times\Gamma(\beta)}{\Gamma(\alpha+1+\beta)} \times \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\times\Gamma(\beta)}$$

$= \frac{\Gamma(\alpha+1)}{\Gamma(\alpha+1+\beta)} \times \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)}$

Also recall that $\Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1)$

$$\frac{B(\alpha+1,\beta)}{B(\alpha,\beta)} = \frac{(\alpha+1-1)\times\Gamma(\alpha+1-1)}{(\alpha+1+\beta-1)\Gamma(\alpha+\beta-1)} \times \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)}$$

$$= \frac{\alpha\Gamma(\alpha)}{(\alpha+\beta)\Gamma(\alpha+\beta)} \times \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)}$$

$$E(\theta|y_i) = \frac{\alpha}{\alpha+\beta} \qquad (18)$$

e) Posterior Variance For Uniform Prior

The posterior variance for uniform prior is given as:

$$Var(\theta|y_i) = E(\theta^2|y_i) - \left(E(\theta|y_i)\right)^2$$

$$E(\theta^2|y_i) = \int_0^1 \theta^2 P(\theta|y_i)d(\theta) \qquad (19)$$

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$$= \int_0^1 \theta^2 \times \frac{\theta^{\alpha-1}(1-\theta)^{\beta-1}}{B(\alpha,\beta)} d(\theta)$$

$$= \frac{1}{B(\alpha,\beta)} \int_0^1 \theta^2 \theta^{\alpha-1}(1-\theta)^{\beta-1} \, d(\theta)$$

$$= \frac{1}{B(\alpha,\beta)} \int_0^1 \theta^{\alpha-1+2} (1-\theta)^{\beta-1} d(\theta)$$

$$\frac{1}{B(\alpha,\beta)} \int_0^1 \theta^{\alpha+1} (1-\theta)^{\beta-1} d(\theta)$$

Recall from Beta function that $B(\alpha,\beta) = \int_0^1 \theta^{\alpha-1}(1-\theta)^{\beta-1} d(\theta)$

$$= \frac{1}{B(\alpha,\beta)} B(\alpha+2,\beta)$$

Recall that $B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$

$$= \frac{1}{B(\alpha,\beta)} B(\alpha+2,\beta) = \frac{\Gamma(\alpha+2)\Gamma(\beta)}{\Gamma(\alpha+\beta+2)} \div \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

$$= \frac{\Gamma(\alpha+2)\Gamma(\beta)}{\Gamma(\alpha+\beta+2)} \times \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$$

$$\frac{\Gamma(\alpha+2)}{\Gamma(\alpha+\beta+2)} \times \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)}$$

Also recall that $\Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1)$

$$= \frac{(\alpha+2-1)\Gamma(\alpha+2-1)}{(\alpha+\beta+2-1)\Gamma((\alpha+\beta+2-1)} \times \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)}$$

$$= \frac{(\alpha+1)\Gamma(\alpha+1)}{(\alpha+\beta+1)\Gamma(\alpha+\beta+1)} \times \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)}$$

$$= \frac{(\alpha+1)(\alpha+1-1)\Gamma(\alpha+1-1)}{(\alpha+\beta+1)(\alpha+\beta+1-1)\Gamma(\alpha+\beta+1-1)} \times \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)}$$

$$= \frac{(\alpha)(\alpha+1)\Gamma(\alpha)}{(\alpha+\beta+1)(\alpha+\beta)\Gamma(\alpha+\beta)} \times \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)}$$

$$E(\theta^2|y_i) = \frac{(\alpha)(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)} \qquad (20)$$

Therefore

$$Var(\theta|y_i) = E(\theta^2|y_i) - \left(E(\theta|y_i)\right)^2$$

$$= \frac{(\alpha)(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)} - \left[\frac{\alpha}{\alpha+\beta}\right]^2$$

$$= \frac{(\alpha)(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)} - \frac{\alpha^2}{(\alpha+\beta)^2}$$

$$\frac{(\alpha+\beta)(\alpha^2+\alpha) - \alpha^2(\alpha+\beta+1)}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

$$= \frac{\alpha^3+\alpha^2+\alpha^2\beta+\alpha\beta-\alpha^3-\alpha^2\beta-\alpha^2}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

$$Var(\theta|y_i) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \qquad (21)$$

D. *Credible Interval*

A credible interval is an interval in the domain of a posterior probability in Bayesian statistics. The frequentist confidence intervals do not need (and in fact, do not require) knowledge of the situation particular prior distribution, but Bayesian credible intervals do

$$C.I = E(\theta|y_i) \pm Z_{\frac{\alpha}{2}} S.D(\theta|y) \qquad (22)$$

IV. RESULTS AND DISCUSSION

A. *Descriptive statistics*

	N	Range	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic
GDP US$	45	3184	383	3567	1411.6	928.8775	0.668412	-0.82965

Table 1: Summary of the data

Interpretation: The statistics overview of the data utilized in this study is shown in the table above. It can be seen that the mean is 1411.6 and the standard deviation is 928.8775, indicating that the data is spread out from the mean. The data appear to be positively skewed to the right, i.e. moderately skewed, and the kurtosis is Platykurtic because it is negative, i.e. the tails are shorter and thinner, and the central peak is often lower and broader.

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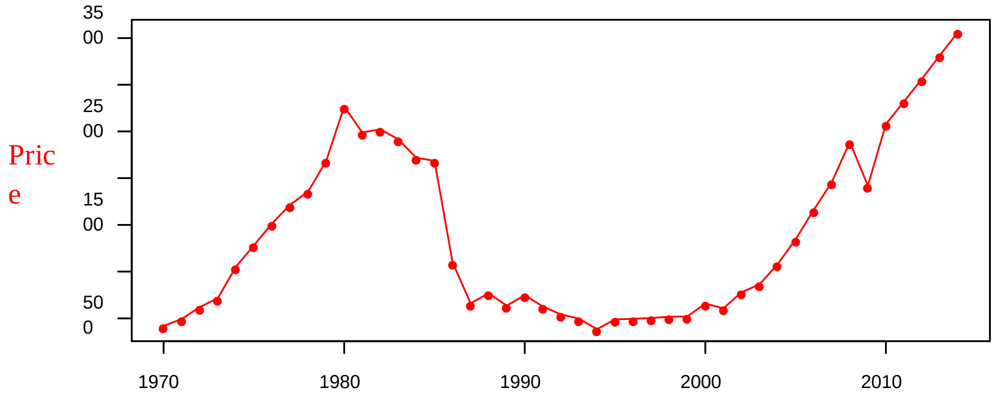
GDP(1970−2014)



Year

Fig. 1: GDP Plot (1970-2014)

GDP(1970−2014)

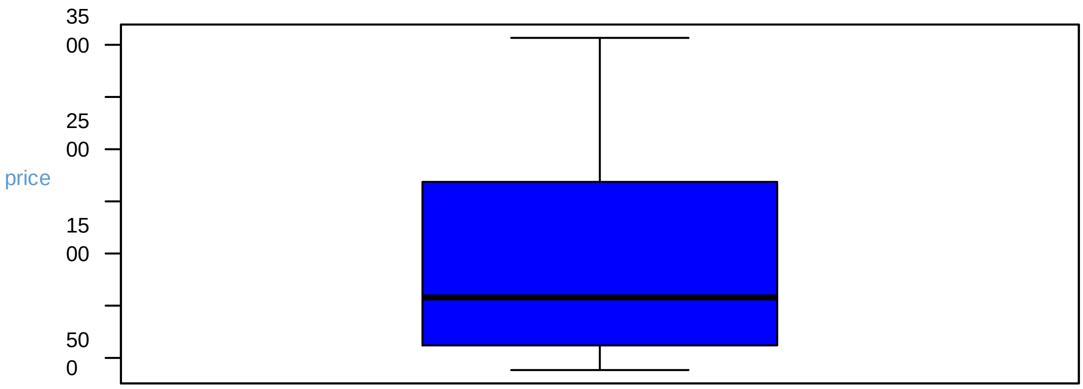


Fig. 2: Box plot for the data

The box plots above illustrate the data's minimum and maximum values, with the minimum being 383 (US$) and the maximum being 3567 (US$). It also displays the upper quartile's value of 2187 dollars and the bottom quartile's value of 620 dollars.

GDP(1970−2014)

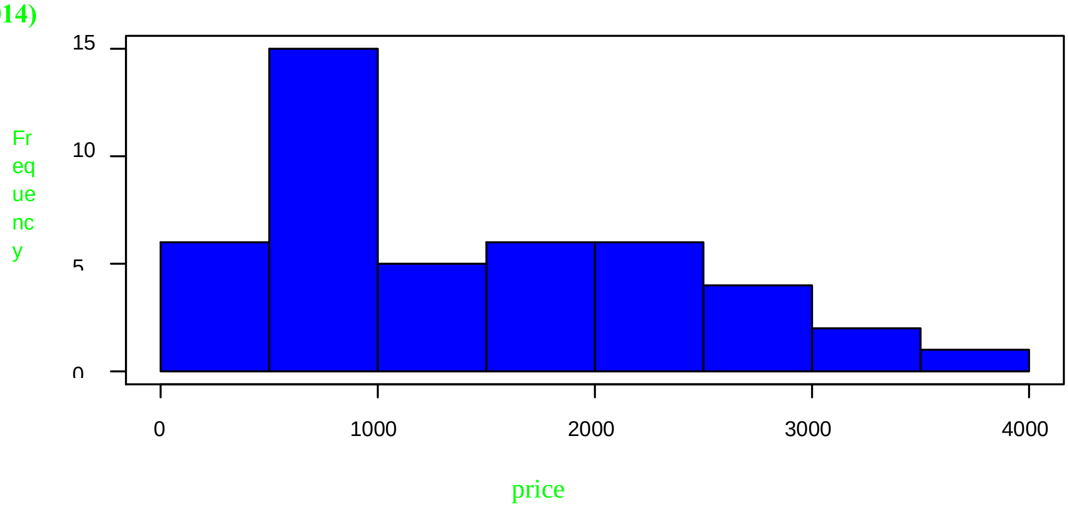


Fig. 3: Histogram for the data

The figure 3shows that the data is positively skewed to the right.

Density plot for GDP

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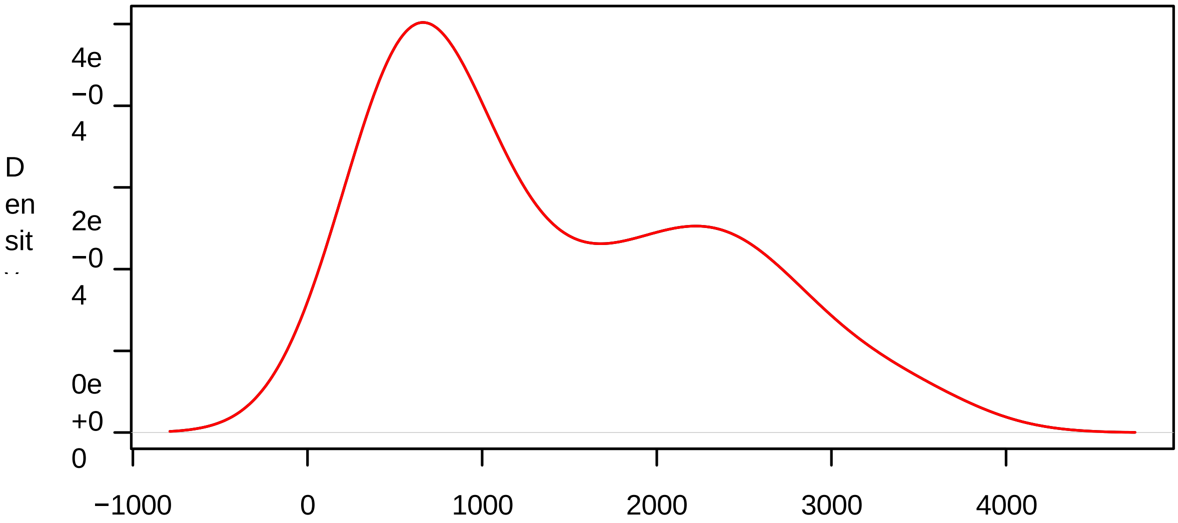

Fig. 4: Density for the GDP

B. *Bayesian Analysis*

a) Beta Prior

Choosing an information prior distribution for θ that reflect the information contained in the data. A flexible choice of prior Distribution for a beta likelihood is beta prior. The prior distribution for the beta distribution under this data set follow a beta distribution with parameter $\alpha 0 \ = \ 4.21 \ and \ \beta 0 = 7.39$.That is $B(\alpha 0, \beta 0 = 7.39) = B(4.21, 7.39)$.

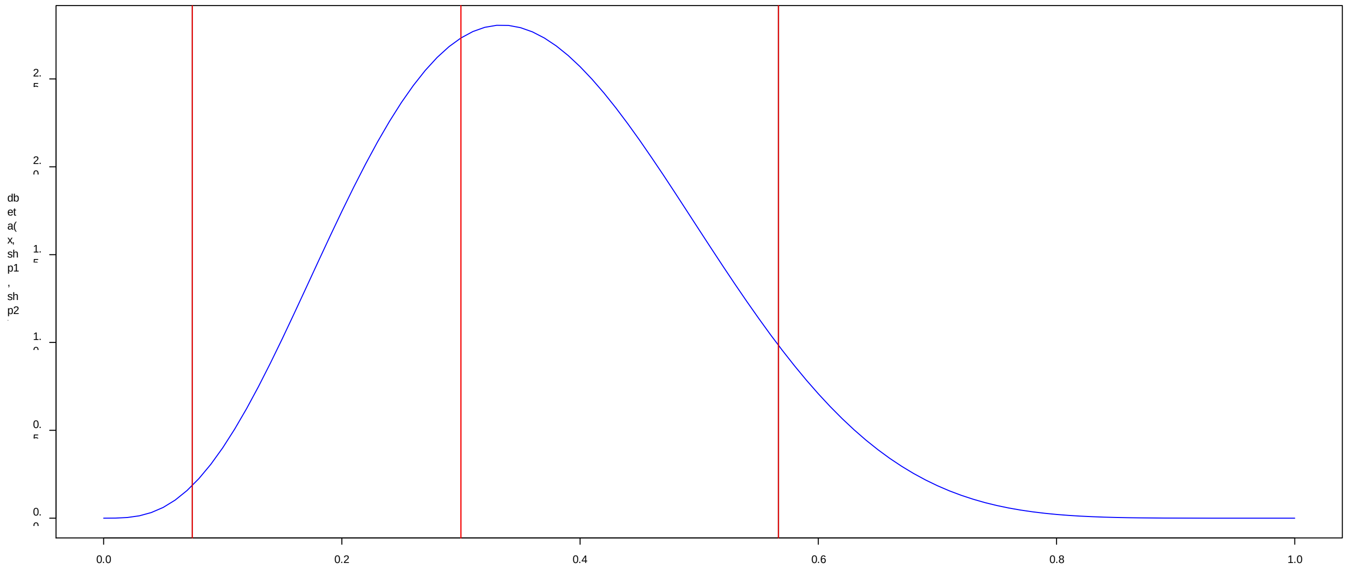

Fig. 5: Plotting of prior distribution

The plot of the prior distribution for this study with parameters α_0=4.21 and β_0 = 7.39, which indicate our prior views about the study variable (GDP), is shown in the above figure, where the peak of the distribution is about 0.35 and the density is between 0.1 and 0.57.

b) Construction of Likelihood

Since there are 45 observations and each of this are independently and identically distributed, the joint density function is the product of the individual Probability Density Functions. Therefore, the likelihood follows a beta distribution Let y_i i= 1, 2,...,45 be the number of observation. The likelihood of the distribution is given as follows;

$$P(y_1, y_2, \ldots, y_{45}|\theta) = \prod_{i=1}^{45} P(y_i|\theta) \propto \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha-1}(1-\theta)^{\beta-1} \quad (23)$$

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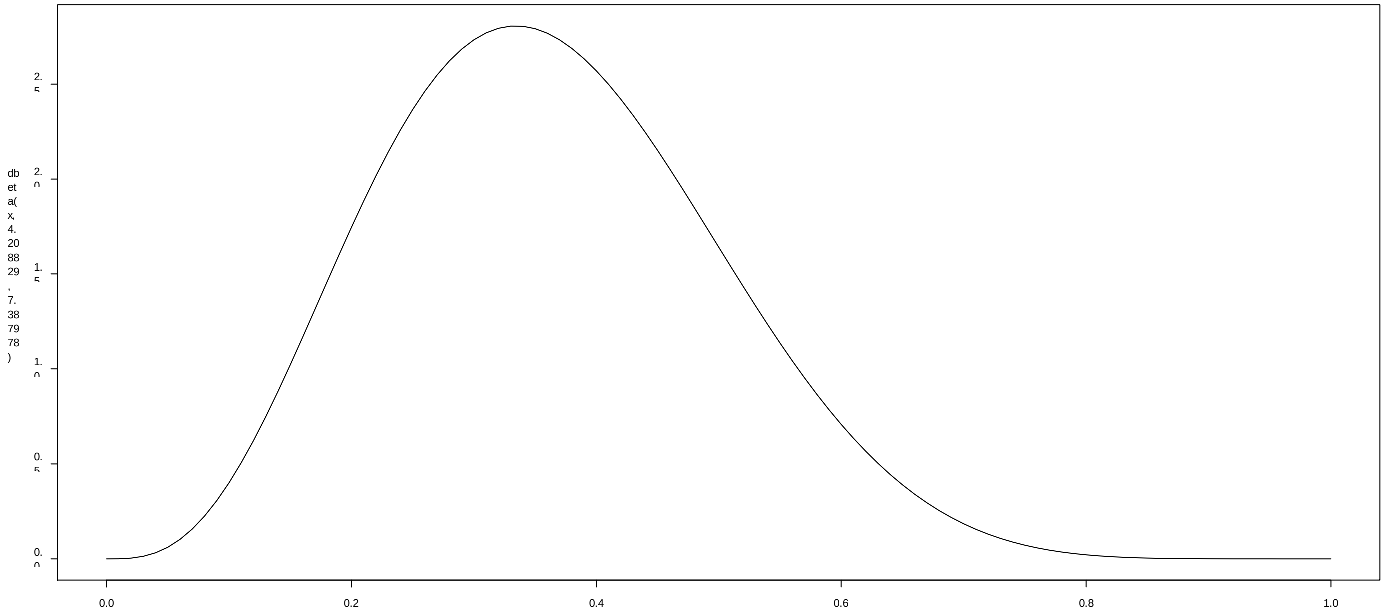

Fig. 6: Plotting of beta likelihood

It can be observed from the plot that the likelihood distribution is at 0.32, the most likely value of the prior given the observed data, is 0.32.

c) Posterior Distribution

After the likelihood function had been determined, then, the posterior can now be estimated as shown below;

$$P(\theta|y_i) \propto P(y_i|\theta) \times P(\theta)$$

$$P(\theta|y_i) = \frac{P(y_i|\theta) \times P(\theta)}{\int_0^1 P(y_i|\theta) \times P(\theta) d(\theta)}$$

The posterior obtained are given below;

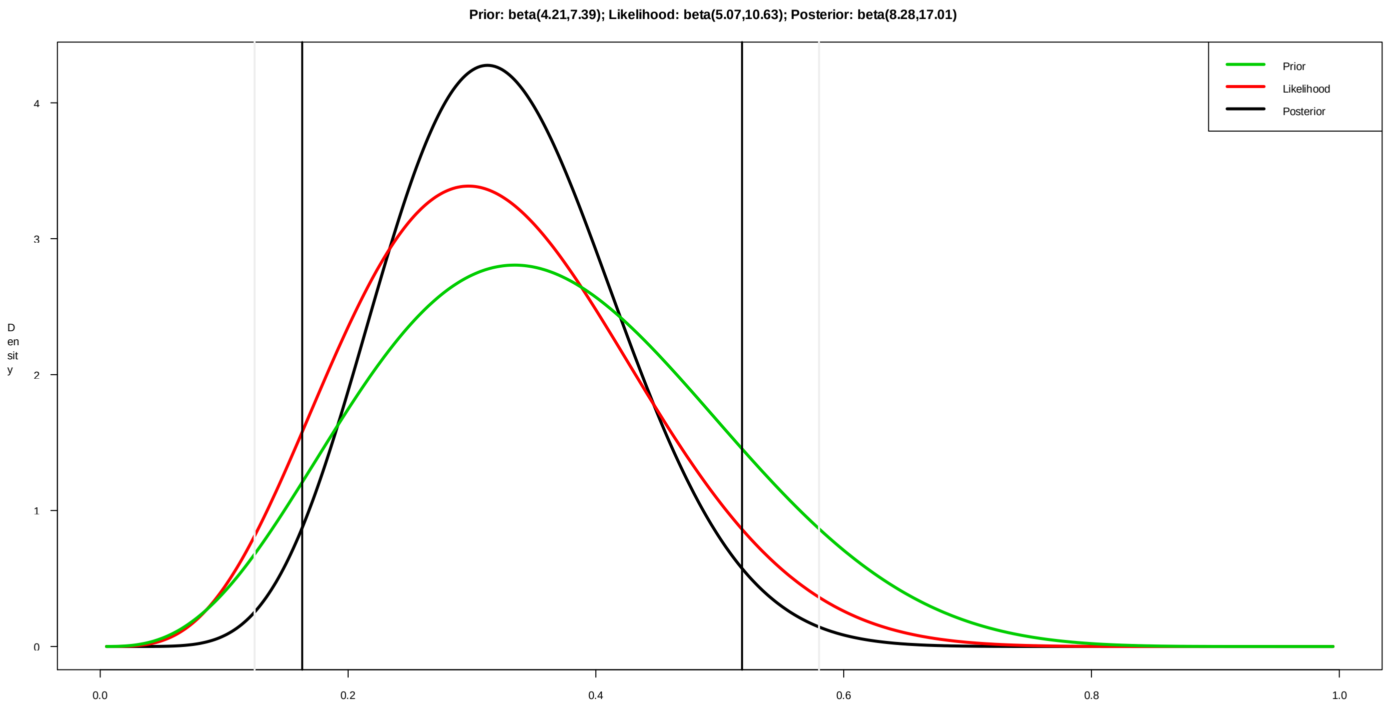

Fig. 7: Posterior Distribution

d) Posterior Mean and Standard Deviation Using beta Prior

Using the beta prior for a beta-likelihood the mean and variance parameters. It is given as:

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Prior Mode = 0.3344	likelihood Mode = 0.2972	Posterior Mode = 0.3125
Prior Mean = 0.3629	likelihood Mean = 0.3231	Posterior Mean = 0.3273
Prior Medan = 0.3546	likelihood Median = 0.3152	Posterior Medan = 0.3227
Prior Variance : 0.0184	likelihood Variance = 0.0131	Posterior Variance = 0.0084
Prior Skewness = 5.652	likelihood Skewness = 10.1626	Posterior Skewness = 16.5043

Table 2: Summary of Bayesian Result

The Posterior Mean (Bayes Estimate) of y is

$$E(\theta|y_i) = \frac{\alpha+\alpha_0-1}{\alpha+\alpha_0+\beta+\beta_0-2} = 0.32723$$

The Posterior Variance which $V(\theta|yi)$ is **0.0084**.

Therefore the Posterior Standard deviation is given as **0.0917**

e) The Credible Interval for the Informative Prior
The credible interval is

$$C.I = E(\theta|y_i) \pm Z_{\frac{\alpha}{2}} S.D(\theta|y)$$

= Posterior Mean of Point Estimate $\pm {}^{Z_{\alpha/2}}\sqrt{Posterior\ Precision.}$

Where $E(\theta|y_i) = 0.3273$ and $S.D(\theta|y) = 0.0084$

And $Z_{\alpha/2}$ = 1.96 (From Normal Distribution table)

Therefore,

[0.31, 0.34] is the 95% credible interval for y (the GDP rate), that is the Minimum interval is 0.31 and the Maximum interval is 0.34 at 95%, with a posterior mean of 0.3273 and a posterior standard deviation of 0.0917.

f) Uniform Prior
Choosing a positive uniform prior (which is a non-informative prior) to allow data to speak for itself, in the sense that we have no notion what the value of before making the observation may be. This shows that all values of the parameter have the same probability. As a result, the likelihood of selecting a GDP for a specific year is.

$$P(\theta) = \frac{1}{45} = 0.0222 \quad \theta \geq 0$$

Since that, we have 45 observations. Therefore, the uniform prior is as follows;

Uniform Prior

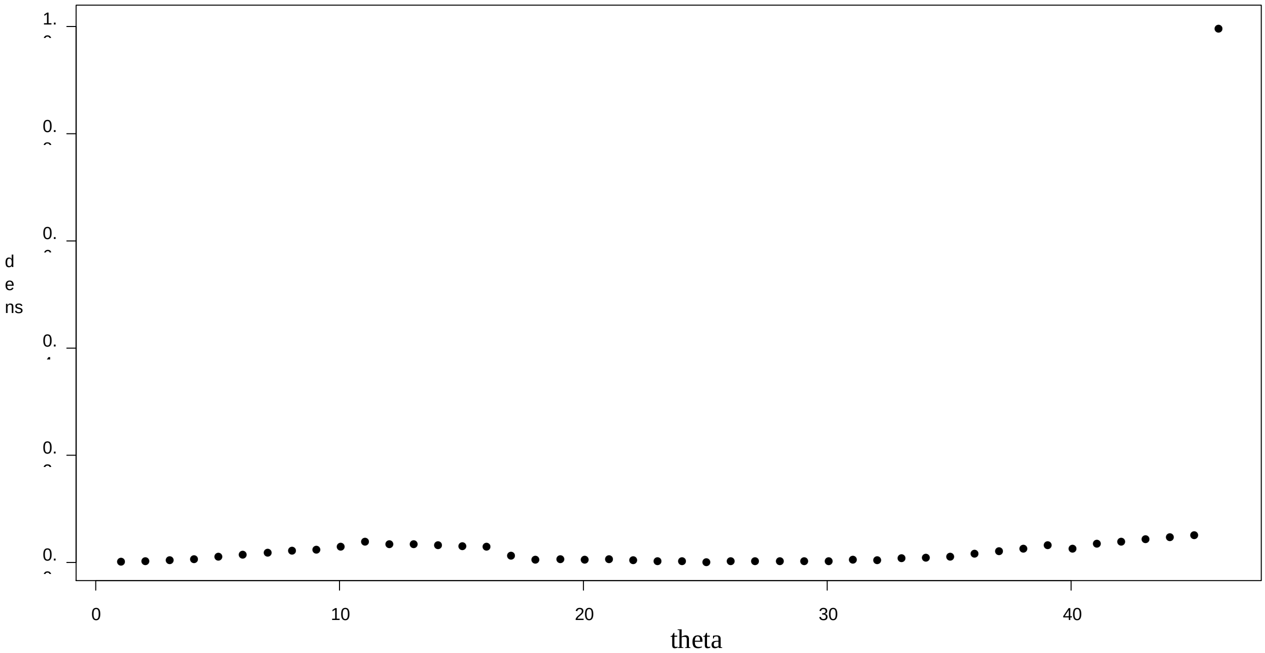

Fig. 8: Plot Using Uniform Prior

S/N	yi	$P(\theta)$	$P(yi\|\theta)$	$P(\theta)\times p(yi\|\theta)$	$p(\theta\|yi)$
1	418	0.0222	0.00658	0.000146	0.00658
2	492	0.0222	0.007745	0.000172	0.00775
3	620	0.0222	0.00976	0.000217	0.00976
4	714	0.0222	0.01124	0.000250	0.01124
5	1051	0.0222	0.016545	0.000368	0.01655
6	1281	0.0222	0.020166	0.000448	0.02017
7	1514	0.0222	0.023834	0.000530	0.02383
8	1715	0.0222	0.026999	0.000600	0.02700
9	1856	0.0222	0.029218	0.000649	0.02922
10	2183	0.0222	0.034366	0.000764	0.03437
11	2768	0.0222	0.043575	0.000968	0.04358
12	2489	0.0222	0.039183	0.000871	0.03918
13	2524	0.0222	0.039734	0.000883	0.03973
14	2415	0.0222	0.038018	0.000845	0.03802
15	2217	0.0222	0.034901	0.000776	0.03490
16	2187	0.0222	0.034429	0.000765	0.03443
17	1099	0.0222	0.017301	0.000384	0.01730
18	660	0.0222	0.01039	0.000231	0.01039
19	767	0.0222	0.012075	0.000268	0.01207
20	635	0.0222	0.009997	0.000222	0.01000
21	747	0.0222	0.01176	0.000261	0.01176
22	626	0.0222	0.009855	0.000219	0.00985
23	541	0.0222	0.008517	0.000189	0.00852
24	496	0.0222	0.007808	0.000174	0.00781
25	383	0.0222	0.006029	0.000134	0.00603
26	488	0.0222	0.007682	0.000171	0.00768
27	494	0.0222	0.007777	0.000173	0.00778
28	503	0.0222	0.007919	0.000176	0.00792
29	517	0.0222	0.008139	0.000181	0.00814
30	519	0.0222	0.00817	0.000182	0.00817
31	658	0.0222	0.010359	0.000230	0.01036
32	606	0.0222	0.00954	0.000212	0.00954
33	778	0.0222	0.012248	0.000272	0.01225
34	863	0.0222	0.013586	0.000302	0.01359
35	1079	0.0222	0.016986	0.000377	0.01699
36	1346	0.0222	0.02119	0.000471	0.02119
37	1659	0.0222	0.026117	0.000580	0.02612
38	1957	0.0222	0.030808	0.000685	0.03081

39	2387	0.0222	0.037578	0.000835	0.03758
40	1919	0.0222	0.03021	0.000671	0.03021
41	2579	0.0222	0.0406	0.000902	0.04060
42	2821	0.0222	0.04441	0.000987	0.04441
43	3065	0.0222	0.048251	0.001072	0.04825
44	3319	0.0222	0.05225	0.001161	0.05225
45	3567	0.0222	0.056154	0.001248	0.05615
Total	63522	1.0000	1.00000	0.022222	1.00000

Table 3: The Posterior Distribution For Uniform Prior

Wherey_i = the rate of GDP per year
$P(\theta)$ = the Prior distribution
$P(yi|\theta)$ = the Likelihood Distribution
$P(\theta|yi)$ = the Posterior Distribution

$$P(\theta|yi) = \frac{P(\theta) \times P(y_i|\theta)}{\sum_{i=1}^{45} P(\theta) \times P(y_i|\theta)}$$

Since equal probabilities are attached to each variable yi, $P(\theta)$, which is the prior distribution is then assumed to be non-informative or otherwise called a flat prior.

g) Posterior Mean For Uniform Prior
The Posterior Mean of y is

$$P(\theta|yi) = \sum_{i=1}^{45} \theta \times P(\theta|y_i) = 0.3229$$

The Posterior Variance of y is

$$Var(\theta|y_i) = E(\theta^2|y_i) - \left(E(\theta|y_i)\right)^2$$

$$E(\theta^2|y_i) = \sum_{i=1}^{45} \theta^2 \times P(\theta|y_i) = 0.11738$$

$$Var(\theta|y_i) = 0.11738 - [0.3229]^2 = 0.01312$$

Also, the Posterior Standard Deviation = $\sqrt{Var(\theta|y_i)}$

$$= \sqrt{0.01312} = 0.114543$$

The credible interval is

$$C.I = E(\theta|y_i) \pm Z_{\frac{\alpha}{2}} S.D(\theta|y)$$

= Posterior Mean of Point Estimate $\pm {}^{Z_{\alpha/2}}\sqrt{Posterior\ Precision}$

Where

$E(\theta|y)$ = 0.3229 and $S.D(\theta|y)$ = 0.114543AndZα/2 = 1.96 (From Normal Distribution table)

As a result, using the Uniform prior, the 95 percent credible interval for y (the rate of GDP) is [0.20836, 0.4374].The Minimum interval is 0.20836 and the Maximum interval is 0.4374 at 95 percent, with a posterior mean of 0.3229 and a posterior standard deviation of 0.114543.

V. SUMMARY, CONCLUSION, RECOMMENDATION

A. *Summary*

The major goal of this study is to determine the distribution of GDP data in Nigeria, and the data was examined using the Bayesian approach, which is based on the Bayes theorem. This data spans 45 years, from 1970 to 2014, and was acquired from the CBN statistical bulletins of 2008, 2011, and 2012, as well as the IMF World Economic Outlook of April 2014 and the Nigeria Economic Report (World Bank may 2013).

The data was subjected to a descriptive analysis, which revealed that the mean and standard deviation were 1411.6 and 928.8775, respectively, and that the data was favorably skewed to the right. The data set's maximum and minimum values were 383 and 3567, respectively. The data set's kurtosis is platykurtic because the value is negative. The lower quartile is 620, and the higher quartile is 2187, according to the box plot.

The data set follows a beta distribution, as seen by the density map. The posterior distribution produces a beta family distribution using an informative prior (conjugate prior). The prior distribution's parameter value is Beta (4.21, 7.39), the likelihood distribution's parameter value is Beta (5.07, 10.83), and the posterior distribution's parameter value is Beta (8.28 and 17.01).

The posterior standard deviation is 0.0917, and the posterior mean (Bayes estimate) using a conjugate is 0.32373. Using the informative prior, the 95 percent credible interval is [0.31, 0.34]. Consider the non-informative prior (uniform prior) for Nigeria's GDP rate: the Posterior Mean (Bayes estimate) is 0.3229, and the Posterior Standard Deviation is 0.114543. Using the uniform prior, the 95 percent credible interval is [0.20836, 0.4374].

B. *Conclusion*

The posterior distribution, which is obtained as a result of two antecedents: the likelihood function, which reflects the information about the parameters contained in the data (the rate of GDP in Nigeria), and the prior distribution, which qualifies what is known about the parameters before observing the data, outlines the basic goal of Bayesian inference. With a little difference, the posterior mean using the uniform prior is greater than the posterior mean using the conjugate prior, hence the two prior distributions can be utilized for the beta distribution. Furthermore, the credible intervals for the two previous distributions diverge. The Posterior standard deviation for conjugate prior gave a better estimate than the Standard deviation of the uniform prior.This means that the conjugate prior perform better than the uniform prior.

C. Recommendation

The following recommendations are given based on the results acquired as far in this study endeavor and the conclusion reached from the Bayesian analysis of Nigeria's GDP rate. The following recommendations are offered in respect to the analysis done so far in this study work.

- When studying Nigeria's GDP rate, an informative Beta prior and beta likelihood should be considered.
- The government should give the resources necessary for local governments to enhance their funding expenditures in relation to GDP.
- To reduce Nigeria's high GDP rate, the government needs create adequate jobs.

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