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Two-Fold Numerical Solutions and Simulation of a Double Pendulum

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Abstract:- In this paper, the re-enactment of a twofold pendulum with mathematical arrangements are examined. The twofold pendulums are organized in static harmony, one of the pendulums takes the upward position, while the subsequent pendulum is in a level position and lays on the cushion. Trademark positions and rakish speeds of both pendulums, just as their energies at every moment of time are introduced. Gotten results ended up being as per the movement of the genuine actual framework. The separation of the two-fold pendulum brings about 4 first ODE's planning the development of the framework.

Keywords:- *Numerical Solution, Simulation, Double Pendulum and Behaviors of the System.*

I. INTRODUCTION

Pendulum is a weight suspended from a turn with the goal that it can swing uninhibitedly. At the point when a pendulum is dislodged sideways from its resting harmony position, it is dependent upon a reestablishing power because of gravity that will speed up it back toward the balance position [1] and [2]. When delivered, the reestablishing power joined with the pendulum's mass makes it waver about the harmony position, swinging to and fro. The ideal opportunity for one complete cycle, a left swing and a right swing, is known as the period. The time frame relies upon the length of the pendulum and furthermore on the abundancy of the swaying. Nonetheless, if the abundancy is little, the period is practically free of the adequacy [3] and [4].

Two-fold pendulum is a mechanical framework that is most generally utilized for showing of the turbulent movement. It is portrayed with two profoundly coupled, nonlinear, 2^{nd} ODE's which makes is exceptionally touchy to the underlying conditions [5] and [6]. Despite the fact that its movement is deterministic in nature, affectability to starting conditions makes its movement capricious or 'turbulent' in the long turn in this paper talks about in the initial segment motivation behind the twofold pendulum [7], in the subsequent segment, the arrangement of directions is introduced and in the third area, the conditions of movement it's mathematical arrangements are examined by utilizing ODE's 45. While in the last segment, conduct of the framework and recreation of the twofold pendulum are examined by this paper and disclose how to linearize the twofold pendulum explore displaying the Linearization Mistake.

The fundamental Point of the research work is to look at twofold pendulum and its application. The objective of this research includes but not limited to the following:

- Provide a simple quantitative description of the motion
- Determine factors affecting the double pendulum
- Determine moment of inertia of the double pendulum
- Demonstrate each of the normal modes in a real double square pendulum
- Demonstrate the appearance of chaos in double pendulum

Statement of Research Problem

This paper isn't just examined the elements of the twofold pendulum framework and talking about the actual framework, yet in addition clarify how the LAGRANGIAN and the HAMILTONIAN conditions of movements are determined, we will dissect and look at between the mathematical arrangement and reproduction, and furthermore change of precise speeds with time for specific framework boundaries at different beginning conditions.

Objective

There have been series of studies on twofold pendulum, that implies it is a framework that albeit the conditions are known, and you are aware of a moment in time the position and speed of the pendulum precise, it is as yet impractical to anticipate how it will act later on. Also, different investigations have zeroed in on twofold pendulum however not a single review has been done on twofold pendulum and its application in Nigeria.

II. METHODOLOGY

The movement of molecule and unbending bodies is overseen by Newton's law. The end goal of the research work, is to determine a substitute methodology, setting Newton's law into a structure especially helpful for quite a long time of opportunity frameworks of frameworks in complex arrange frameworks. This methodology brings about a bunch of conditions called Lagrange's conditions and some piece of Runge-kutta technique. There is the start of a mind boggling, more numerical way to deal with mechanics called scientific elements. In this research we will just arrange this technique at a rudimentary level. Indeed, even at this improved level, obviously extensive rearrangements happen in determining the conditions of movement for complex frameworks. These two methodologies Newton's law and Lagrange's conditions are absolutely viable. No new actual laws result for one methodology versus the other. Many have contended that Lagrange's conditions, in light of preservation of energy, are a more major assertions of the laws administering the movement of particles and unbending bodies.

III. SYSTEM COORDINATES

The two-fold pendulum is delineated in Diagram1. It is advantageous to characterize the directions as far as the points between every pole and the vertical. In this outline, and address the mass, length and the point from the typical of the inward weave (m_1,L_1 and θ_1) and, and represent the mass, length, and the point from the ordinary of the external sway (m_2, L_2 and θ_2). The straight-forward kinematics conditions address in next area to determine conditions of movement by utilizing Lagrange conditions.

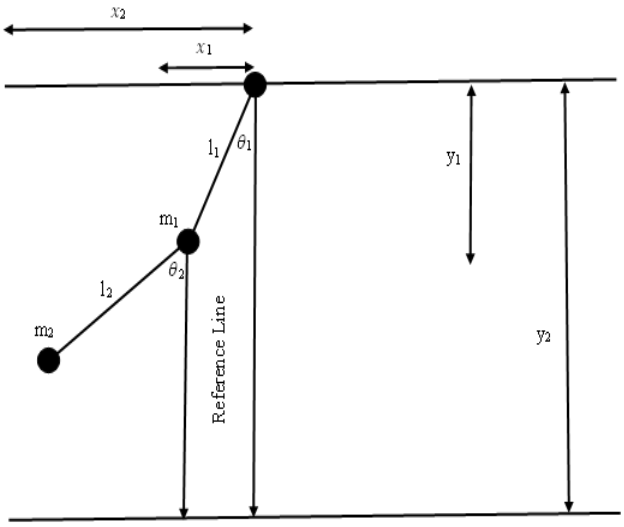

Diagram 1

IV. EQUATIONS OF MOTION

In this segment, the primary thought of the framework facilitates depends on settling these amounts onto level and vertical parts as in the diagram 1, we acquire the situation of the focal point of mass of the two bars, where (x_1, y_1) are the points of the inward bounce and (x_2, y_2) is the points of the external weave. To just our mathematical investigation, let us right off the bat talk about particularly situation when $m_1 = m_2 = m$ and $L_1 = L_2 = l$. That is, we consider two indistinguishable bars with ($I = \frac{1}{12}ml^2$).

Accept that masses of poles can be dismissed yet their snapshot of idleness ought to be incorporated to more readily mirror the actual framework they address.

$$x_1 = \frac{l}{2}sin(\theta_1) \quad ---------\rightarrow (1)$$

$$x_2 = l(sin(\theta_1) + \frac{1}{2}sin(\theta_2) ---------\rightarrow (2)$$

$$y_1 = -\frac{l}{2}cos(\theta_1) \quad -------------\rightarrow (3)$$

$$y_2 = -l(cos(\theta_1) + \frac{1}{2}cos(\theta_2) --------\rightarrow (4)$$

The Lagranian is given by

$$L = Kinetic\ Energy - potential\ Engery$$

$$L = \frac{1}{2}m(v_1^2 + v_2^2) + \frac{1}{2}I(\dot{\theta}_1^2 + \dot{\theta}_2^2) - mg(y_1 + y_2) --\rightarrow (5)$$

$$L = \frac{1}{2}m(\dot{x}_1^2 + \dot{y}_1^2 + \dot{x}_2^2 + \dot{y}_2^2) + \frac{1}{2}I(\dot{\theta}_1^2 + \dot{\theta}_2^2) - mg(y_1 + y_2) \quad ------\rightarrow (6)$$

The initial term is the direct active energy of the focal point of mass of the bodies and the subsequent term is the rotational dynamic energy around the focal point of mass of every pole. The last term is the possible energy of the bodies in a uniform gravitational field.

Substituting the above coordinates, yielded

$$L = \frac{1}{6}ml^2[\dot{\theta}_2^2 + 4\dot{\theta}_1^2 + 3\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - \theta_2)] + \frac{1}{2}mgl(3cos\theta_1 + cos\theta_2) \rightarrow (7)$$

There is just one rationed amount (the energy), and no saved momenta. The two momenta might be composed as

$$p_{\theta_1} = \frac{\partial L}{\partial \dot{\theta}_1} = \frac{1}{6}ml^2[8\dot{\theta}_1 + 3\dot{\theta}_2\cos(\theta_1 - \theta_2)] \rightarrow (8)$$

$$p_{\theta_2} = \frac{\boldsymbol{\partial L}}{\boldsymbol{\partial \dot{\theta}_2}} = \frac{\mathbf{1}}{\mathbf{6}}ml^2[2\dot{\theta}_2 + 3\dot{\theta}_1\cos(\theta_1 - \theta_2)] \rightarrow (9)$$

These articulations might be modified to get

$$\dot{\theta}_1 = \frac{6}{ml^2}\frac{2p_{\theta_1} - 3cos(\theta_1 - \theta_2)p_{\boldsymbol{\theta_2}}}{16 - 9\cos^2(\theta_1 - \theta_2)} ----\rightarrow (10)$$

$$\dot{\theta}_2 = \frac{6}{ml^2}\frac{8p_{\theta_2} - 3cos(\theta_1 - \theta_2)p_{\boldsymbol{\theta_1}}}{16 - 9\cos^2(\theta_1 - \theta_2)} ----\rightarrow (11)$$

The excess conditions of movement for energy are

$$\dot{p}_{\theta_1} = \frac{\partial L}{\partial \theta_1} = -\frac{1}{2}ml^2[\dot{\theta}_1\dot{\theta}_2\sin(\theta_1 - \theta_2) + 3\frac{g}{l}sin\theta_1] \rightarrow (12)$$

$$\dot{p}_{\theta_2} = \frac{\partial L}{\partial \theta_2} = -\frac{1}{2}ml^2\left[-\dot{\theta}_1\dot{\theta}_2\sin(\theta_1 - \theta_2) + \frac{g}{l}sin\theta_2\right] \rightarrow (13)$$

How about we now expect that $l = 1$. This provides us with a bunch of four conditions that can be utilized to reproduce the conduct of the twofold pendulum

$$\dot{\theta}_1 = 6\frac{2p_{\theta_1} - 3cos(\theta_1 - \theta_2)p_{\boldsymbol{\theta_2}}}{16 - 9\cos^2(\theta_1 - \theta_2)} ------\rightarrow (14)$$

$$\dot{\theta}_2 = 6\frac{8p_{\theta_2} - 3cos(\theta_1 - \theta_2)p_{\boldsymbol{\theta_1}}}{16 - 9\cos^2(\theta_1 - \theta_2)} ------\rightarrow (15)$$

$$\dot{p}_{\theta_1} = -\frac{1}{2}[\dot{\theta}_1\dot{\theta}_2\sin(\theta_1 - \theta_2) + 3gsin\theta_1] -\rightarrow (16)$$

$$\dot{p}_{\theta_2} = -\frac{1}{2}[-\dot{\theta}_1\dot{\theta}_2\sin(\theta_1 - \theta_2) + gsin\theta_2] -\rightarrow (17)$$

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The rationed amount, energy work, is given by the Hamiltonian=Kinetic Energy +Potential Energy

$$H = \theta_i p_i - L = ml^2\dot{\theta}_1^2 + \frac{1}{2}ml^2\dot{\theta}_2^2 + ml^2\dot{\theta}_1\dot{\theta}_2 cos(\theta_1 - \theta_2) - 2mglcos\theta_1 - mglcos\theta_2 \rightarrow (18)$$

Substituting the above solved equations for $\dot{\theta}_1$ and $\dot{\theta}_2$we obtain

$$H = \frac{3ml^2{p_{\theta_2}}^2 - 2ml^2 p_{\theta_1}p_{\theta_2}cos(\theta_1 - \theta_2)}{2ml^4[m + m\sin^2(\theta_1 - \theta_2)]} - 2mglcos\theta_1 - mglcos\theta_2 \rightarrow (19)$$

Again, when $m = l = 1$,

$$H = \frac{3{p_{\theta_2}}^2 - 2p_{\theta_1}p_{\theta_2}cos(\theta_1 - \theta_2)}{2[1 + \sin^2(\theta_1 - \theta_2)]} - 2gcos\theta_1 - gcos\theta_2 \quad ------\rightarrow \quad (20)$$

We presently take general case. The initial segment of addressing this framework is inferring the conditions for position. The situation of each mass m_1 *and* m_2 can be given by:

$$\left.\begin{array}{c} x_1 = l_1 sin(\theta_1) \\ x_2 = l_1 sin(\theta_1) + l_2 sin(\theta_2) \\ y_1 = -l_1 cos(\theta_1) \\ y_2 = -l_1 cos(\theta_1) - l_2 cos(\theta_2) \end{array}\right\} \quad ------\rightarrow (21)$$

In these situations, θ_1 and θ_2 are allotted from the negative y-pivot as displayed in Figure.1

Then, energy conditions are utilized to track down the dynamic and likely energies of the framework.

$$KE = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$
$$PE = m_1 g y_1 + m_2 g y_2$$

Then, at that point, the conditions can be controlled utilizing the position conditions above just as the way that $v = r * \omega$ where $r = l$ and $\omega = \theta'$.

$$v_1 = {\dot{x}_1}^2 + {\dot{y}_1}^2, \; v_2 = {\dot{x}_2}^2 + {\dot{y}_2}^2$$
$$\dot{x}_1 = l_1\dot{\theta}_1 cos(\theta_1)$$
$$\dot{x}_2 = l_1\dot{\theta}_1 cos(\theta_1) + l_2\dot{\theta}_2 cos(\theta_2)$$
$$\dot{y}_1 = l_1\dot{\theta}_1 sin(\theta_1)$$
$$\dot{y}_2 = l_1\dot{\theta}_1 sin(\theta_1) + l_2\dot{\theta}_2 sin(\theta_2)$$
$$v_1 = l_1^2{\dot{\theta}_1}^2$$
$$v_2 = l_1^2{\dot{\theta}_1}^2 + l_2^2{\dot{\theta}_2}^2 + 2l_1 l_2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - \theta_2)$$
$$KE = \frac{1}{2}(m_1 + m_2)l_1^2{\dot{\theta}_1}^2 + \frac{1}{2}m_2(l_2^2{\dot{\theta}_2}^2 + 2l_1 l_2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - \theta_2)) \rightarrow (22)$$

The Lagrange is the distinction between the dynamic and expected conditions of the framework. It is utilized when a framework is expressed as a bunch of summed-up facilitates instead of speeds. For this situation the directions of the framework depend on θ_1 and θ_1:

$$L = KE - PE$$

$$L = \frac{1}{2}(m_1 + m_2)l_1^2{\dot{\theta}_1}^2 + \frac{1}{2}m_2(l_2^2{\dot{\theta}_2}^2 + 2l_1 l_2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - \theta_2)) + (m_1 + m_2)gl_1 cos(\theta_1) + m_2 g l_2 cos(\theta_2) \quad ----\rightarrow (23)$$

The second piece of the Lagrange condition incorporates taking most of the way of the above Lagrange condition with respect to the summarized works with. This will give two new conditions

$$\frac{d}{dt}.\frac{\partial L}{\partial\dot{\theta}_i} - \frac{\partial L}{\partial\dot{\theta}_i} = 0, i = 1{,}2 ----------\rightarrow (24)$$

From Equation 23:

$$\left.\begin{array}{l} \frac{\partial L}{\partial\dot{\theta}_1} = (m_1 + m_2)l_1^2\dot{\theta}_1 + m_2 l_1 l_2\dot{\theta}_2\cos(\theta_1 - \theta_2) \\ \frac{d}{dt}.\frac{\partial L}{\partial\dot{\theta}_1} = (m_1 + m_2)l_1^2\ddot{\theta}_1 + m_2 l_1 l_2[\ddot{\theta}_2\cos(\theta_1 - \theta_2) \\ \quad -\dot{\theta}_2(\dot{\theta}_1 - \dot{\theta}_2)\sin(\theta_1 - \theta_2)] \\ \frac{\partial L}{\partial\theta_1} = -m_2 l_1 l_2\dot{\theta}_1\dot{\theta}_2\sin(\theta_1 - \theta_2) \\ \quad -(m_1 + m_2)l_1.g.\sin(\theta_1) \end{array}\right\} \rightarrow (25)$$

Then, at that point, setting the conditions 25 into conditions 24, conditions 26 is created:

$$(m_1 + m_2)l_1^2\ddot{\theta}_1 + m_2 l_1 l_2\ddot{\theta}_2\cos(\theta_1 - \theta_2) + m_2 l_1 l_2 {\dot{\theta}_1}^2\sin(\theta_1 - \theta_2) + (m_1 + m_2)l_1.g.\sin(\theta_1) = 0 --------\rightarrow (26)$$

Similarly, with θ_2 using equation 24:

$$\left.\begin{array}{l} \frac{\partial L}{\partial\dot{\theta}_2} = m_2 l_2^2\dot{\theta}_2 + m_2 l_1 l_2\dot{\theta}_1\cos(\theta_1 - \theta_2) \\ \frac{d}{dt}.\frac{\partial L}{\partial\dot{\theta}_2} = m_2 l_2\ddot{\theta}_2 + m_2 l_1 l_2\ddot{\theta}_1\cos(\theta_1 - \theta_2) \\ \quad -m_2 l_1 l_2\dot{\theta}_1(\dot{\theta}_1 - \dot{\theta}_2)\sin(\theta_1 - \theta_2) \\ \frac{\partial L}{\partial\theta_2} = m_2 l_1 l_2\dot{\theta}_1\dot{\theta}_2\sin(\theta_1 - \theta_2) - m_2 l_2.g.\sin(\theta_2) \end{array}\right\} - \rightarrow (27)$$

Replacing the above conditions into equation 24:

$$m_2 l_2^2\ddot{\theta}_2 + m_2 l_1 l_2\ddot{\theta}_1\cos(\theta_1 - \theta_2) - m_2 l_1 l_2{\dot{\theta}_1}^2\sin(\theta_1 - \theta_2) + m_2 l_2.g.\sin(\theta_2) = 0 \quad -- \rightarrow (28)$$

Condition 26 and condition 28 are utilized to depict the movement of the pendulum's framework and are 2nd ODE's conditions. This can't yet be utilized in MATLAB in light of the fact that there are four (4) missing variables. The framework movement should be depicted in first-request differential conditions before ODE45 can be utilized. Momenta Conditions. The momenta, p_1 and p_2, are found by taking the fractional of the Lagrange concerning θ_1 and θ_2, individually.

The momenta conditions are portrayed as the halfway subsidiary of the Lagrange concerning the precise speeds. In this way:

$$\left.\begin{aligned} p_1 = \frac{\partial L}{\partial \dot\theta_1} = (m_1 + m_2) l_1^2 \dot\theta_1 + m_2 l_1 l_2 \dot\theta_2 \cos(\theta_1 - \theta_2) \\ p_2 = \frac{\partial L}{\partial \dot\theta_2} = m_2 l_2^2 \dot\theta_2 + m_2 l_1 l_2 \dot\theta_1 \cos(\theta_1 - \theta_2) \end{aligned}\right\} \rightarrow$$
(29)

The Hamiltonian equation is $H = \theta_i p_i - L$ for $i = 1$ *or* 2.The Hamiltonian will be used to put the equations in terms of four initial conditions:

$$H = \theta_i p_i - L = \frac{1}{2}(m_1 + m_2) l_1^2 \dot\theta_1^{\,2} + \frac{1}{2} m_2 (l_2^2 \dot\theta_2^{\,2} + 2 l_1 l_2 \dot\theta_1 \dot\theta_2 \cos(\theta_1 - \theta_2)) - (m_1 + m_2) g l_1 cos(\theta_1) - m_2 g l_2 cos(\theta_2) \;\; --------\rightarrow 30$$

The momenta equations in equation 29 are then solved for $\dot\theta_1$ and $\dot\theta_2$.These two equations are then placed into equation 30 and the following equation is derived.

$$H = \frac{m_2 l_2^2 p_2^2 + (m_1 + m_2) l_1^2 p_2^2 - 2 m_2 l_1 l_2 p_1 p_2 \cos(\theta_1 - \theta_2)}{2 m_2 l_1^2 l_2^2 [m_1 + m_2 \sin^2(\theta_1 - \theta_2)]} - (m_1 + m_2) g l_1 cos(\theta_1) - m_2 g l_2 cos(\theta_2) \;\; --\rightarrow (31)$$

1st ODE. Since the Hamiltonian condition has the four introductory conditions remembered for it, it very well may be fallen to pieces into 4 first ODE's by utilizing the accompanying separation conditions:

$$\left.\begin{aligned} \frac{\partial H}{\partial p_i} = \frac{d\theta_i}{dt} = \dot\theta_i \\ -\frac{\partial H}{\partial \theta_i} = \frac{dp_i}{dt} = \dot p_i \end{aligned}\right\} -------------\rightarrow (32)$$

$$\left.\begin{aligned} \dot\theta_1 &= \frac{l_2 p_1 - l_1 p_2 \cos(\theta_1 - \theta_2)}{l_1^2 l_2 [m_1 + m_2 \sin^2(\theta_1 - \theta_2)]} \\ \dot\theta_2 &= \frac{l_1 (m_1 + m_2) p_2 - l_2 m_2 p_1 \cos(\theta_1 - \theta_2)}{l_1 l_2^2 m_2 [m_1 + m_2 \sin^2(\theta_1 - \theta_2)]} \\ \dot p_1 &= -(m_1 + m_2).g.l_1 \sin(\theta_1) - A + B \\ \dot p_2 &= -m_2.g.l_2 \sin(\theta_2) + A - B \end{aligned}\right\} ---\rightarrow (33)$$

$$\left.\begin{aligned} A &= \frac{p_1 p_2 \sin(\theta_1 - \theta_2)}{l_1 l_2 [m_1 + m_2 \sin(\theta_1 - \theta_2)]^2} \\ B &= \frac{l_2^2 m_2 p_1^2 + l_1^2 (m_1 + m_2) p_2^2 - l_1 l_2 m_2 p_1 p_2 \cos(\theta_1 - \theta_2)}{2 l_1^2 l_2^2 [m_1 + m_2 \sin^2(\theta_1 - \theta_2)]^2} \sin[2(\theta_1 - \theta_2)] \end{aligned}\right\}$$
$$-------------------------\rightarrow (34)$$

V. RESULT FOR FINDING NUMERICAL SOLUTION

Sorting the Function M-File from the above, the information, can be formed in MATLAB with:

$$\left.\begin{aligned} u(1) &= \theta_1 \\ u(2) &= \theta_2 \\ u(3) &= p_1 \\ u(4) &= p_2 \end{aligned}\right\} ---------------\rightarrow (35)$$

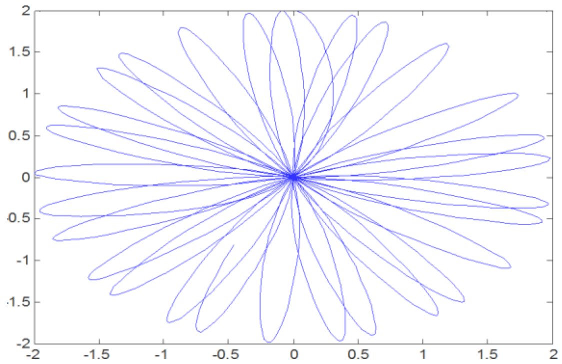

Fig 1: quasiperiodic behaviour

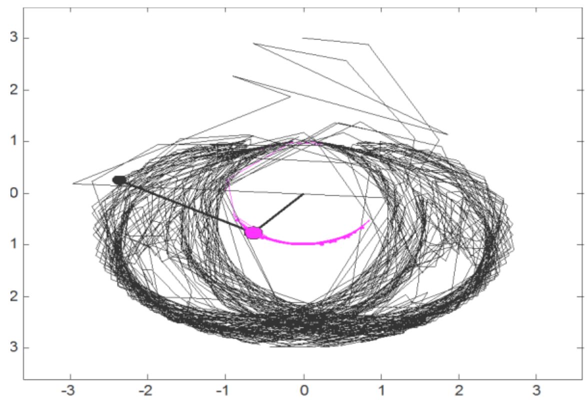

Fig 2: Chaotic behaviour

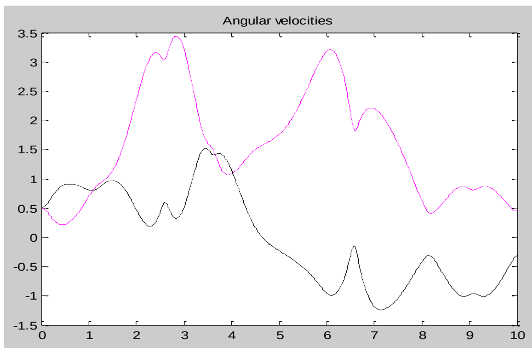

Fig 3: Angular velocity of graph and energy of the system = 10.4250

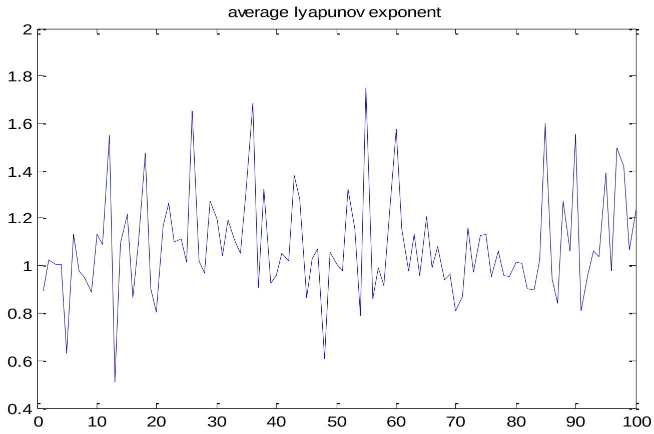

Fig 4: lyapunov exponent

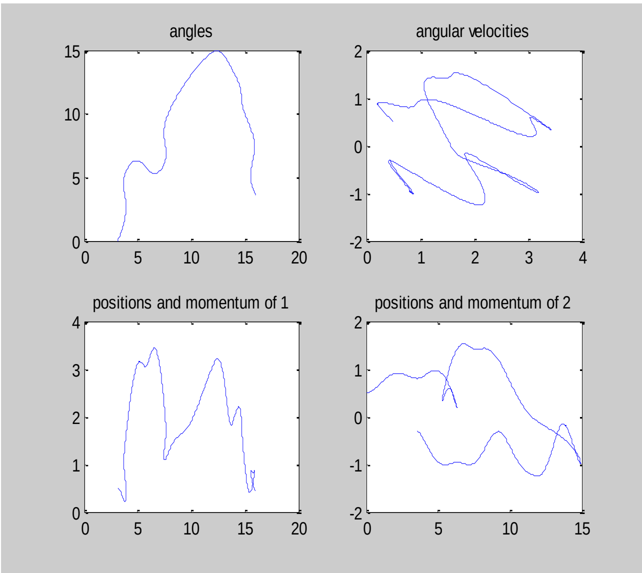

Fig 5: Plots of input variables against each other

➢ *Result For Simulation of Double Pandulum*

By having two swinging poles eased back by tacky contact, let us fabricate a model, this model shows a determined twofold pendulum with friction on both revolute joints. The rakish speed plots show the locking and opening of the joints. In chart (1) is addressed twofold pendulum [7].

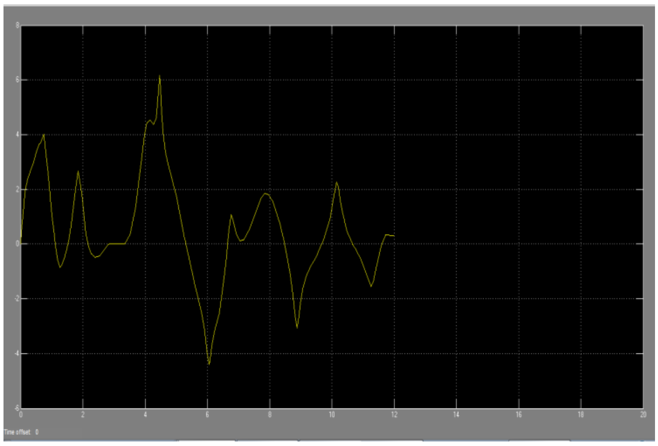

Fig 6: less speed velocity

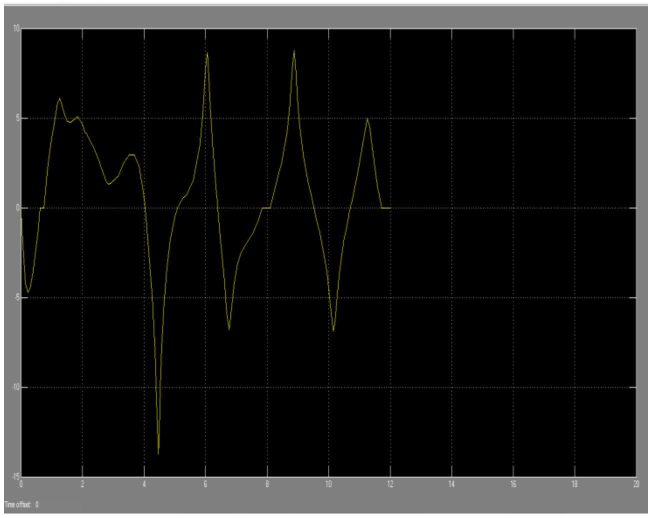

Fig 7: high speed Velocity

VI. CONCLUSION

The Twofold pendulum is an extremely complicated framework. Because of the intricacy of the framework there are numerous suspicions, in case there was grinding, and the framework was non-moderate, the framework would be tumultuous. Turmoil is a condition of obvious problem and inconsistency. Turmoil after some time is profoundly touchy to beginning conditions and can just happen in non-moderate frameworks. The hour of this movement is known as the period, the period doesn't rely upon the mass of the twofold pendulum or on the size of the circular segments through which they swing. One more factor associated with the time of movement is, the speed increase because of gravity.

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