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Modeling the Performance of Rayleigh and Weibull Distributions in Estimating Rainfall in Ondo State, Nigeria

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Abstract:- In this study we aimed to examine the performance of Rayleigh and Weibull distributions in relative to the distribution that will perform better in fitting the data on rainfall in Ondo State from 2005- 2019. We used method of maximum likelihood to estimate the parameters of the distributions. The distributions were compared using the criteria for selecting the best model fit such as log-likelihood function, Akaike information criterion (AIC), and Bayesian information criterion (BIC). The results obtained show that the Rayleigh distribution performed better than the Weibull distribution in comparison in terms of model fit; this indicated that the Rayleigh distribution is efficient in analyzing the rainfall data other than the Weibull distribution.

Keywords:- *AIC, BIC, Maximum Likelihood, Log-Likelihood, Rayleigh Distribution, Weibull Distribution.*

I. INTRODUCTION

Rainfall is an important aspect of the hydrological cycle and changes in its pattern. Rain is a major factor of the water cycle and is answerable for depositing most of the fresh water on the Earth. Rainfall is measured as the main source of domestic water for living as well as for agriculture. The analysis of the rainfall series of the State would improve the running of water resources as well as optimize its uses. One of the major difficult tasks with rainfall data is to deal with interpreting past records of rainfall events in terms of future probabilities of existence. Therefore, knowing the rainfall distribution that causes flood might play a crucial role for the ecological development and preservation of natural resources of the state. Assessing a statistical distribution which provides a better fit to yearly rainfall has long been a concerning issue for hydrologists, meteorologists, and other water resource personnel. Knowing the rainfall distribution is essential for stochastic modeling, rainfall frequency analysis, and rainfall trend analysis. Rainfall is considered as one of the key natural resource which plays a crucial part in sustaining human life be it in agriculture, industry and domestics therefore, scarcity or extreme rainfall might be very harmful as there will be food shortage, water pollution, flooding and telecommunication problems, etc. All of this can lead to economical beating in a region. Researches in many areas of life has made use of probability distributions in modeling rainfall data while many studies and research are continuously in progress and efforts are being made to model and predict future drifts of rainfall to improve planning such as Ayeni *et al* (2019) Ayeni *et al* (2020a), Ayeni *et al* (2020b), Olumide *et al* (2013), Omotosho, *et al* (2013), Sabarish *et al* (2017), Vikram and Shanu (2017), Kalita et al (2017), Esberto (2018), and Mohita et al (2010).

Therefore this study aimed to compare the performance of Rayleigh distribution and Weibull distribution on rainfall data in Ondo State from 2005- 2019 using the model selection criteria like the Akaike information criterion (AIC), Bayesian information criterion (BIC), and the log-likelihood function (l)

II. METHODOLOGY

Several studies has laid emphasis on how to study rainfall pattern throughout the universe and one of the major concerns in the analysis of rainfall data is with making a valid inference about the past data in terms of future probabilities of occurrences. A good number of probability distribution methods have been used in estimating rainfall data and they have been proven to be useful for rainfall studies and surveys.

The analysis of most rainfall data largely depends on its distribution pattern. It has now long been a focus of interest in the field of hydrology in establishing a probability distribution that provides a good fit to most rainfall data. In this study, we, therefore, aim to fit Rayleigh and Weibull distribution to rainfall data in Ondo State from 2005- 2019. The method of maximum likelihood will be used to estimate their parameters, while the log-likelihood, Bayesian information criterion (BIC) and Akaike information criterion (AIC) goodness of fit test will be employed to determine their goodness of fit and R-software would be used for data analysis.

A. Rayleigh Distribution

The pdf of the Rayleigh distribution is given as;

$$f(x;\sigma) = \frac{x}{\sigma^2} e^{-x^2/2\sigma^2} \quad x \geq 0, \sigma > 0 \tag{1}$$

where: σ is a scale parameter

B. Weibull Distribution

The 2-parameter Weibull pdf is given by;

$$f(x) = \frac{k}{c}\left(\frac{x}{c}\right)^{k-1} \exp\left(\frac{x}{c}\right)^{k} \quad x \geq 0, c > 0, k > 0 \tag{2}$$

Where the c is the scale parameter and k is the shape parameter.

C. Maximum Likelihood Estimator (MLE)

Let $x_1, x_2 ... x_n$ be a random sample of size *n* drawn from a pdf $f(x;\theta)$, the likelihood function is defined as;

$$f(x_1, x_2 ... x_n;\theta) = \prod_{i=1}^{n} f(x_i;\theta) \quad (3)$$

Now, we estimate the parameters of both the Rayleigh and Weibull distributions using maximum likelihood estimation as follows,

D. Rayleigh Distribution

The likelihood function is given as;

$$f(x_i;\theta) = \prod_{i=1}^{n}\left(\frac{x_i}{\theta^2}\right)\exp\left(-\frac{x_i}{2\theta^2}\right) \quad (4)$$

To determine the parameters that maximize the likelihood, we take the natural logarithm of (3.4) and solve the equations as follows;

$$InL(\theta) = \left[\sum_{i}^{n} In(x_i) - 2nIn(\theta) - \frac{1}{\theta^2}\sum_{i=1}^{n}\frac{x_i^2}{2}\right] \quad (5)$$

Differentiating (5) with respect to θ then, the maximum likelihood of the Rayleigh distribution is obtained as;

$$\hat{\theta} = \left(\frac{1}{2n}\sum x_i^2\right)^{1/2} \quad (6)$$

E. Weibull distribution

The likelihood function is given as;

$$\mathrm{L}(x,k,c) = \prod_{i=1}^{n}\left(\frac{k}{c}\right)\left(\frac{x_i}{c}\right)^{k-1}\exp\left(-\frac{x_i}{c}\right)^{k} \quad (7)$$

On taking the natural logarithms of equation (7) and differentiating with respect to k and c in turn equating the resulting equation to zero then we have;

$$\frac{\partial InL}{\partial k} = \frac{n}{k} + \sum_{i=1}^{n} Inx_i - \frac{1}{c}\sum_{i=1}^{n} x_i^k Inx_i = 0 \quad (8)$$

$$\frac{\partial InL}{\partial c} = -\frac{n}{c} + \frac{1}{c^2}\sum_{i=1}^{n} x_i^k Inx_i = 0 \quad (9)$$

On estimating *c* from either of the two above equations and simplifying we get

$$\frac{\sum_{i=1}^{n} x_i^k Inx_i}{\sum_{i=1}^{n} x_i^k} - \frac{1}{k} - \frac{1}{n}\sum_{i=1}^{n} Inx_1 = 0 \quad (10)$$

which may be solved to get the estimate of *k* and this can be accomplished by using the Newton Raphson method.

III. RESULTS AND DISCUSSION

This work used secondary data collected based on the monthly amount of rainfall data in Ondo State from 2005-2019. The Rayleigh and Weibull probability were applied to the data to determine the best fit distribution for the data. The data were analyzed using the R software package. The measures of goodness were determined by using selection criteria such as the Akaike information criterion (AIC), Bayesian information criterion (BIC), and the log-likelihood function (l). The model with the least Akaike information criterion (AIC), Bayesian information criterion (BIC), or the highest log-likelihood function (l) value is considered the best model, and the results are obtained as follows;

Table I: summary of the data

Statistic	Value
n	168
Mean	184.2917
Var.	30837.07
SD	175.604
Skewness	1.415975
Kurtosis	2.296287

➢ *Interpretation*

The results from table 1 above indicated that the distribution of the data is skewed to the right with skewness 1.415975. Also, it was observed that the kurtosis is 2.296287 which is lesser than 3. This implies that the distribution of the data has a shorter and lighter tail with a light peakedness when compared to that of the Normal distribution.

Table II: The Maximum Likelihood Estimates for the Distributions

Distribution	Rayleigh	Weibull	
Parameters	**Sigma(σ)**	**Shape (k)**	**Scale (c)**
Values	1.824804	1.009116	190.307573
Standard Error	0.0714665 5	0.0601674 5	15.2761402 6

Table III: The Log-Likelihood Value for the Two Distributions

Goodness of Fit	Rayleigh	Weibull
Log-Likelihood	-350.2398	-1017.943

➢ *Interpretation*

The log-likelihood value for the data is presented in Table 3. It was observed that the Rayleigh distribution provides a better fit as compared to Weibull distributions since it has the highest value of log-likelihood (l). Hence, the Rayleigh distribution performed better than the Weibull distribution.

Table IV: The AIC Value for the Two Distributions

Goodness of Fit	Rayleigh	Weibull
AIC	702.4795	2039.886

➢ *Interpretation*

The AIC value for the data is presented in Table 4. It was observed that the Rayleigh distribution provides a better fit as compared to Weibull distributions since it has a smaller value of AIC. Hence, the Rayleigh distribution performed better than the Weibull distribution.

Table V: The BIC Value for the Two Distributions

Goodness of Fit	Rayleigh	Weibull
BIC	705.5733	2046.074

➢ *Interpretation*

The BIC value for the data is presented in Table 5. It was observed that the Rayleigh distribution provides a better fit as compared to Weibull distributions since it has a smaller value of BIC. Hence, the Rayleigh distribution performed better than the Weibull distribution.

IV. CONCLUSION

The probability distribution in which the AIC and BIC value is the lowest and the log-likelihood is the highest respectively was selected; it was discovered that the Rayleigh distribution is noticeably low and high values in term of the AIC, BIC and log-likelihood values respectively. Therefore, the superiority of fit of the Rayleigh distributions was established without distinguishing superiority. Equally, the suitability of the Rayleigh distribution for life data analysis has been shown in advanced studies. Therefore, for higher precision in rainfall data in Ondo State, the use of Rayleigh distribution is highly recommended and also purposefully useful in various fields where analysis of such data is crucial.

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