ISSN No:-2456-2165

Optimal Design of Water Pipeline Infrastructure Project based on 0-1 Planning and Minimum Spanning Tree Model

Shi-gang Zhou,
School of Microelectronics,
Northwestern Polytechnical University,
Xi'an 710072, China

Yuan Gao
Computer Inst of Northwestern
Polytechnical University Xi'an,
China

Abstract:- In rural China, piped water projects have many advantages over traditional methods of drilling wells. Tap water projects are energy efficient, low cost, effective and simple to operate, but they also have certain limitations. In this paper, the 0-1 planning model and the minimum spanning tree model are used to study and optimize the water pipeline laying problem encountered in infrastructure projects. The 0-1 planning model is established and the Prim algorithm of the minimum spanning tree theory is used to divide different water supply stations to achieve certain minimum pipeline mileage, minimum total mileage or upgraded water supply stations under different conditions and iterative constraints, and the corresponding pipeline planning diagrams are obtained.

Keywords:- *Hierarchical pipe laying, 0-1 planning, minimum spanning tree, prim algorithm.*

I. INTRODUCTION

Along with the rapid development of China's economy, people's quality of life has been generally improved, for the rural construction has also improved the relevant living infrastructure, the effect is more significant is the rural tap water engineering construction. Through a large number of practice shows that in the process of design and transformation of water pipes often encounter many problems, especially in some of the more complex terrain of the mountainous areas in the construction process, has a very large construction difficulties, which hinders the improvement of rural living facilities, but also reduce the living standards of rural people. Therefore, the relevant construction personnel should make more efforts to continuously optimize the construction technology, solve the relevant problems, and accelerate the modernization of rural construction[1].

Since it does not require the use of pumping equipment, it is widely used in dry rural water projects. This paper mainly analyzes the main points of pipeline design and engineering construction technology for rural tap water based on specific tap water project construction examples and relevant construction experience, hoping to play some reference role for rural tap water project construction. How to scientifically plan the path of water supply pipeline, so as to better improve the utilization rate of resources and economy, for the construction of new It is important to build a new rural area and realize the overall well-off.

The main work of this paper is:

- We adopted the minimum spanning tree theory prim algorithm based on 0-1 planning to solve the established model problem, which can be divided into different areas of the characteristics applicable to the route planning of graded water supply stations. The implementation of the model procedure is relatively simple and mature.
- The prim algorithm model using 0-1 planning carries out different types of objective constraints according to different practical needs. Realistic adaptability is strong.
- The visualization of route planning can be better realized, which enhances the practicality of the model. The practicality of the model is enhanced. It saves the time of engineering drawing.

II. PROBLEM STATEMENT

A. Related Information

A central water supply station, 12 primary water supply stations and 168 secondary water supply stations are built in a certain area, and water is piped from the central station A to the primary and secondary water supply stations. The pipeline from the central station A to the primary water supply station is a type I pipeline, and the pipeline from the primary water supply station to the secondary water supply station is a type II pipeline, and A cannot be directly connected to the secondary water supply station. According to the specific requirements of pipeline laying, the pipeline path is planned scientifically. The following figure (Fig. 1) shows the relationship between the water supply stations.

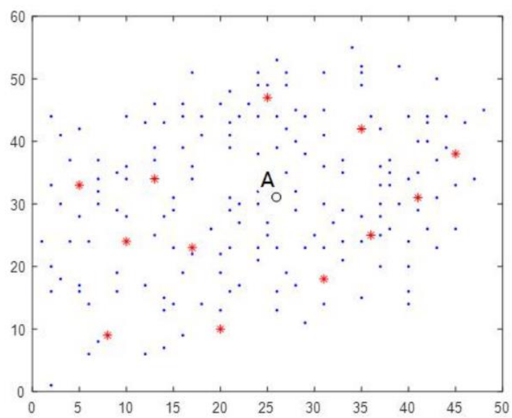

Fig. 1: Coordinate map of the location of each water supply station

B. Problems that need to be solved

Problem 1: Starting from the central water supply station A, the total distance of pipelines was minimized while meeting the water supply conditions.

Problem 2: Upgrade two secondary stations to primary stations due to insufficient supply in the Type II pipeline market. Determine which two secondary stations will be upgraded to primary stations so as to minimize the use of Type II pipelines. Find the amount of Type II piping that will be reduced compared to the first question.

Problem 3: Based on Question 1, consider the fact that in reality, due to power, the pipeline from a Class I station can only supply up to 40 km of water, and that in order to supply water to all stations, several Class II stations need to be upgraded to Class I stations, but the pipeline from that station can only supply up to 40 km of water after the upgrade. What is the minimum number of secondary water supply stations that can be upgraded to supply water to all water supply stations? What is the minimum total number of kilometers of pipeline to be laid in this configuration?

C. Problem Analysis

In order for each water supply station to be available, all stations should be connected and should form a network. At the same time, in order to use the least amount of pipes, planning the closest network becomes the key to this problem.

Analysis of problem 1: Problem 1 requires that the pipeline starts from A and has the shortest total pipeline mileage. To solve this problem, we create a planar point-set area model based on a minimum spanning tree to centralize the water supply stations[2]. According to the location coordinate data given in Table 1, two-dimensional coordinates are created, and a central water supply station A is taken as a point set C with only one element, all the points of primary water supply stations are included in point set D, and all the points of secondary water supply stations are included in point set E. In this way, the problem is transformed into one that can be described graphically. After that, we find the element Dx in the nearest point set D and transform it into the element in C by circular enumeration according to the prim algorithm of minimum spanning tree, until all the elements in D are transformed into the elements in C, that is, we get L1; similarly, we get L2, and the sum of the two planning distances is obtained[3]. Finally, we find the optimal solution based on Kruskal's algorithm and give the optimal The optimal pipeline planning diagram is given.

Analysis of problem 2: Problem 2 requires upgrading two secondary water supply stations to reduce the consumption of type II pipes, which can be directly connected to the central water supply station point A. This leads to the reduction of two type II pipes to increase two type I pipes. Based on the solution of problem 1, we can find the longest type II pipeline, through the prim algorithm we used above, we deduce that the longest two paths are the shortest type II pipeline that can be connected to one of the points, then after upgrading the other pipelines connected to it remain unchanged, so the elimination of the longest two type II pipelines can be the optimal planning. Then, according to the shortest path 0-1 planning, we calculate the optimal solution result and its graphical solution.

Analysis of question 3: In order to satisfy the constraint that the total distance of the secondary tubes from each primary station is less than 40 km, we have solved the first problem on the basis of the first problem. In problem one we have partitioned the stations using the algorithm of minimum spanning tree and created a matrix using 0-1 planning to record the internal connectivity[4]. Considering that each secondary site may have branches connecting to other secondary sites, we use a recursive function call method with enumeration to determine the pipeline connectivity. When two sites are connected, the distance between the sites is added to the total distance of the partition, and the function itself is called to determine the connectivity to other sites. Each time the recursive call is made to calculate the total distance, the total distance needs to be judged, and if it has exceeded 40 km, the location of the point is recorded. When all the points that do not meet the requirements are obtained, the nearest station to the central water supply station is selected within each partition and upgraded to a primary water supply station, which is then brought into the model of the first question for solution, and a new partition can be obtained. The new partition is then brought into the recursive model of question three to verify the correctness, and if the final result meets the requirements, the shortest conforming track length is obtained.

III. MODEL BUILDING AND SOLVING

A. Concept

- Prim Algorithm：It can be called the "additive point method". Each iteration selects the point corresponding to the least costly edge and adds it to the minimum spanning tree[5]. The algorithm starts from a certain vertex s, and gradually grows to cover all vertices of the entire connected network.
- The set of all vertices of the graph is V, the initial set u={s}, v = {V- u} ={s}. Among the edges that can be formed by the two sets u and v, choose the edge with the least cost (u0, v0), add it to the minimum spanning tree, and merge v0 into the set u. Repeat the above steps until the minimum spanning tree has n-1 edges or n vertices.
- Spanning tree: A spanning tree of a connected graph is a connected subgraph that contains all n vertices in the graph, but only enough n-1 edges to form a tree. A spanning tree with n vertices has n-1 edges and only n-1 edges, and if another edge is added to the spanning tree, it must become a ring.
- Minimum spanning tree: Among all spanning trees of the connected network, the spanning tree with the smallest cost sum of all edges is called minimum spanning tree.

ISSN No:-2456-2165

B. Explanation of terms and symbols

Alphabet	Definition
L_1	Make the central water supply water station to the first level water supply station shortest route Planning
L_2	Make primary water supply water supply station to the second water supply station Shortest route Planning
A	Central water supply station
C	Regional point set consisting of a central water supply stations
D	Regional point set consisting of primary water supply stations
E	Regional point set consisting of secondary water supply stations
Dx	Elements of the set D
Ex	Elements of the set E

Table 1: Explanation of terms and symbols

C. Problem 1

a) Analysis and Modeling of Problem 1.
How should the water main be laid from the central water supply station A to minimize the total mileage of the pipeline? The layout is given graphically, and the total number of miles of Type I and Type II pipes is given. It is easy to see that there are three different water supply stations in the problem, the highest level central station A, the first level station, and the lowest level second level station, the A level station must be connected to at least one of the first level stations and the first level station must be connected to at least one of the second level stations. The shortest route is determined by laying the shortest total pipeline mileage. After analysis, this problem is solved by the prim algorithm of minimum spanning tree theory.

b) Solving of the model.
Since it must be connected to at least one primary water supply station, we can assume that A is already connected to a primary water supply station, so according to the rule that water supply stations of the same level can be connected to each other, when we continue to connect the third water supply station, we can equate the primary water supply station connected to the central water supply station A to a central water supply station A. Next, we analyze the primary water supply station and the secondary water supply station, the primary water supply station When a secondary water supply station is connected to a primary water supply station, the secondary water supply station can be "upgraded and equated" to a point equivalent to the primary water supply station. This is the basis for the circular enumeration we will do next. It is useful to equate the different levels of water supply stations as points in a planar right-angle coordinate system, with the coordinates of each point already given. The distance between stations is the Euclidean distance between the coordinates of two points.

By dividing the three levels of stations into three different point sets, we take a central station A as a point set with only one element C, all the points of the primary stations in point set D, and all the points of the secondary stations in point set E. According to our analysis above, when point 2 in point set D is connected to point 1 in C, we can connect the points in point set D to the points in point set E. According to our analysis above, when point 2 in the set of D is connected to point 1 in C, we can treat point 2 as a point in the set of C as well.

First, we consider the connection of the central water supply station, A, to at least one primary water supply station, and we require We need to lay the pipeline at the shortest distance, i.e., the shortest Euclidean distance, so the traversal compares the shortest distance and connects them, and then Then, these two connected points can be included in the point set C together, and the remaining 11 points are still included in the point set B. Then, by traversing, we find the point set D. Then, by traversal, we find the shortest distance from the remaining points in point set D to any point in point set C, and connect this shortest line segment The shortest line segment is connected, and one endpoint x of this line segment belongs to point set C, and the other endpoint y belongs to point set D. The endpoint y is also is included in the C-point set, and the remaining 10 points are still included in the D-point set. By iterating and comparing the distances, we We finally include all the points in the C-point set. This result is equivalent to the fact that we have completed the set of points between the central water supply station A and all the primary water supply stations. This result means that we have completed the connection between the central water supply station A and all the primary water supply stations, and the pipeline distance is the shortest.

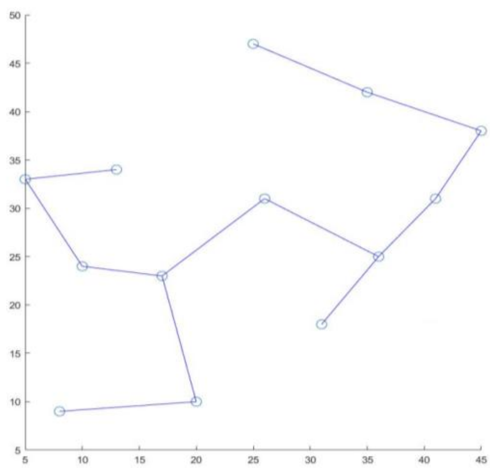

Fig. 2: The path of A to the primary water supply station obtained by the Prim algorithm

We solve for the shortest distance between the primary and secondary water supply stations. According to the prim algorithm, similar to the method above for the central water supply station A and the primary water supply station, when finding the shortest distance between the primary and secondary water supply stations, the points in the point set E, which represents the secondary water supply stations connected to the primary water supply station set D, can be considered as equivalent points to the primary water supply stations and planned into the point set D.

By enumeration, the distance between each point in E and each point in D is calculated one by one, and the point in point set E with the closest distance to point set D is found, which can be regarded as the point equivalent to the point in D as shown above, and taken from E to be included in D.

The nearest point in the "point set E consisting of points from the remaining secondary water supply stations" to the "point set D with the addition of one point" is found by enumeration again. By circular enumeration, the shortest path can be found, which is equivalent to the shortest path between the primary and secondary water supply stations that we want to find.

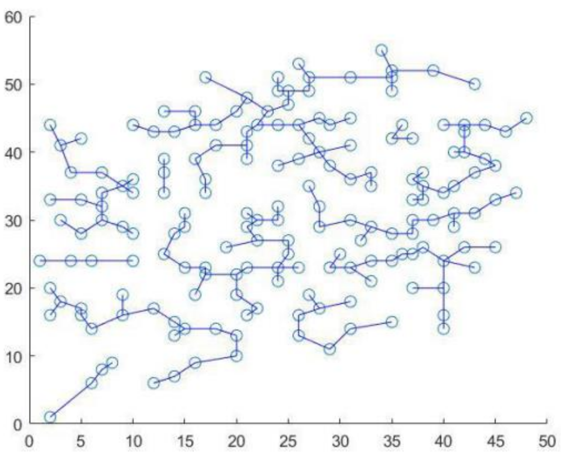

Fig. 3: Paths of primary and secondary water supply stations obtained by the Prim algorithm

D. Problem 2

a) Analysis and Modeling of Problem 2.

How to select two secondary water supply stations so that the reduced length of Type II pipeline is maximized? Using the solution of the first question as a basis, the results are analyzed by graphical calculations. Since the question states that type II pipeline is only used for connection between primary and secondary water supply stations or type 2 water supply stations, and we have already found the plan with the shortest total distance in the first question, we need to optimize on this basis. From the prim algorithm we used above can be inferred: each secondary water supply station is connected to the nearest secondary or primary water supply station connected to each other, so if a secondary water supply station is upgraded to a primary water supply station, the original path of the secondary water supply station connected to it will remain unchanged, but only need to increase its route to the central water supply station A or the nearest primary water supply station can be, from this out, the secondary pipeline is reduced by The longest two secondary pipelines can be found and adjusted to obtain the optimal solution. Use prim algorithm with shortest distance 0-1 planning model to solve the problem 2:

Let a n-point line network with starting point 1 and ending point n. Introduce the 0-1 variable X_{ij}, if arc (i, j) is on the shortest path, then X_{ij} =1. For any vertex except the starting point and ending point, if $\sum_{j=1}^{n} x_{ij}$ =1 states that one of all arcs penalized from i must be on the shortest path, i.e., the shortest route passes through that vertex, then one of the arcs from the other vertices to that vertex must be on the shortest path, so there is $\sum_{j=1}^{n} x_{ji}$ =1. for points 1 and n, then necessarily satisfies $\sum_{j=1}^{n} x_{1j}$ =1, $\sum_{j=1}^{n} x_{in}$ =1.The resulting 0-1 planning shortest path model is:

$$\sin z = \sum_{(i,j)\in E} w_{ij}X_{ij},$$

$$s.t. \begin{cases} \sum_{(i,j)\in E} X_{ij} = \sum_{(i,j)\in E} X_{ji} \\ \sum_{(i,j)\in E} X_{1j} = 1 \\ X_{1j} = 0 \text{ 或 } 1 \end{cases} \qquad (1)$$

b) Solving of the model.

The problem 2 requires the shortest class II pipeline, so we only consider the connection between the secondary station and the primary station. Since the second question is a modification of the first one, and we have already found the distance of each pipe in the first question, we can directly refer to the corresponding result of the first question in the second question.

First, after a simple round-robin comparison, we find the two longest type II pipes. By calculating and plotting, we find that one of the two points connected to each of these two pipes happens to be no longer connected to the other pipe after the elimination of the pipe. According to the theoretical basis of the first question, since each point is connected to the nearest point, this removed pipe is the smallest type II pipe that can be connected to that point. Then these two type-two pipes should be removed, and these two points are the points that need to be upgraded. We remove these two points from the set E and move them to the set D. Get rid of the corresponding connection lines to get the initial results follows, the two red solid points are the points that need to be upgraded for this problem:

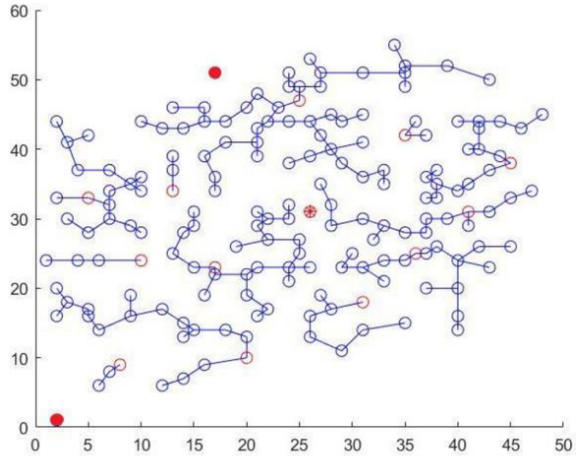

Fig. 4: Preliminary processing results

After the initial treatment, we just need to find the shortest type I pipe to make the connection. Since the Type I pipe only connects the primary water supply station to the primary water supply station or the primary water supply station to the central water supply station A, we again find the shortest Type I pipe that connects to the two points that need to be upgraded, based on the prim algorithm.

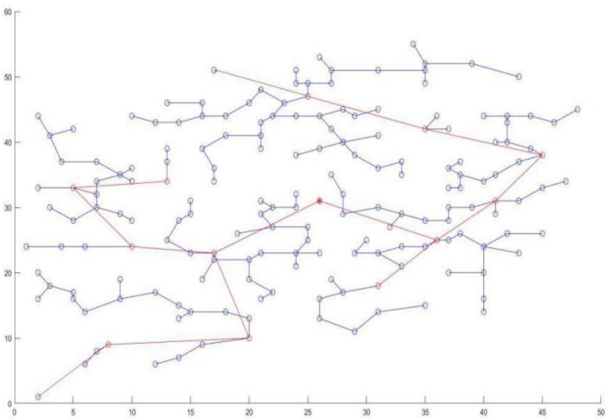
Fig. 5: Improved connection diagram

E. Problem 3

Minimum distance model based on 01 planning and using iterative algorithms: In order to satisfy the constraint that the total distance of the secondary tubes from each primary station is less than a certain number of kilometers as proposed in the problem, the stations are partitioned using a minimum spanning tree algorithm and a matrix is created using the 0-1 planning method to record the internal connectivity with the following operational model equation.

$$W(i,j)=\begin{cases}0, \text{Sites } i,j \text{ are connected without tubes}\\ 1, \text{ Sites } i,j \text{ are connected with tubes}\end{cases} \quad (2)$$

Considering that each secondary site may have branches that connect to other secondary sites, we use the method of recursive function calls and enumeration to determine the pipeline connectivity.

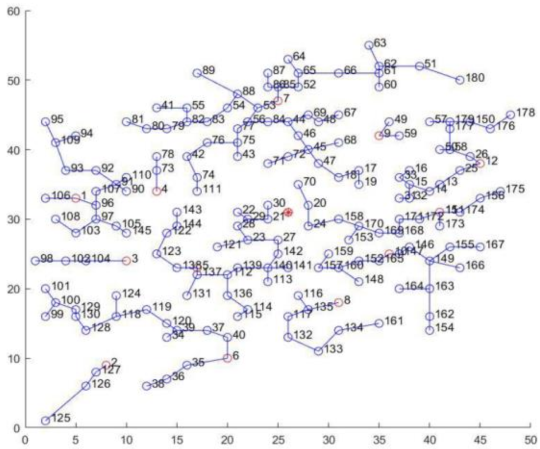
Fig. 6: Connection between points

Considering that each secondary site may have branches that connect to other secondary sites, we use the method of recursive function calls and enumeration to determine the pipeline connectivity. When two sites are connected, the distance between the site is added to the total distance of the partition and called itself to determine the connectivity with other sites. Each time the recursive call is made to calculate the total distance, the total distance needs to be judged, and if it has exceeded 40 km, the location of the point is recorded.

$$totallength = \sum length(i,j) \quad (3)$$

i, *j* are two points connected within a partition.

When all the points that do not meet the requirements are obtained, the nearest station to the central water supply station is selected within each sub-area and upgraded to a primary water supply station, which is then brought into the model of the first question for solution.

Let the reduced distance due to the new level *i* be $x(i)$, the increased level $y(i)$, and the length of the partition be $length(i)$.

$$\min \sum [y(i)-x(i)]$$

$$s.t.\ length(i) \le 40 \quad (4)$$

A new partition can be obtained. The new partition is then brought into the recursive model of Problem 3 for verification, and if the final result meets the requirement, the shortest conforming track length is obtained.

Let the distance between two points *i*, *j* be $x(i, j)$, the connectivity be $w(i, j)$, the total distance be totlength, and the distance of the partition to which *i* belongs be $length(i)$.

$$\min\ totallength \sum w(i,j) * x(i,j)$$

$$s.t.\ length(i) \le 40 \quad (4)$$

ISSN No:-2456-2165

F. Model Assumptions

- without considering external factors such as topography, assuming that all water supply pipelines can be perfectly straight, the length of the pipeline between two points can be considered as the Euclidean distance.
- The solution made according to the principle of shortest path and other requirements of the question can meet the requirements of water supply quality and equipment maintenance, etc.

IV. EXPERIMENTAL ANALYSIS

A. Problem 1

Adding the two shortest distances we found before, the result is the required shortest distance. As follows:

- Total mileage between level 1 and level 2: 403.6307.
- Total mileage between the terminus and the primary station: 120.9412.
- Total mileage of Type I and Type II pipelines: 524.5719.

The calculated final shortest path is shown in Fig 1.

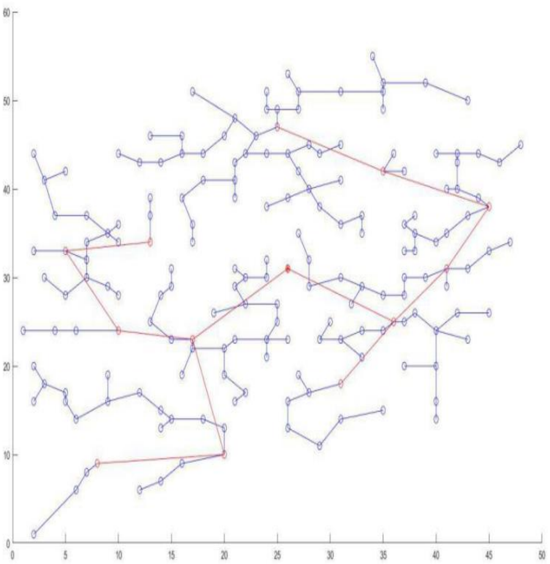

Fig. 7: Pipeline planning diagram

B. Problem 2

The coordinates of the secondary water supply stations that need to be upgraded are: P113(2,1), P77(21,43). The type II pipes removed are the connections between P113, P114 and P77, P76, reducing the total number of type II pipes Number of miles: 11.4031.

C. Problem 3

After bringing the result of the first question into the recursive function that calculates the total length of the partition, the following first-level water supply stations are obtained in the partition does not meet the requirements: V_5, V_6, V_7, V_{10}. The model is used to calculate the selected points, and P_{107}, P_{18}, P_{41} and P_{140} are selected as the secondary water supply stations that need to be upgraded, which are solved in the first question.

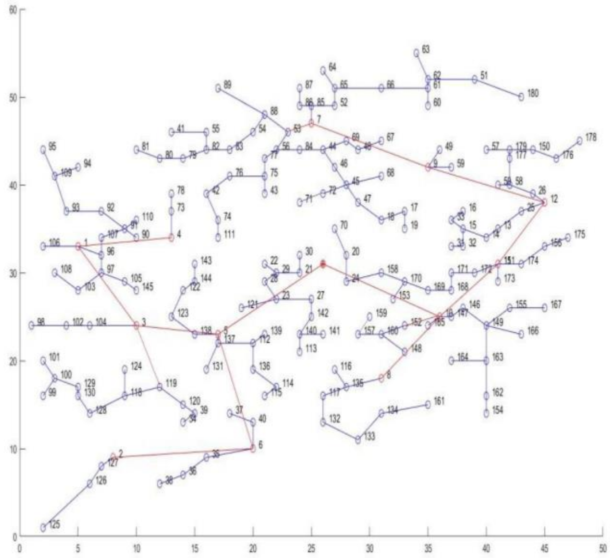

Fig. 8: Graph of experimental results of the new partition.
Calculation results:

The path of the primary pipe is 146.4259 km; the path of the secondary pipe is 393.4047 km; the total distance is 539.8306 km.

In the recursive model, it is found that the partition in P41 still does not satisfy the restriction, so P72 is selected as the primary water supply station again by the point selection model in the third question, and the following results are obtained by bringing the model into the first question again.

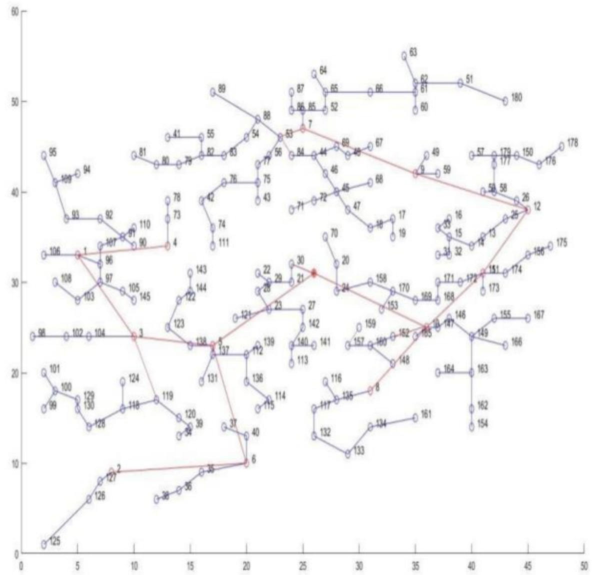

Fig. 9: Validation experimental results graph

D. Calculation results:

The path of the primary pipe is 148.6620 km; the path of the secondary pipe is 391.4047 km; the total distance is 540.0667 km. The problem is solved when the recursive model is validated. So at least 5 water supply stations need to be upgraded, with the serial numbers P107, P18, P41, P140, P72, at which time The shortest distance is 540.0667 km.

ISSN No:-2456-2165

V. CONCLUSION

In this paper, we abstract the water supply stations at all levels as points in the plane right-angle coordinate system, and the paths are connected as 0-1 plans, and the shortest paths are solved by the prim algorithm of minimum spanning tree theory. The graphical solution of the plan can be realized by matlab. The model can be applied to real-life water pipeline planning and design to save cost, as well as to further upgrade the water supply station and optimize the design again. However, more constraints should be applied to optimize the model considering the actual conditions.

REFERENCES

[1.] Gruber, Martin, and Günther Raidl. A new 0-1 ILP approach for the bounded diameter minimum spanning tree problem. na, 2005.
[2.] Gao, Wei, et al. "Modelling and application of fuzzy adaptive minimum spanning tree in tourism agglomeration area division." Knowledge-Based Systems 143 (2018): 317-326.
[3.] Held, Michael, and Richard M. Karp. "The traveling-salesman problem and minimum spanning trees." Operations Research 18.6 (1970): 1138-1162.
[4.] Xu, Zichang. "Optimal model of channel layout based on minimum spanning trees." Transactions of the Chinese Society of Agricultural Engineering 33.1 (2017): 124-130.
[5.] Katagiri, Hideki, Kosuke Kato, and Takashi Hasuike. "A random fuzzy minimum spanning tree problem through a possibility-based value at risk model." Expert Systems with Applications 39.12 (2012): 10639-10646.