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Fuzzy Set based Multi-Attribute Decision-Making, its Computing Implementation and Application

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Abstract:- This paper reflects results of research related to the analysis of models of multi-attribute decision-making in a fuzzy domain (within the so-called <X, R> models). These results can be used independently or within the framework of a general project of multicriteria decision making beneath conditions of uncertainty. The considered techniques for fuzzy preference modeling are directed at evaluating, comparing, choosing, prioritizing, and/or ordering alternatives. These techniques have served for developing a system for multi-attribute decision making MDMS2. It has been implemented in the C# programming language, utilizing the framework .NET. MDMS2 permits one to present preferences of decision makers in one of five preference formats. These formats as well as quantitative estimates are reduced to non-reciprocal fuzzy preference relations to provide homogeneous preference information for procedures of decision making. The paper results are of a general character and are illustrated by an example related to the problem of choosing an alternative of an energy source to be installed in an isolated system demonstrates the paper results.

Keywords:- *fuzzy preference modeling; preference formats; non-reciprocal fuzzy preference relations;<X, R> models; energy sources.*

I. INTRODUCTION

In the subject of systems analysis and operational research, some researchersconsent that, from the general perspective, the uncertainty of goals (related to a multi-criteria character of many decision problems) is harder to get overin view of the fact that "we simply do not know what we want" [1].As a matter offact, this kind of uncertainty are unable be successfullyapprehended solely in accordance with the application of academic models, likeoccasionally the only sources of knowledge are the persons that make decisions.

It is feasible to point out two maincategory of circumstances[1, 2] that needsthe employment of a multicriteria technique:

- The primarycategory is related to situations in which the consequences of the solution are unable be estimated using a unique criterion: those situations are related to the evaluation of models that includephysical andeconomicevidence (when the alternatives areunable be decreased to a comparableform) and besides because of the demand to contemplateevidence whose cost estimates are arduous or impractical;
- The second category is connected with problems that can be resolved in accordance with a unique or multiple criteria. Nevertheless, if the uncertainty of information restricts obtaining unique solutions, at this moment, it is viable to minimize those situations to multi-criteria decision-making by adding new supplementary criteria, including qualitative ones, in which its use considers the experience, intuition and knowledge of specialists involved.

Considering the above, it's then necessary to discriminate two forms of criteria: attributes andobjectives. Thus, multicriteria decision-making problems can be categorized into a pair of classes. [1, 2]:

- multi-objective decision-making;
- multi-attribute decision-making.

In general, multi-objective decision-making is recognized as the continuous type of multi-criteria decision-making and its main feature are that the decision maker (DM) needs to accomplishseveraltargets while these goals are non-equivalent andin discordance with each other. Multi-objective decision models include a vector of decision parameters (which can be continuous and/or discrete), objective functions that expresstargets and restrictions. The MD tries to maximize or minimize the objective functions.

Multi-attribute decision-making is associated with preference decision-making (ie, ordering, choice, prioritization, and/or comparison) over accessible options that are distinguished by numerous, often opposite, essence. The major uniqueness of multi-attribute decision problems is that there iscommonly a finitequantity of preset alternatives, which are related with an attribute range level. The decision must be made based on the attributes.

Analogous to this categorization, two groups of models can be built:<*X*, *F*> models (as multi-objective models) and <*X*, *R*> models (as multi-attribute models) [3]. The currentstudy is devoted to analyzing <*X*, *R*> models. However, it is necessary to indicate that the analysis of <*X*, *F*> and <*X*, *R*> models serve as a segment of anextensive plan for multicriteria decision making under conditions of uncertainty [1, 4, 5]. This planis related with a conception of the standardprocedure to taken into consideration the unpredictability of information [6] to multi-criteria problems, established on evaluating exceptional combinations of characteristic estimates applied within the choice criteria [1, 4]. Its essential aspect is to operate accessible quantitative information to the highest degree to diminish uncertainty decision regions.If the problem-solving scope related to quantitative information processing doesn't permit obtaining individual solutions, the general plan presupposes the usage of qualitative information (within <*X*, *R*> models [1, 3]) based on specialists intuition, knowledge and experience.

II. <*X*, *R*> MODELS AND THEIR ANALYSIS

Suppose we receive a group$X$ of alternatives from the decision unpredictability dimension and/or preset alternatives, which must be inspected by q criteria of a quantitative and/or qualitative essence. The problem of decision-making may be introduced as a couple<*X*, *R*>where $R = \{R_1, R_2, \ldots, R_p, \ldots, R_q\}$is a vector of fuzzy predilectionrelations [3, 7] which can be introduced as

$$R_p = [X \times X,\ \mu_{R_p}(X_k, X_l)],\ p = 1, 2, ..., q,\ X_k, X_l \in X \quad (1)$$

where $\mu_{R_p}(X_k, X_l)$is a membership function of the pth fuzzy preference relation.

In (1), R_p(also named a non-strict fuzzy predilectionrelation or fuzzy fragilepredilectionrelation in papers [3, 8]) is defined as a fuzzy set of all combination of the Cartesian product$X \times X$, such that the membership function$\mu_{R_p}(X_k, X_l)$ represents the degree to which $X_k$weakly dominatesX_l, i.e., the degree to which $X_k$is not worse than $X_l$for the pth criterion. In a somewhat loose sense, $\mu_{R_p}(X_k, X_l)$ also represents the degree of truth of the statement "$X_k$is preferred overX_l").

A common and compelling procedure to building fuzzy preference relations (particularly, non-reciprocal fuzzy preference relations [3, 9]) R_p is presented in [10]. To better understand this approach, let us consider$F(X_k)$ and $F(X_l)$as fuzzy sets considering estimates of attribute$F$ for alternatives X_k and X_l, respectively. Then, the quantity $\eta\{\mu[F(X_k)], \mu[F(X_l)]\}$is the degree of preference $\mu[F(X_k)] \succcurlyeq \mu[F(X_l)]$, while $\eta\{\mu[F(X_l)], \mu[F(X_k)]\}$is the level of preference$\mu[F(X_l)] \succcurlyeq \mu[F(X_k)]$. Then, the membership functions of the generalized predilection relations$\eta\{\mu[F(X_k)], \mu[F(X_l)]\}$ and $\eta\{\mu[F(X_l)], \mu[F(X_k)]\}$take [3, 11] the following forms:

$$\eta\{\mu[F(X_k)], \mu[F(X_l)]\} = \sup_{F(X_k), F(X_l) \in F} \min\{\mu[F(X_k)], \mu[F(X_l)], \mu_R[F(X_k), F(X_l)]\} \quad (2)$$

$$\eta\{\mu[F(X_l)], \mu[F(X_k)]\} = \sup_{F(X_k), F(X_l) \in F} \min\{\mu[F(X_l)], \mu[F(X_k)], \mu_R[F(X_l), F(X_k)]\} \quad (3)$$

where $\mu_R[F(X_k), F(X_l)]$and $\mu_R[F(X_l), F(X_k)]$are the membership functions of the reciprocal fuzzy predilection relations that, properly, indicate the principle of the predilections ofX_k over X_l and of X_l over X_k and (for illustration, "more attractive", "more flexible", etc.).

When Fcan be estimated on a numerical scale, if the principle of predilectionat the back of relation$R$is rational with the natural order ($\leq$) along the axis of estimated values of$F$, then (2) and (3), correspondingly, are shorten to the succeedingformulations:

$$\eta\{\mu[F(X_k)], \mu[F(X_l)]\} = \sup_{\substack{F(X_k), F(X_l) \in F \\ F(X_k) \leq F(X_l)}} \min\{\mu[F(X_k)], \mu[F(X_l)]\}$$

(4)

$$\eta\{\mu[F(X_l)], \mu[F(X_k)]\} = \sup_{\substack{F(X_k), F(X_l) \in F \\ F(X_l) \leq F(X_k)}} \min\{\mu[F(X_k)], \mu[F(X_l)]\}$$

(5)

If F has a maximization nature, the correspondence (4) and (5) have to be formulated for $F(X_k) \geq F(X_l)$ and $F(X_l) \geq F(X_k)$, correspondingly.

The correspondence (4) and (5) agree with some acknowledged fuzzy numericgrouping index[1].Illustrations of their application are specified in [1, 3]. The usage of (4) and (5) is well established. Take heed of that it is necessary to indicate that in plain language there are occasions where the fuzzy quantities$F(X_k)$ and $F(X_l)$ have trapezoidal membership functions [3] that are revealed in such aapproach that it is not viable to differentiate$X_k$and X_l[10]. For example, can be said that alternatives $X_1$and X_2 shown in Fig. 1 are two of a kind since

$$\eta\{\mu[F(X_1)], \mu[F(X_2)]\} = \eta\{\mu[F(X_2)], \mu[F(X_1)]\} = \alpha \quad (6)$$

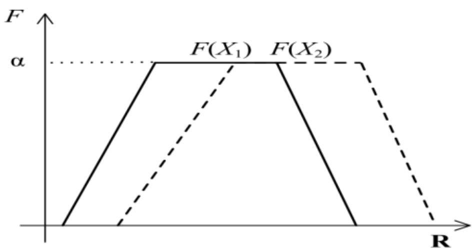

Fig. 1: Comparison of alternatives with trapezoidal membership functions

Let us return to construction matrices R_p.
The availableness of fuzzy or linguistic estimates of alternatives $F_p(X_k)$, $p = 1, 2, ..., q$, $X_k \in X$with the membership functions $\mu[F_p(X_k)]$, $p = 1, 2, ..., q$, $X_k \in X$allow one, using (3) and (4), to buildR_p, $p = 1, 2, \ldots, q$bearing on the following relationships:

$$\mu_{R_p}(X_k, X_l) = \sup_{\substack{X_k, X_l \in X \\ F_p(X_k) \leq F_p(X_l)}} \min\{\mu[F_p(X_k)], \mu[F_p(X_l)]\} \quad (7)$$

$$\mu_{R_p}(X_k, X_l) = \sup_{\substack{X_k, X_l \in X \\ F_p(X_l) \leq F_p(X_k)}} \min\{\mu[F_p(X_k)], \mu[F_p(X_l)]\} \quad (8)$$

Another method to creating matrices R_p on the report of preference functions is assumedin [12]. Our latest developments on creating fuzzy preference relations are delivered in [13].If estimates $F_p(X_k)$,p=1,2,...,q,$X_k \in X$ are stipulated on a unit value scale, these solution allow to acquire$\mu_{R_p}(X_k, X_l)$,p=1,2,...,q,$X_k, X_l \in X$as follows:

$$\mu_{R_p}(X_k, X_l) = 1 - \delta_p(X_l, X_k)$$
(9)

where $\delta_p(X_l, X_k)$correlate to the quantity of all positive discrepancy in the middle of the unfavorable outcomes of $F_p(X_l)$ and the favorable outcomes of $F_p(X_k)$.

The solution of [13] are acceptable for handle with both fuzzy and crisp appraisal on the equal state while embracing the predilectionconjecture on interval scale.

Fuzzy preference relations are not a single form of preference illustration. For example, the authors of [14] indicate eight formats that can be used to put in place preferences among studied alternatives. Among these formats, one can distinguish non-reciprocal fuzzy preference relations discussed above, additive reciprocal fuzzy preference relations, ordering of alternatives, utility values, fuzzy estimates, and multiplicative preference relations [1, 3], which cover all practical situations of preference elicitation. Besides, in many cases, deterministic information should also be taken into account. Although the possibility to use diverse preference formats provides psychological comfort for experts, it creates a need to convert all utilized formats to a single one which can be refined and studied. Taken into consideration that the benefits and coherence of employing non-reciprocal fuzzy preference relations for purpose, the outcome of [1, 3, 9] allow using presumed transformation functions to adapt different preference formats to non-reciprocal fuzzy preference relations, providing congruent preference information for decision making procedures.

Considering the situation of establishing an exclusive fuzzy non-strict preference relation R. It may be prepared[11] to build a fuzzy strict preference relation as

$$R^S = R \setminus R^{-1} \quad (10)$$

where R^{-1} is the inverse relation.

The membership function corresponding to (10) is:

$$\mu_R^S(X_k, X_l) = \max\{\mu_R(X_k, X_l) - \mu_R(X_l, X_k), 0\} \quad (11)$$

It can be used as the premise for the decision operation introduced in [11]. Its attributes in addition to the questions of its axiomatic characterization are debated, for example, in [15].

Using (11) allow to build a set of non-dominated alternatives with the membership function.

$$\mu_R^{ND}(X_k) = \inf_{X_l \in X}[1 - \mu_R^S(X_l, X_k)] = 1 - \sup_{X_l \in X}\mu_R^S(X_l, X_k) \quad (12)$$

which allows the evaluation of the magnitude of non-dominance of all alternative X_k. Whereas it is instinctive to elect alternatives providing the powerful magnitude of non-dominance, choose alternatives X^{ND}can be chosen as

$$X^{ND} = \{X_k^{ND} | X_k^{ND} \in X, \mu_R^{ND}(X_k^{ND}) = \sup_{X_k \in X}\mu_R^{ND}(X_k)\} \quad (13)$$

The expressions (11)-(13) are applicable to resolve problems of choice along with different situations, associated with the prioritization, comparison, evaluation and/or allocation of alternatives with an exclusive criterion. These formulation scan also be used when R is a vector of fuzzy preference relations, beneath different methods to the multi-attribute analysis.

In particular, when R is a vector of fuzzy preference relations, the formulations (11)-(13) can serve as a support for the **first technique** for multi-attribute decision-making in a fuzzy approach if we take $R = \bigcap_{p=1}^{q} R_p$, i.e.,

$$\mu_R(X_k, X_l) = \min_{1 \le p \le q}\mu_{R_p}(X_k, X_l), \; X_k, X_l \in X \quad (14)$$

When applying the intersection (14), the set X^{ND} fulfills the function of a Pareto set [11]. Its contraction is viable based on adapting the importance of $R_p, p = 1, 2, ..., q$using the following convolution (of the combination of mono-objective fuzzy preference relations) as follows:

$$\mu_T(X_k, X_l) = \sum_{p=1}^{q}\lambda_p\,\mu_{R_p}(X_k, X_l), \; X_k, X_l \in X \quad (15)$$

Where$\lambda_p \ge 0, p = 1, 2, ..., q$are weights (importance factors) for the corresponding criteria normalized as

$$\sum_{p=1}^{q}\lambda_p = 1 \quad (16)$$

The elucidation of $\mu_T(X_k, X_l), X_k, X_l \in X$ allows to achieve the membership function $\mu_T^{ND}(X_k)$of the set of non-dominated alternatives as stated in a formulation analogous to (12). The intersection of $\mu_R^{ND}(X_k)$and $\mu_T^{ND}(X_k)$defined as

$$\mu^{ND}(X_k) = \min\{\mu_R^{ND}(X_k), \mu_T^{ND}(X_k)\}, \; X_k \in X \quad (17)$$

provides us with

$$X^{ND} = \{X_k^{ND} | X_k^{ND} \in X, \mu^{ND}(X_k^{ND}) = \sup_{X_k \in X}\mu^{ND}(X_k)\} \quad (18)$$

The expressions (12) and (13) can additionallyfunction as support for creating the **second technique**, which is of a lexicographic aspect. It is correlated with one step at a timeinstitution of criteria for set a side-by-side alternatives. The technique allows us to create an order$X^1, X^2, ..., X^q$so that $X \supseteq X^1 \supseteq X^2 \supseteq ... \supseteq X^q$with the use of the following expressions:

$$\mu_{R_p}^{ND}(X_k) = \inf_{X_l \in X^{p-1}}[1 - \mu_{R_p}^S(X_l, X_k)] = 1 - \sup_{X_l \in X^{p-1}}\mu_{R_p}^S(X_l, X_k), \; p = 1, 2, ..., q \quad (19)$$

$$X^p = \{X_k^{ND,p} | X_k^{ND,p} \in X^{p-1}, \mu_{R_p}^{ND}(X_k^{ND,p}) = \sup_{X_l \in X^{p-1}}\mu_{R_p}^{ND}(X_k)\} \quad (20)$$

It must be recognized that if R_p is transitive [3, 10], we can ignore the pairwise correlationof alternatives at the pth step. In this case, the correlation can be done on a

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sequentialsource (the direct use of (7) and (8)) with memorizing the best alternatives.

Finally, the **third technique** implemented within MDMS2 is analogous with the following application. The use of (12) represented in the form

$$\mu_{R_p}^{ND}(X_k)=1-\sup_{X_l \in X} \mu_{R_p}^{S}(X_l,X_k),\ p = 1, 2, ..., q \qquad (21)$$

Allow us to create the membership functions of the set of non-dominated alternatives for each fuzzy preference relation.

The membership functions $\mu_{R_p}^{ND}(X_k)$, $p = 1, 2,...,q$ perform similar to membership functions substituting objective functions $F_p(X)$, $p = 1,2,...,q$in resolving traditional multi-objective problems [1, 3] conforming to modifying the Bellman-Zadeh proposition to decision making in a fuzzy conditions [16]. For that reason, it is feasible to establish

$$\mu^{ND}(X_k)=\min_{1\le p\le q}\mu_{R_p}^{ND}(X_k) \qquad (22)$$

to obtain X^{ND}.

If required to discriminatethe emphasis of different preference relations, it is workable to convert (22) to

$$\mu^{ND}(X_k)=\min_{1\le p\le q}\,[\mu_{R_p}^{ND}(X_k)]^{\lambda_p} \qquad (23)$$

The applicationof (23) makes no attempt todemand the normalization of λ_p,p=1,...,$q$in the way similar to (16).

III. COMPUTING IMPLEMENTATION

The MDMS2 has been developed in the C# programming language and is executed in the graphical environment of the Microsoft Windows Operating System. In this section, we list several typical windows that appear in the process of multi-attribute decision making.

The window MDMS2 (see Figure 2) is divided in tabs. The use of tab "Descrição do problema" permits one to select the technique to be used to solve the problem, to indicate the numbers of alternatives and criteria, and to show the solution results.

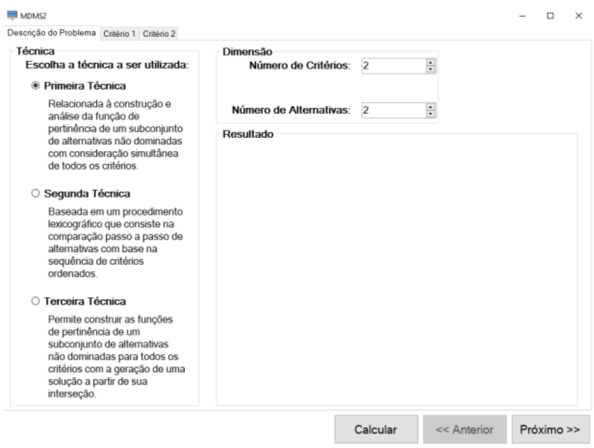

Fig. 2: MDMS2 window and "Descrição do problema" tab

After the number of criteria is set, tabs are created: each criterion taken into account corresponds to a separate tab. Each "Critério N" tab is designed to choose the preference format (non-reciprocal fuzzy preference relations,utility values,classification of alternatives, fuzzy appraisal or multiplicative preference relations) for the criteria and also to load information on each alternative.

IV. CHOICE OF AN ALTERNATIVE OF AN ENERGY SOURCE

As an clarifying example, below we appraised the solution of the problem of choosing an alternative of an energy source to be installed in an isolated system.

A mining site is located in an isolated location remote from the interconnected power system. The mine is to have a total demand of 20 MW of assured energy.

To meet this demand, a study is to be carried out on the choice of one of the following energy sources: diesel generation source (Alternative 1), wind energy source (Alternative 2, and solar energy source (Alternative 3).

In this choice, the following criteria of a quantitative character are to be taken into account:

- Capacity factor;
- Levelized Cost of energy;
- Deployment Time;
- Space Requirement;
- Plant Lifetime
- Greenhouse Gases Emissions

Besides the following criteria of a quantitative character are to be considered:

- Environmental Risk
- Corporate Image Risk
- Technological Maturity

Attributes		Diesel Generation	Wind Generation	Solar Generation
Installed Power	kW	23.000,00	42.600,00	69.000,00
Capacity Factor	%	87%	47%	29%
Assured Energy	kW	20.010,00	20.022,00	20.010,00
Levelized Cost of Energy	$/kW	37,00	36,93	0,08
Deployment Time	Months	24,00	30,00	22,00
Space Requirement	m2/kW	4,00	43,00	23,00
Lifespan	Years	15,00	30,00	25,00
Greenhouse Gas Emissions	tCO2/MWh	0,76	-	-
Environmental Risk		High	Low	High
Corporate Image Risk		High	Low	Low
Technological Maturity		High	High	Medium

Table 1: Assessing alternatives according to the criteria under consideration

Among the considered criteria, Levelized Cost of Energy, Deployment Time, Space Requirement, Greenhouse Gases Emission, Environmental Risk, and Corporate Image Risk are related to minimization. On the other hand, the criteria of the Capacity Factor, Plant Lifetime, and Technological Maturity are related to maximization.

Thus, the analysis is associated with 9 criteria and 3 alternatives. This information is to be included in the "Descrição do Problema" (Figure 2).

Then, for each criterion, the corresponding preference format should be chosen. For the criteria of a quantitative character, numerical scales (as the utility values format) are utilized. At the same time, for the criteria of a qualitative character, the fuzzy estimates are used.

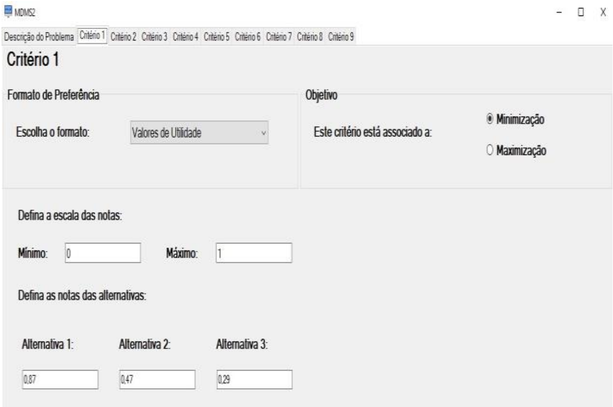

Fig. 3: Estimates for the first criterion (Capacity Factor)

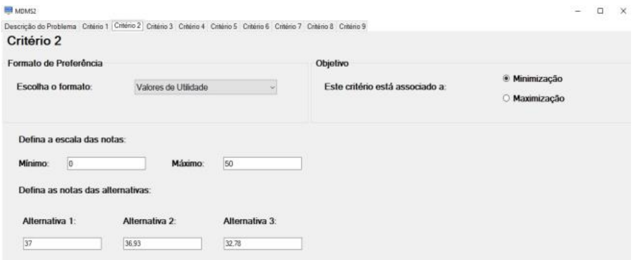

Fig. 4: Estimates for the second criterion (Levelized Cost of Energy)

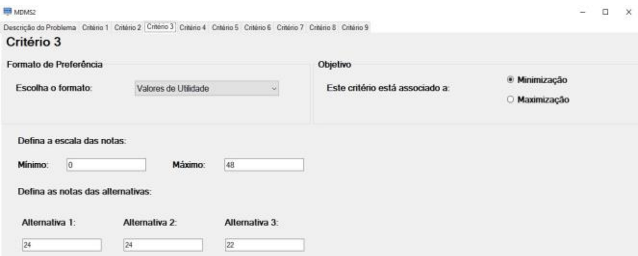

Fig. 5: Estimates for the third criterion (Deployment Time)

Fig. 6: Estimates for the fourth criterion (Space Requirement)

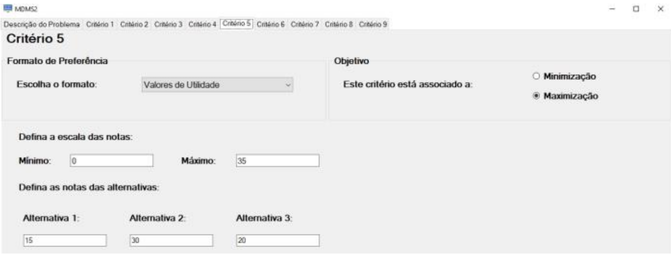

Fig. 7: Estimates for the fifth criterion

(Plant Lifetime)

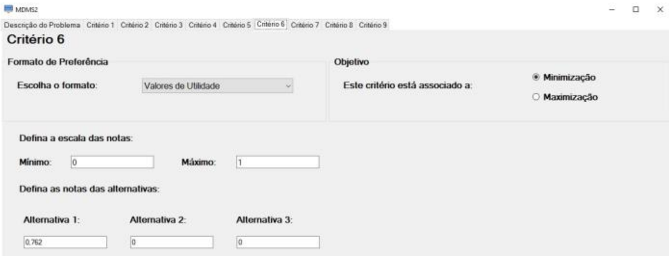

Fig. 8: Estimates for the sixth criterion

(Greenhouse Gases Emissions)

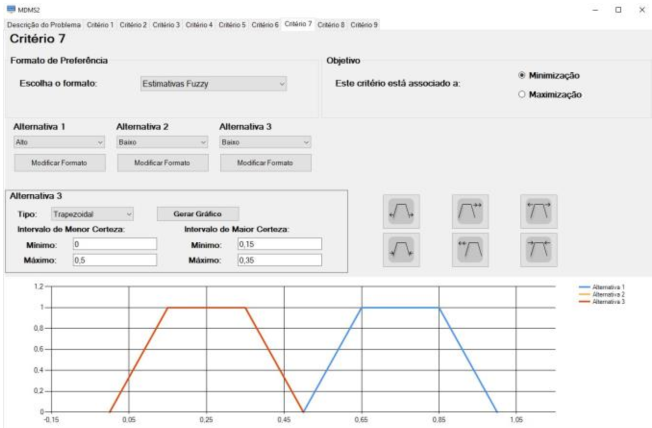

Fig. 9: Estimates for the seventh criterion (Environmental Risk)

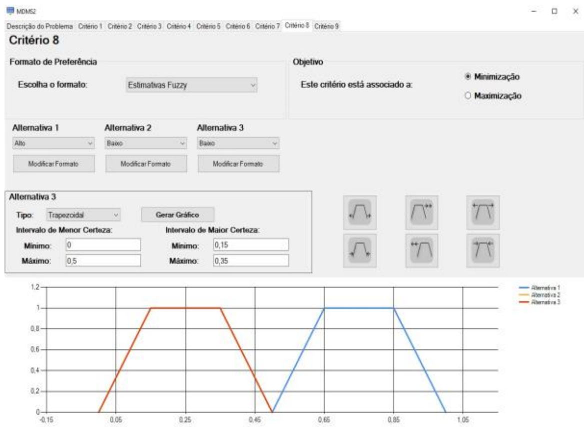

Fig. 10: Estimates for the eighth criterion (Corporate Image Risk)

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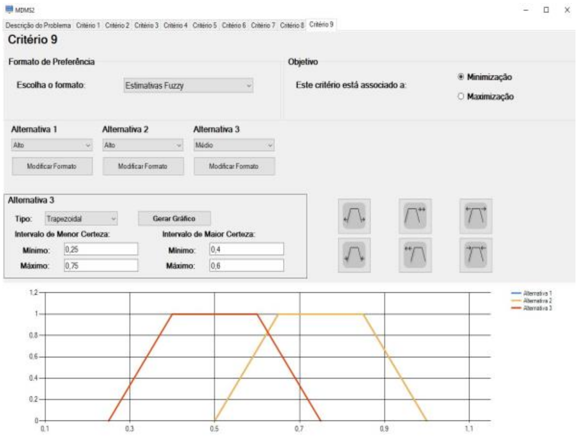

Fig. 11: Estimates for the ninth criterion (Technological Maturity)

The application of the results of [1] to the estimates given in Figures 3-8 allow us to create the corresponding non-reciprocal fuzzy preference relations represented in Figure 12. At the same time, the use of the results of [3, 10] to the estimates of Figures 9-11 provides the non-reciprocal fuzzy preference relations represented in Figure 12.

Resultado

Relação de Preferência do 1º Critério:

[1,000 1,000 1,000]
[0,600 1,000 1,000]
[0,420 0,820 1,000]

Relação de Preferência do 4º Critério:

[1,000 1,000 1,000]
[0,220 1,000 0,600]
[0,620 1,000 1,000]

Relação de Preferência do 7º Critério:

[1,000 0,000 0,000]
[1,000 1,000 1,000]
[1,000 1,000 1,000]

Relação de Preferência do 2º Critério:

[1,000 0,999 0,916]
[1,000 1,000 0,917]
[1,000 1,000 1,000]

Relação de Preferência do 5º Critério:

[1,000 0,571 0,857]
[1,000 1,000 1,000]
[1,000 0,714 1,000]

Relação de Preferência do 8º Critério:

[1,000 0,000 0,000]
[1,000 1,000 1,000]
[1,000 1,000 1,000]

Relação de Preferência do 3º Critério:

[1,000 1,000 0,958]
[1,000 1,000 0,958]
[1,000 1,000 1,000]

Relação de Preferência do 6º Critério:

[1,000 0,238 0,238]
[1,000 1,000 1,000]
[1,000 1,000 1,000]

Relação de Preferência do 9º Critério:

[1,000 1,000 0,833]
[1,000 1,000 0,833]
[1,000 1,000 1,000]

Fig. 12: Non-reciprocal fuzzy preference relations

The results of implementing the first technique to the non-reciprocal fuzzy preference correlations represented in Figure12 (execution of (14), (11), and (12), respectively) are given in Figure 13. These results generate the Alternative 3 as the problem solution.

Interseção entre as matrizes de preferência fuzzy:

[1,000 0,000 0,000]
[0,220 1,000 0,600]
[0,420 0,714 1,000]

Relação de Preferência Fuzzy Estrita da Interseção:

[0,000 0,000 0,000]
[0,220 0,000 0,000]
[0,420 0,114 0,000]

Conjunto Não Dominado das Alternativas:

[0,580 0,886 1,000]

Alternativa Escolhida: 3

Fig. 13: The problem solution on the basis of the first technique

The implementation of the second technique presupposes the ordering of the criteria. Considering the employment of the second technique with the following ordering of the criteria: 6, 7, 8, 9, 2, 3, 4, 5, and 1.

The process of decision making is reflected by Figure 14. In particular, the processing of the criterion "Greenhouse Gases Emissions", applying (11), (19), and (20), permits us to cut the Alternative 1. The application of the criterion "Environmental Risk" to analyze the Alternatives 2 and 3 does not permit us to reduce the decision uncertainty region (this criterion has no resolution power to distinguish the analyzed alternatives). Then, go to the analysis of the Alternative 2 and 3, applying the criterion "Corporate Image Risk". The use of (11), (1), and (20) generates the Alternative 2 as the problem result.

Finally, applying the third technique. The process of its utilization is reflected by Figure 15. The use of (11) and (21) permits us to establish the membership functions of the set of non-dominated alternatives for all considered criterion. Their intersection in accordance with (22) provides the following solution: Alternative 3.

Thus, the first and third techniques indicate the Alternative 3 as the problem solution. At the same time, the second technique indicates the Alternative 2 as the problem solution. With all things considered, worth bearing in mind that the fact of the circumstance to achievedivergent solutions based ondistinctmethods is natural, and the selection of the method is a privilege of the decision maker.

Relação de Preferência Fuzzy Estrita adicionando o 6º Critério:

[0,000 0,000 0,000]
[0,762 0,000 0,000]
[0,762 0,000 0,000]

Conjunto Não Dominado das Alternativas adicionando o 6º Critério:

[0,238 1,000 1,000]

Relação de Preferência Fuzzy Estrita adicionando o 7º Critério:

[0,000 0,000]
[0,000 0,000]

Conjunto Não Dominado das Alternativas adicionando o 7º Critério:

[1,000 1,000]

Relação de Preferência Fuzzy Estrita adicionando o 8º Critério:

[0,000 0,000]
[1,000 0,000]

Conjunto Não Dominado das Alternativas adicionando o 8º Critério:

[0,000 1,000]

Alternativa Escolhida: 2

Fig. 14: The problem solution on the basis of the second technique

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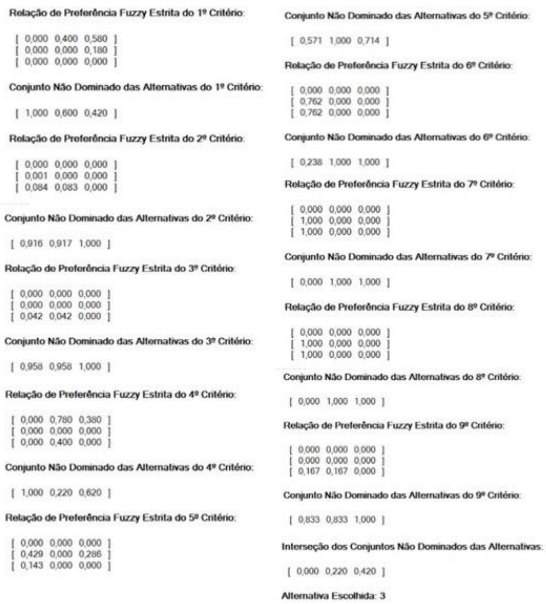

Relação de Preferência Fuzzy Estrita do 1º Critério:

[0,000 0,400 0,580]
[0,000 0,000 0,180]
[0,000 0,000 0,000]

Conjunto Não Dominado das Alternativas do 1º Critério:

[1,000 0,600 0,420]

Relação de Preferência Fuzzy Estrita do 2º Critério:

[0,000 0,000 0,000]
[0,001 0,000 0,000]
[0,084 0,083 0,000]

Conjunto Não Dominado das Alternativas do 2º Critério:

[0,916 0,917 1,000]

Relação de Preferência Fuzzy Estrita do 3º Critério:

[0,000 0,000 0,000]
[0,000 0,000 0,000]
[0,042 0,042 0,000]

Conjunto Não Dominado das Alternativas do 3º Critério:

[0,958 0,958 1,000]

Relação de Preferência Fuzzy Estrita do 4º Critério:

[0,000 0,780 0,380]
[0,000 0,000 0,000]
[0,000 0,400 0,000]

Conjunto Não Dominado das Alternativas do 4º Critério:

[1,000 0,220 0,620]

Relação de Preferência Fuzzy Estrita do 5º Critério:

[0,000 0,000 0,000]
[0,429 0,000 0,286]
[0,143 0,000 0,000]

Conjunto Não Dominado das Alternativas do 5º Critério:

[0,571 1,000 0,714]

Relação de Preferência Fuzzy Estrita do 6º Critério:

[0,000 0,000 0,000]
[0,762 0,000 0,000]
[0,762 0,000 0,000]

Conjunto Não Dominado das Alternativas do 6º Critério:

[0,238 1,000 1,000]

Relação de Preferência Fuzzy Estrita do 7º Critério:

[0,000 0,000 0,000]
[1,000 0,000 0,000]
[1,000 0,000 0,000]

Conjunto Não Dominado das Alternativas do 7º Critério:

[0,000 1,000 1,000]

Relação de Preferência Fuzzy Estrita do 8º Critério:

[0,000 0,000 0,000]
[1,000 0,000 0,000]
[1,000 0,000 0,000]

Conjunto Não Dominado das Alternativas do 8º Critério:

[0,000 1,000 1,000]

Relação de Preferência Fuzzy Estrita do 9º Critério:

[0,000 0,000 0,000]
[0,000 0,000 0,000]
[0,167 0,167 0,000]

Conjunto Não Dominado das Alternativas do 9º Critério:

[0,833 0,833 1,000]

Interseção dos Conjuntos Não Dominados das Alternativas:

[0,000 0,220 0,420]

Alternativa Escolhida: 3

Fig. 15: The problem solution on the basis of the third technique

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