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Holography and Dark Energy from Action Principle Literature Review and Generalizations

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Abstract:- In this paper we have briefly reviewed how holographic principle, which led the AdS/CFT correspondence, can be used to formulate an action principle for dark energy furthermore, a generalized action using f(R) gravity is also derived.

Keywords:- *Holographic Principle, Dark Energy, Action Principle.*

I. HOLOGRAPHIC PRINCIPLE

The exploration of black hole thermodynamics has led to a remarkable observation about the nature of our universe. Among them is the holographic principle (HP) first proposed by Hooft [1] which states that all information trapped inside a volume of a space can be represented as a hologram, which corresponds to the information being in-coded on a lower-dimensional boundary of that space. This means we can study the information trapped inside a volume in a space by studying the information trapped on its boundary. The first evidence to support this claim is the Bekenstein bound $s \geq \frac{2\pi\kappa RE}{\hbar c}$ where S is the entropy of the system, K is the Boltzmann's constant and R is the radius [2]. As a direct consequence of this bound, one can conclude that the entropy of a black hole is proportional to the area of a black hole. A more successful realization of the HP is the Maldacena duality famously known as the AdS/CFT correspondence [3] which states that large N limits of certain conformal field theories in d dimensions can be described in terms of supergravity on the product of $d+1$ -dimensional AdS space with a compact manifold.

Many attempts in the past two decades are made to incorporate the observable universe with the HP and one of them is the Holographic Dark Energy (HDE) which aims to solve the problem of Dark Energy (DE) using HP [4]. In this short research paper, we are going to focus on a particular approach to HDE from the action principle. In section 2 we provide a review of some progress made in HDE using the action principle and then in section 3 we briefly present the modified HDE action and try address the future scope with this approach followed by a conclusion in section 4.

II. HOLOGRAPHIC DARK ENERGY FROM ACTION PRINCIPLE

The first HDE model from action principle was introduced by Miao Li et al [5] to the solve the causality problem and circular logic problem of HDE. They did so by introducing a Lagrangian multiplier λ into the action. For Robertson-Walker metric

$$ds^2 = -N^2(t)dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \quad (1)$$

consider the following action

$$S = \frac{1}{16\pi}\int dt\left\{\sqrt{g}\left(R - \frac{2c}{a^2(t)L^2(t)}\right) - \lambda(t)\left(\dot{L}(t) + \frac{N(t)}{a(t)}\right)\right\} + S_m \quad (2)$$

Here L has the dimensions of length and is a component of constraint equation enforced by λ, i.e. $\dot{L} = -1/a$. Thus, as a consequence we do not need to assume the future event horizon as the infrared cut-off. So, the present cut-off depends on the future event horizon but and the future event horizon is completely determined by present cut-off. From action (2), we get the following Friedman's equations:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{c}{3a^2L^2} + \frac{\lambda}{6a^4} + \frac{8\pi}{3}\rho_M, \quad (3)$$

$$\frac{2\ddot{a} + \dot{a}^2 + k}{a^2} = \frac{c}{3a^2L^2} - \frac{\lambda}{6a^4} - 8\pi p_M \quad (4)$$

With

$$\dot{L} = -\frac{1}{a}, \quad L = \int_t^{\infty} \frac{dt'}{a(t')} + L(\infty), \qquad (5)$$

and

$$\dot{\lambda} = -\frac{4ac}{L^3}, \quad \lambda = -\int_0^t dt' \frac{4a(t')c}{L^3(t')} + \lambda(0). \quad (6)$$

Analysing the above equations, we see that the future event horizon is aL and $\lambda(0)$ behaves as radiation and thus can be a candidate for DE. For the case $c, k, \rho_m \propto a^{-3}$ and $\rho_r \propto a^{-4}$ there exists a de-Sitter solution with $a = e^{Ht}, L = e^{-Ht}/H, \lambda = -6H^2 e^{-4Ht}$ and $c = 6, k = 0, \rho_M = p_M = 0$. Here M denotes the matter field, m denotes matter without pressure and r denotes radiation. For large times, we see that the state parameter of DE varies as $w = \left(-3 + 2c + \sqrt{9+12c}\right) / 3\left(-3 - 2c + \sqrt{9+12c}\right)$.

Thus, $w = -1, -1 < w < -1/3, w < -1$ for $c = 6, c > 6, c < 6$ respectively. And thus, DE is dominant with large times. For an exact solution with matter $\left(\rho_m = 3b/4\pi a^4, P_m = 0\right)$ and radiation $\left(\rho_r = 3d/4\pi a^4, P_r = 1/3\rho_r\right)$, we get $\rho_M = \rho_m + \rho_r, P_M = d/8\pi a^4$. Here b and d are parameters which will be related with scale factor a. Now redefine L as $L = -\eta$ where η is the conformal time. Then, for $c > 6$ we see that $-1 < w \to \left(-3 + 2c + \sqrt{9+12c}\right) / 3\left(-3 - 2c + \sqrt{9+12c}\right) < -1/3$ as $L \to 0$ and $w \to -1/3$ as $L \to L_0$. And for $0 < c < 6$ we see that $w \to \left(-3 + 2c + \sqrt{9+12c}\right) / 3\left(-3 - 2c + \sqrt{9+12c}\right) < -1$ as $L \to 0$ and $w \to -1/3$ as $L \to L_0$.

The above model can be generalized to set particle horizon and the Hubble horizon as the cut-off by modifying the action. But it is observed that they give non-accelerating solutions of universe [5].

Chunshan Lin in [6] constructed a general covariant local field theory for HDE using a similar action as (2) using Stueckelberg field and concluded that the low energy effective theory of the holographic dark energy is the massive gravity theory whose graviton has 3 polarisations, including one scalar mode and two tensor modes. Now, having gained enough information about HDE. Now we present a brief review of how we could construct a modified version of action (2)

III. A GENERALIZED MODEL FOR HDE

$f(R)$ theories have played a crucial role in describing the universe with dark sector and it is through these theories that we are able to explode many underlying structures of General Relativity (GR). If we modify action (2) by replacing R with $f(R)$ and solve for S, then we get following equations of motion for Robertson-Walker metric:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} - \frac{1}{3f'}\left\{\frac{f - Rf'}{2} - 3H\dot{R}f''\right\}$$

$$= \frac{c}{3a^2L^2} + \frac{\lambda}{6a^4} + \frac{8\pi}{3}\rho_M, \qquad (7)$$

$$\frac{2\ddot{a}a + \dot{a}^2 + k}{a^2} + \frac{1}{f'}\left\{2\left(\frac{\dot{a}}{a}\right)\dot{R}f'' + \ddot{R}f'' + \dot{R}^2 f''' - \frac{f - Rf'}{2}\right\}$$

$$= \frac{c}{3a^2L^2} - \frac{\lambda}{6a^4} - 8\pi\rho_M \qquad (8)$$

where the prime ' represents a derivative with respect to R and dot represents a derivative with respect to time. This can be thought as a modified version of the known theory of HDE from action principle. The general idea has been presented however we refrain in this report to assert anything about solving Eqn. (7) and (7) because that would involve specialize techniques. By inserting different values of $f(R)$, we could get new insights into the model.

IV. CONCLUSION

In this paper, we studied how an effective and working action principle can be constructed from the holographic principle. Furthermore, a generalized action for dark energy from holographic principle is derived in the framework of $f(R)$ gravity.

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