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Optimization Approach for Common Steel Open Section under Applying Direct Torque at Warpless End of a Cantilever Beam

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Abstract:- The main objective of this manuscript is to offer an optimization approach for the common thin-walled I, Z and channel-section beams subjected to direct torsion. The displacement limitations for the incident angle of twist are only considered: Using the Lagrange multiplier method, the mass of the supposed element is assumed to be the objective function, the deduced equations have solutions can represent the optimal values of the ratios between dimensions of the formed parts of the considered sections with respect to the global length of element.

Keywords:- *Direct torsion, Optimization Approach, Optimal dimensions, cold formed sections.*

I. INTRODUCTION

Many reports have been carried out for the optimization purposes treating the situations where geometrical configurations of structures are specified and only the formulated dimensions of parts, such as masses in order to attain the minimum structural weight or cost[1], [2].Many approaches have been followed for the localized of the most minimum point for the optimization issues[3]Common cross-sections, especially in steel industries are the I, Z and channel sections. A series of exertions appear where the optimization parameters of various common cross-sections, such as I-section [4], channel-section [5]or Z-section beams [6]have been determined by Lagrange's multipliers technique. The preliminary concern through the formulation of the basic mathematical relations are the assumptions of the thin-walled beam theory, on one side [7]and the rudimentary assumptions of the optimum design on the other. Thin-walled steel sections in particular with the open nature, are commonly susceptible to large warping stresses and excessive twist when applying torque. Therefore, a familiar practice was presented to vanish twisting moments in steel assemblies involving steel open sections whenever it is possible. However, in a number of practical applications, twisting cannot be avoided and the designer is compelled to count on the torsional resistance of these members. The classical formulation for open thin-walled sections subjected to torsion was developed by Vlasov [7], the Vlasov formulation is based on two fundamental kinematic assumptions: (a) In-plane deformations of the section are negligible, and (b) shear strains along the section mid-surface are negligible.

A. Major Hypotheses

The formulation is limited to be involved when analysis of open section thin-walled beams under the effect of torsional moment.

In the considered case, cantilever beam has a length L under the effect of direct torque at its warpless end. The studied cross-sectional profile is indicated below inFig. 1). It is assumed to be formed from identical flanges` widths with considering unchanged thickness for all parts.

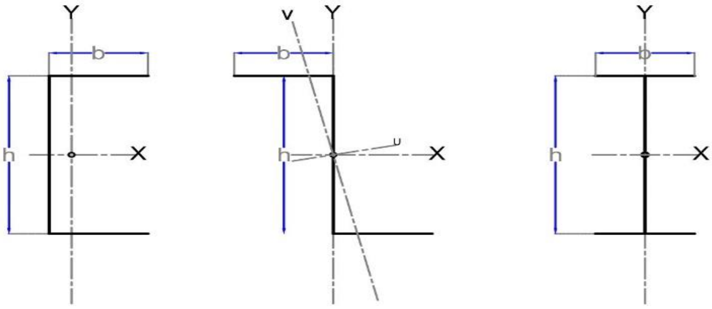

Fig. 1: Considered section in the optimization process

B. Lagrange principle

The most popular method to implement the required objective to find out the optimal dimensions of cross-sections. Lagrange method need an objective function, which can be supposed the total mass of the studied prismatic element ($\text{mass} = \text{mass}_{\min}$, $A = A_{\min}$). On the other hand, a constraint that govern our design process, would be the limitation of the incident angle of twist that causing the permitted deflection$\phi \leq \phi_{(\delta_{\max})}$.

C. Deflection constraint:

The deflection is limited for cantilever beams and set in most design codes of practice by L/180 for live load and L/90 for the combination of dead &live load. These values can be connected to the incident twisting angle corresponding to the applying torque as follows:

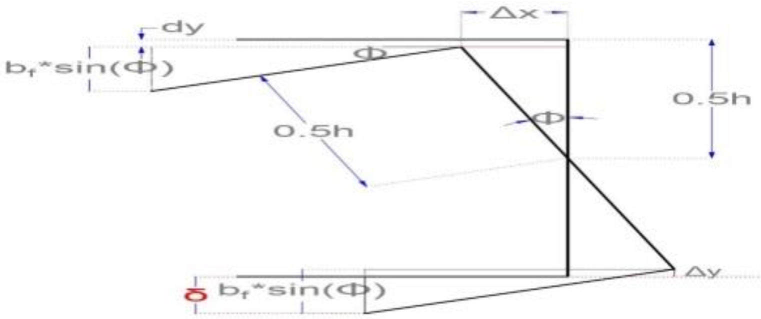

Fig. 2: Relation between the incident twisting angle and the corresponding deflection

$$\delta = b * \sin(\phi) - \Delta y$$
$$\Delta y = 0.5h(1 - \cos(\phi))$$
$$\delta = b * \sin(\phi) - 0.5h(1 - \cos(\phi)) \quad (1)$$

This relation can be written in the following form with considering a small value for $\phi(\sin(\phi) \cong \phi, \cos(\phi) \cong 1)$:
Therefore, the constraint for twisting angle can be noticed for $\phi_{all} = \frac{L}{n*b}$.

Where, n= 90, 180 as indicated above.

Using the induced equation in AISC representing the incident twisting angle (ϕ) W.R.T. the applying torque at a warpless end of a cantilever beam with using a specific section having individual properties of $(G, E, C_w, J, \lambda, L)$ as follows:

$$\phi = \left(1 - \left(\frac{2(\cos(\lambda L) - 1)}{\lambda L \sin(\lambda L)}\right)\right) * \frac{ML}{GJ} \quad (2)$$

Where:
E is the modulus of elasticity.
G is the shear modulus of elasticity, can be determined from $\frac{E}{2+2\vartheta}$.
ϑ is the poisson ratio = 0.3 for common steels.
C_w is the warping constant.
J is the torsional constant.
λ is the torsional parameter and is determined from $\sqrt{\frac{GJ}{C_w}} = 0.62\sqrt{\frac{J}{C_w}}$.

For C section:
As listed in AISC design guide for torsion[8], the determination of torsional constant, warping constant and the torsional parameter for the C-section could be as follows:

$$J = (2b + h) * \frac{t^3}{3}$$
$$C_w = \frac{tb^3h^2}{12}\frac{(6b + h)}{3b + 2h}$$
$$\lambda = \frac{31 * \left(\frac{t^2*(2*b + h)*(6*b + h)}{b^3*h^2*(3*b + 2*h)}\right)^{\frac{1}{2}}}{25}$$

D. Methodical implementation
The objective function $A = (2b + h)t$ to be minimized.
The constraint equation to be limited below the permitted deflection:

$$\left(1 - \left(\frac{2(\cos(\lambda L) - 1)}{\lambda L \sin(\lambda L)}\right)\right) * \frac{ML}{GJ} \leq \frac{L}{nb} \quad (3)$$

Therefore, with a Lagrange multiplier is denoted by Y to avoid the confliction with the torsional parameter, the general equation including the objective function with the constraint equation in the form of F(b,h,t,L,Y) would be as follows:

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$$t(2*b+h)+Y\left(\frac{L}{180b}+\frac{3LM\left(\frac{25\left(2\cosh\left(\frac{31*L*\left(\frac{t^2*(2*b+h)*(6*b+h)}{b^3*h^2*(3*b+2*h)}\right)^{\frac{1}{2}}}{25}\right)-2\right)}{31L*\sinh\left(\frac{31L*\left(\frac{t^2*(2*b+h)*(6*b+h)}{b^3*h^2*(3*b+2*h)}\right)^{\frac{1}{2}}}{25}\right)\left(\frac{t^2(2*b+h)*(6*b+h)}{b^3*h^2*(3*b+2*h)}\right)^{\frac{1}{2}}}-1\right)}{Gt^3(2*b+h)}\right)$$

To execute the optimization approach, it is a must to apply the Lagrange multiplier to a vector depends on each parameter concerning all dimension (b,h,t,L,Y) as indicated above.It is worth noting that the relationships deduced from these derivations will be complex enough to bring this issue to its face.When noticing those resulting equations, using the last two developed equations will give the prerogativeto to use the last equation arising from the last derivation $(\frac{\partial F}{\partial Y})$ would be used to devastate the subsequent equation $(\frac{\partial F}{\partial L})$ and the following can be extracted:

$$\frac{\partial F}{\partial Y}=\frac{L}{n*b}+\frac{3ML\left(\frac{25*\left(2*\cosh\left(\frac{31*L*\left(\frac{t^2*(2*b+h)*(6*b+h)}{b^3*h^2*(3*b+2*h)}\right)^{\frac{1}{2}}}{25}\right)-2\right)}{31*L*\sinh\left(\frac{31*L*\left(\frac{t^2*(2*b+h)*(6*b+h)}{b^3*h^2*(3*b+2*h)}\right)^{\frac{1}{2}}}{25}\right)*\left(\frac{t^2*(2*b+h)*(6*b+h)}{b^3*h^2*(3*b+2*h)}\right)^{\frac{1}{2}}}-1\right)}{G*t^3*(2*b+h)}=0\ldots\ldots(4)$$

Therefore:

$$\frac{\partial F}{\partial L}=\frac{25*\left(2*\cosh\left(\frac{31*L*\left(\frac{t^2*(2*b+h)*(6*b+h)}{b^3*h^2*(3*b+2*h)}\right)^{\frac{1}{2}}}{25}\right)-2\right)}{31*L*\sinh\left(\frac{31*L*\left(\frac{t^2*(2*b+h)*(6*b+h)}{b^3*h^2*(3*b+2*h)}\right)^{\frac{1}{2}}}{25}\right)*\left(\frac{t^2*(2*b+h)*(6*b+h)}{b^3*h^2*(3*b+2*h)}\right)^{\frac{1}{2}}}-1-1+\frac{\cosh\left(\frac{31*L*\left(\frac{t^2*(2*b+h)*(6*b+h)}{b^3*h^2*(3*b+2*h)}\right)^{\frac{1}{2}}}{25}\right)*\left(2*\cosh\left(\frac{31*L*\left(\frac{t^2*(2*b+h)*(6*b+h)}{b^3*h^2*(3*b+2*h)}\right)^{\frac{1}{2}}}{25}\right)-2\right)}{L\sinh^2\left(\frac{31*L*\left(\frac{t^2*(2*b+h)*(6*b+h)}{b^3*h^2*(3*b+2*h)}\right)^{\frac{1}{2}}}{25}\right)}=0\ldots\ldots(5)$$

Hereafter, the following relation can be concluded:

$$\frac{31*\left(\frac{t^2*(2*b+h)*(6*b+h)}{b^3*h^2*(3*b+2*h)}\right)^{\frac{1}{2}}}{25*\tanh\left(\frac{31*L*\left(\frac{t^2*(2*b+h)*(6*b+h)}{b^3*h^2*(3*b+2*h)}\right)^{\frac{1}{2}}}{25}\right)}=\frac{\left(1+\frac{G*t^3*(2*b+h)}{(n*b)*3*M}\right)}{\left(1-\frac{G*t^3*(2*b+h)}{(n*b)*3*M}\right)}\ldots\ldots(6)$$

By bearing in mind that ($r_1=\frac{h}{b}, r_2=\frac{t}{b}, R=\frac{L}{h}$) ($G=810\ t/cm^2$ for most steels) , ($n=180$) to give more appropriately to the design process, the relation can be formed to:

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$$\frac{\tanh\left(1.24\frac{R}{r_2}*\left(\frac{(2+r_1)*(6+r_1)}{(3+2*r_1)}\right)^{\frac{1}{2}}\right)}{1.24\frac{R}{r_2}*\left(\frac{(2+r_1)*(6+r_1)}{(3+2*r_1)}\right)^{\frac{1}{2}}}=\frac{\left(1-\frac{1.5t^3*(2+r_1)}{M}\right)}{\left(1+\frac{1.5t^3*(2+r_1)}{M}\right)}$$

For more simplifying:

$$\frac{1.5t^3*(2+r_1)}{M}=\frac{\left(1.24*\frac{R}{r_2}*\left(\frac{(2+r_1)*(6+r_1)}{(3+2*r_1)}\right)^{\frac{1}{2}}-\tanh\left(1.24*\frac{R}{r_2}*\left(\frac{(2+r_1)*(6+r_1)}{(3+2*r_1)}\right)^{\frac{1}{2}}\right)\right)}{\left(1.24*\frac{R}{r_2}*\left(\frac{(2+r_1)*(6+r_1)}{(3+2*r_1)}\right)^{\frac{1}{2}}+\tanh\left(1.24*\frac{R}{r_2}*\left(\frac{(2+r_1)*(6+r_1)}{(3+2*r_1)}\right)^{\frac{1}{2}}\right)\right)} \quad \text{......(7)}$$

This relation presents the optimal solution for the optimal ratios for all global dimensionfor C section. But the single solution to this equation would be very complicated to be catch, therefore, it is possible to be solved numerically or solved graphically using some logical assumptions. By noting many texts of codes of practice and the common industrial cross-sections, the thickness, t could be ranged between 1.25 mm to 4 mm, R could be ranged between 5 to 40, r_2= 0.1 and λL could be ranged between 0.1 to 10 as texted in AISC guide for torsion. Consequently, the most convenient suggestions for r_1 would be illustrated form Fig. 3) to Fig. 5):

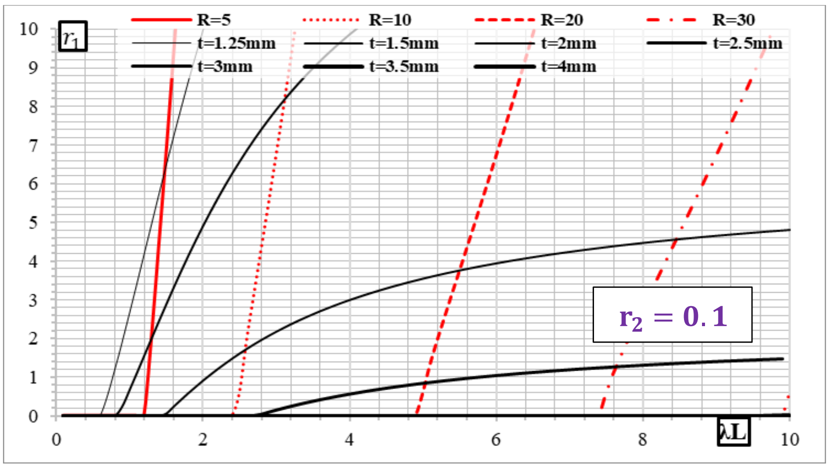

Fig. 3: The suggested values for $\boldsymbol{r_1}$ for C sections, whenapplying a direct torque of 0.1 t.cm

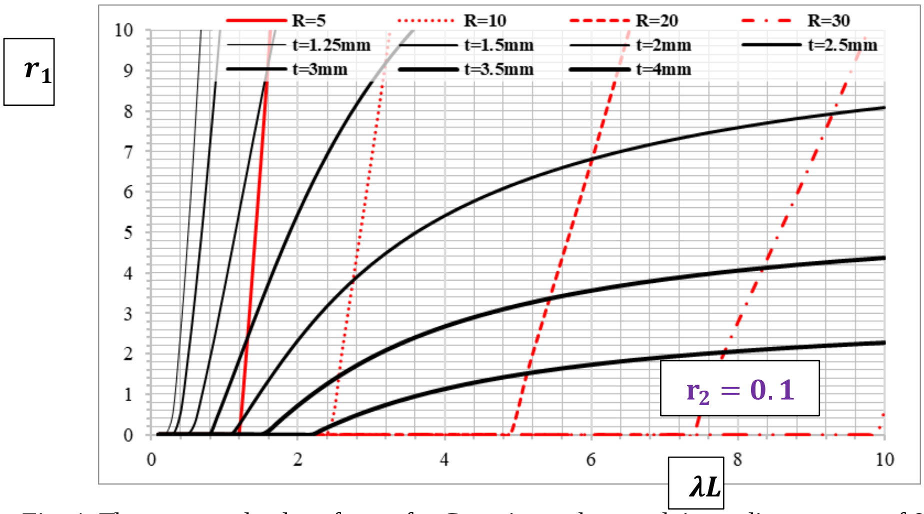

Fig. 4: The suggested values for $\boldsymbol{r_1}$ for C sections,when applying a direct torque of 0.5 t.cm

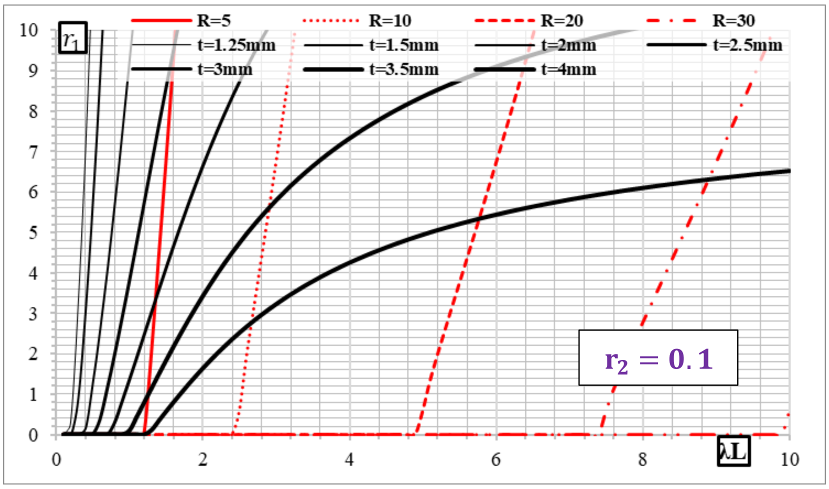

Fig. 5: The suggested values for r_1 for C sections, when applying a direct torque of 1 t.cm

For Z section:

$$C_w = \frac{tb^3h^2}{12}\frac{(b+2h)}{2b+h}$$

$$J = (2b+h) * \frac{t^3}{3}$$

$$\lambda = \frac{31 * L * \left(\frac{t^2*(2*b+h)^2}{b^3*h^2*(b+2*h)}\right)^{\frac{1}{2}}}{25}$$

Following the same approach, this can be accomplished that:

$$\frac{1.5t^3*(2+r_1)}{M_{t.cm}} = \frac{\left(1.24*R*\left(\frac{(2+r_1)^2}{r_2^2*(1+2r_1)}\right)^{\frac{1}{2}}\right)-\tanh\left(1.24*R*\left(\frac{(2+r_1)^2}{r_2^2*(1+2r_1)}\right)^{\frac{1}{2}}\right)}{\left(1.24*R*\left(\frac{(2+r_1)^2}{r_2^2*(1+2r_1)}\right)^{\frac{1}{2}}\right)+\tanh\left(1.24*R*\left(\frac{(2+r_1)^2}{r_2^2*(1+2r_1)}\right)^{\frac{1}{2}}\right)} \quad \text{.......}(8)$$

Therefore, the corresponding values for r_1 can be established from the following figures:

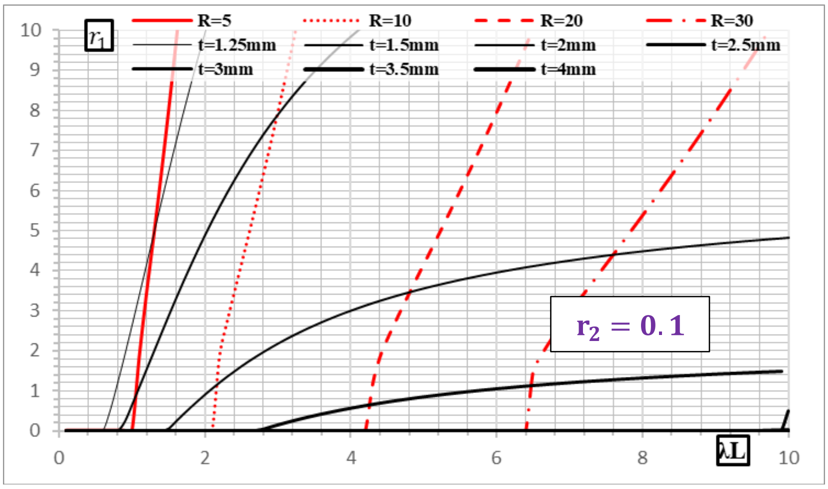

Fig. 6: The suggested values for r_1 for Z sections, when applying a direct torque of 0.1 t.cm

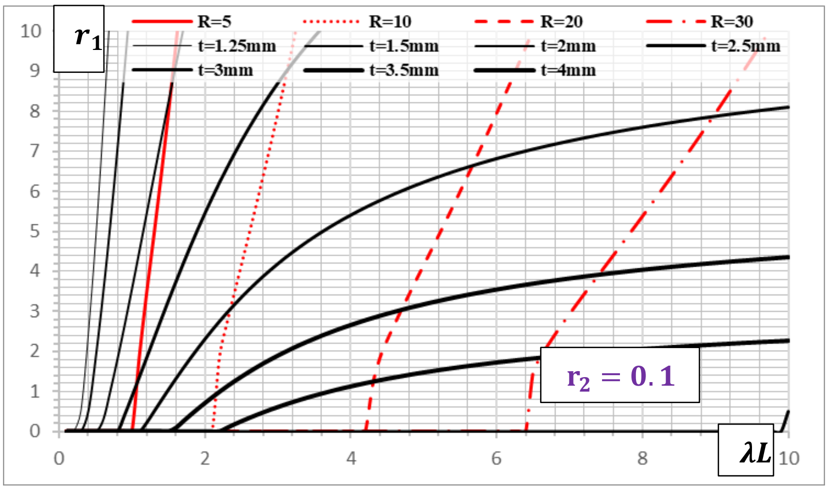

Fig. 7: The suggested values for r_1 for Z sections, when applying a direct torque of 0.5 t.cm

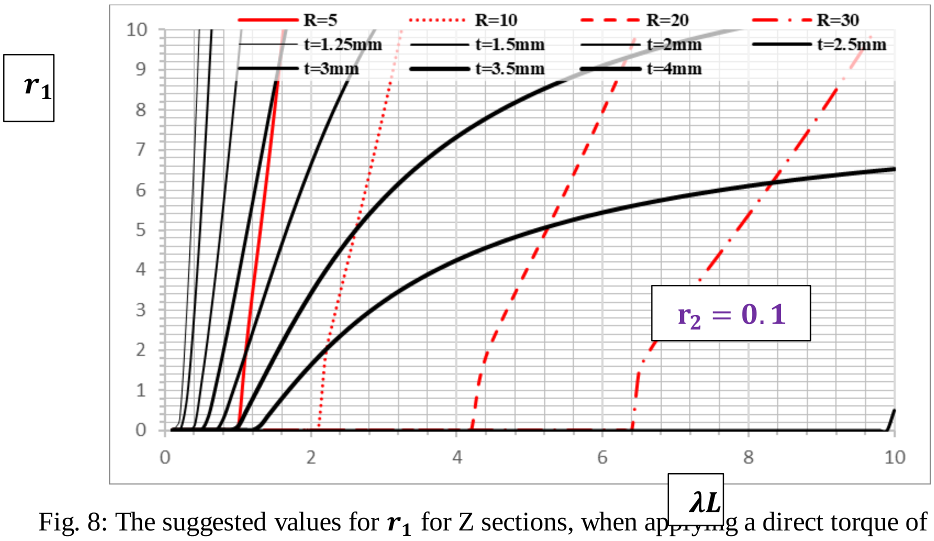

Fig. 8: The suggested values for r_1 for Z sections, when applying a direct torque of 1 t.cm

For I section:

$$C_w = \frac{tb^3h^2}{24}$$

$$J = (2b + h) * \frac{t^3}{3}$$

$$\lambda = 1.7536 \left(\frac{t^2 * (2 * b + h)}{b^3 * h^2}\right)^{\frac{1}{2}}$$

Following the same approach, this can be accomplished that:

$$\frac{1.5t^3(2 + r_1)}{M_{t.cm}} = \frac{\frac{1.75R}{r_2}\sqrt{(2 + r_1)} - \tanh\left(\frac{1.75R}{r_2}\sqrt{(2 + r_1)}\right)}{\frac{1.75R}{r_2}\sqrt{(2 + r_1)} + \tanh\left(\frac{1.75R}{r_2}\sqrt{(2 + r_1)}\right)} \quad \text{.......(9)}$$

Therefore, the corresponding values for r_1 can be established from the following figures:

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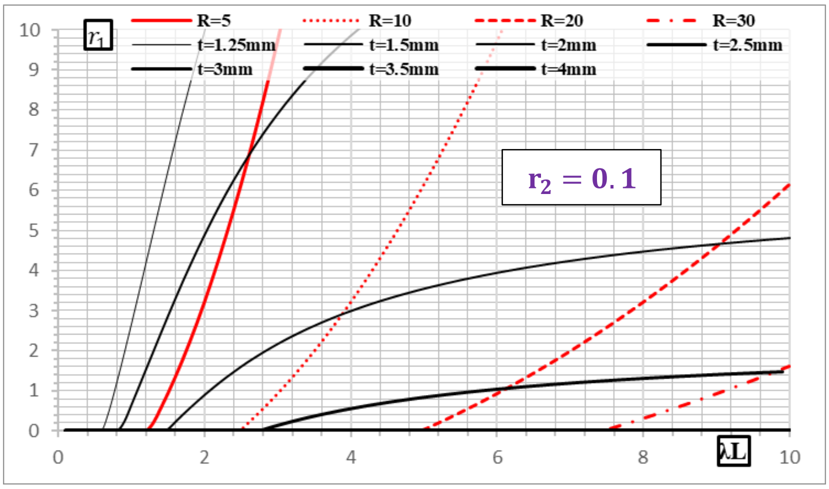

Fig. 9: The suggested values for r_1 for I sections, when applying a direct torque of 0.1 t.cm.

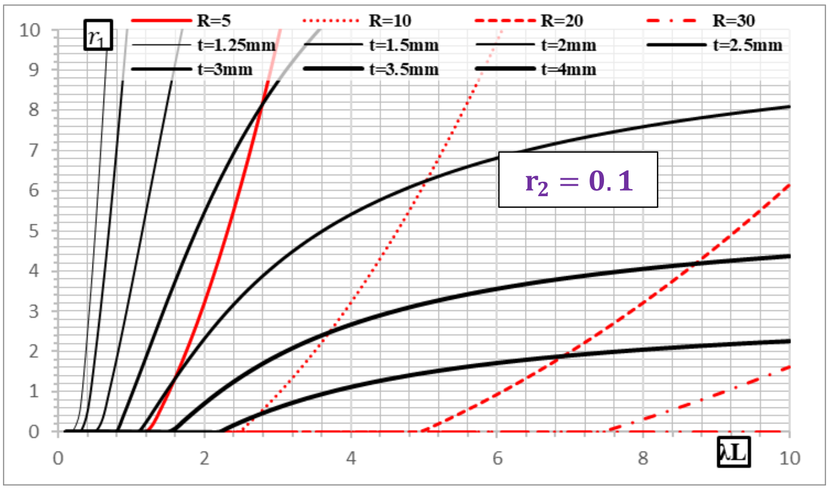

Fig. 10: The suggested values for r_1 for I sections, when applying a direct torque of 0.5 t.cm.

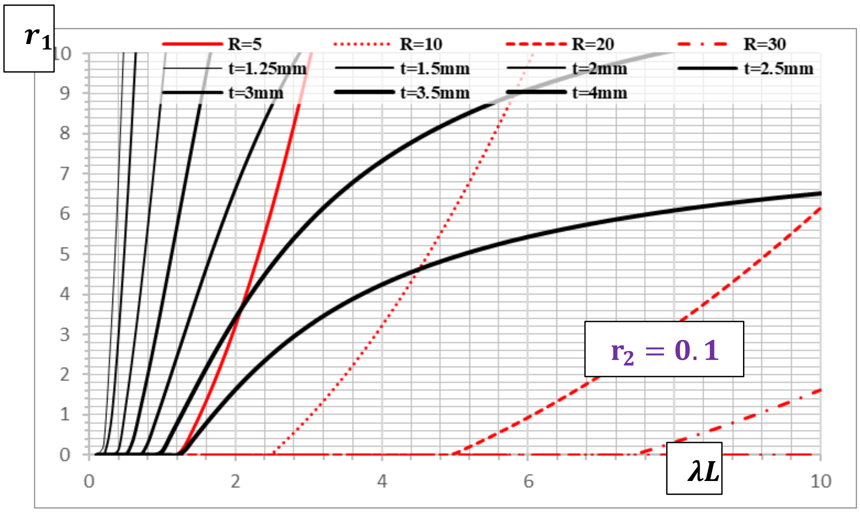

Fig. 11: The suggested values for r_1 for I sections, when applying a direct torque of 1 t.cm.

II. EVALUATION AND DISCUSSIONS

The following extraction can be found out from the noticed relations:

$\frac{1.5t^3*(2+r_1)}{M_{t.cm}} = \left(\frac{\lambda L - \tanh(\lambda L)}{\lambda L + \tanh(\lambda L)}\right)$ would be the mother relation can achievethe optimal ratios for any cantilevered beam with any cross-sectional profile, when applying a direct torque at its warpless end.

The calculation is carried out for the cantilever beam of preferred section of the thickness 1.25 mm $\leq$ t$\leq$ 4mm, $r_2 = 0.1$

Value of the span to depth ratio can be limited within 5 to 30, and cannot be used 40, because this would need preposterous values for λL with it is corresponding value for thickness.

III. CONCLUSION

This manuscript presents a methodology to optimize of thin-walled open-section cantilever beams with a warpless end, handling the Lagrange multiplier approach. Choosing the cross-section area as the objective function and vertical deflection for constraint functions, optimal ratios of cross-section individual parts (webs and flanges) are illustrated from fig. (3) to fig. (11) by following the perpendicular line from pointing of λL till the required thickness, then by simple interpolation, the value of the corresponding R, and the value of r_1 can be determined easily.

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