

Learning Analytic Geometry with the aid of Wolfram Alpha

Maria Gabriela Campuzano,
Universidad Estatal Península de Santa
Elena – UPSE, La Libertad, Ecuador

Tania Crisanto,
Universidad de las Fuerzas
Armadas – ESPE, Sangolquí, Ecuador

Abstract:- The active use of ICT for math education offers known advantages like creating interactive environments to facilitate learning, improved monitoring and detailed feedback. The objective of this study is to determine the effectiveness of using Wolfram Alpha tool on academic achievement and students' attitude towards it when learning analytic geometry. Wolfram Alpha web resource was chosen because it has a good visual representation, shows step-by-step solutions, returns detailed outcome and is easy to use. A quasi-experimental methodology was used dividing students into experimental and control groups. Academic achievement was assessed with two sample t-test to pre and post tests' scores. Questionnaires were applied to Wolfram Alpha group to gather students' perception of using this tool and tabulated under descriptive statistics. Research was applied to an online course of pre-calculus during disruption of face-to-face classes due to COVID spread. Results suggest that Wolfram Alpha improves students' academic achievement generating positive and interactive conditions where students can benefit from the use of ICT in different ways. Students have optimistic attitudes about this tool and support it to be included in regular pre-calculus courses. Teachers should consider the use of Wolfram Alpha for their math courses.

Keywords:- analytic geometry, Wolfram Alpha, conics, parabola, quasi-experiment.

I. INTRODUCTION

As Information and Communication Technologies (ICT) keeps developing, its use for education keeps growing. Worldwide, schools, colleges and universities have invested on computers, tablets, software, internet and training[1]. It has been demonstrated that when computers and mobile devices are properly used, academic achievement can be increased by the use of ICT at all education levels [2]. ICT can provide more flexibility, improve feedback and monitoring, create interactive learning and motivate students [3,4]. The advantages and disadvantages of using ICT on education depends on the subject and how it is implemented. The impact technology will have on students achievement is not only influenced by infrastructure and software but also influenced by pedagogical practices [5]. Face-to-face classes disruption due to COVID outbreak forced ICT into education in unexpected ways. Students were forced to attend classes online using computers, tablets or mobile phones and teachers to provide the best tools available for a high-quality education under adverse circumstances. Math teachers worldwide had to adapt to the new reality and develop rapidly and with little training their distance courses [6].

An important area of mathematics is analytic geometry which is also called coordinate geometry. Usually, it is studied in pre-calculus because it involves key concepts which will aid single variable and multivariate calculus understanding. Mathematical representation is a key concept for learning analytic geometry and complex problems can be solved easily with the right representation [7]. The use of ICTs can help to represent elements of analytic geometry. Math software used for math classes may play a key role for a meaningful permanent learning. Previous research for math learning includes the following software or web resources: GeoGebra, Mathematica, Matlab, Cabri Geometry and Wolfram Alpha [8-15].

This study was performed during a distance learning course of pre-calculus due to pandemic. Wolfram Alpha web resource was selected for teaching analytic geometry because this tool requires little programming knowledge and offers visualization and step-by-step resolution for math problems. Despite that Wolfram Alpha has been evaluated for mathematics before by other studies, it has not been reported its use for analytic geometry before. How students' academic achievement is influenced was determined using a quasi-experimental methodology. Students' perception towards Wolfram Alpha was assessed using questionnaires.

II. WOLFRAM ALPHA

Wolfram Alpha is a search engine which computes answers and provides knowledge. Using algorithms and expert knowledge, it automatically answers questions, performs analysis and creates reports. [16,17]. This unique online computation tool was built on Wolfram language and launched in 2009 by Wolfram Research company. It can be accessed by typing <https://www.wolframalpha.com> on a web browser, through its mobile application or by Mathematica software. Wolfram Alpha's interface is presented in Fig. 1. Wolfram Alpha is used for mathematics, science and technology, society, culture, everyday life among others areas [18]. This powerful tool can solve math exercises of various topics as geometry, limits, differentiation, integrals and even Laplace transformations for differential equations [19-21]. Wolfram Alpha is also used for physics, for instance to convert units, calculate solid and liquid properties, calculate forces and search physical constants [22]. Step-by-step solution is offered for math, chemistry and physics problems (Fig. 1).



line through (1,2) to (2,1)

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WolframAlpha Step-by-Step Solution

Equation forms:

STEP 1

Find the equation of the line in slope-intercept form with the properties:

point: $p_1 = (1, 2)$
point: $p_2 = (2, 1)$

Hint: Compute the slope of the line.

The slope of the line through points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$:

$$m = \frac{1 - 2}{2 - 1} = -1$$

Fig. 1: Wolfram Alpha step-by-step solution

III. RELATED WORK

A. Analytic Geometry Teaching with Math Software

The following section presents a literature review of previous studies for teaching analytic geometry with GeoGebra and Cabri software. Most of previous re-search has been performed using Geo Gebra software.

Geo Gebra was studied with a quasi-experimental methodology in analytic geometry courses for first-year university students when studying the circle topic. Pre and post tests' scores analysis demonstrated that when students learnt using GeoGebra they increased their achievement and understanding. Surveys performed to students showed that students have positive attitude towards the software and that their spatial visualization improved due to the software use [23]. Researchers performed a quasi-experimental study where students from the experimental group were taught with the aid of GeoGebra for learning circles. T-tests on scores revealed that GeoGebra aided students to perform better than control group. Surveys to students evidenced positive attitude towards this software [24]. Using a similar methodology, GeoGebra was used for a geometry course with positive outcomes for post-tests for experimental group; however, in a delayed post-test, students who were taught under traditional methodology scored better than the ones taught with software [25]. GeoGebra has been applied to higher secondary analytic geometry too. Twelve grade students were divided into control and experimental groups where pre and post treatment evaluations were applied. T test revealed that students instructed with GeoGebra have betters scores specially for low achievers [26]. GeoGebra was studied for learning cylindrical and spherical coordination in analytic geometry courses for pre-service teachers. Most of the studies reviewed were performed using a quasi-experimental methodology analyzing pre and post test scores; however, this study applied a different methodology. Semi-structured interviews were analyzed to assess the contribution of the software to improve students' understanding. Results showed

how pre-service teachers increased significantly their understanding of the subject [27].

Cabri 3D was analyzed for a spatial analytic geometry for prospective math teachers. All participants had previously worked with Cabri 2D. Using the work-sheets created for this study and the software, participants successfully solved all the problems. This study concluded that Cabri 3D aids students to understand spatial analytic geometry as it offers a good visualization and generates positive attitudes towards learning this subject [28].

B. Wolfram Alpha Studies

Despite that Wolfram Alpha has not been studied for analytic geometry it has been studied for other math areas. The following literature review relates to Wolfram Alpha studies applied to math courses.

Researchers reported having taught mathematics with the aid of Wolfram Al-pha for two years, in the Czech Republic. The university created a system called "TRIAL" where mathematics problems can be retrieved. Examinations took place using Wolfram Alpha and the most common mistakes were related to syn-tax errors and incorrect analysis of results [29]. Wolfram Alpha was evaluated for single variable calculus course with the aid of questionnaires. Students responses aided the study to conclude that this software adds interaction to class, motivates students to learn and it should be used permanently for math courses [30]. Wolfram Alpha was utilized for a calculus course. Students answered questionnaires to assess the effectiveness of this complementary tool. Results indicate that most of the students believed Wolfram Alpha to be a useful resource to learn and prepare for exams [15]. Researchers reported the feasibility of replacing traditional computer algebraic system (CAS) with Wolfram Alpha for differential and integral calculus; however, they considered this tool not powerful enough to fully replace CAS in multivariate calculus. Surveys demonstrated that students prefer to use Wolfram Alpha as it has natural language programming and is al-ways updated. When this tool showed deficiencies, laboratory practices were complemented with content from Wolfram Demonstration Project and Math-World [31]. Wolfram Alpha was compared to MathCad, analyzing their functionality and examples for function integration. MathCad needs students to have prior knowledge to present results in an adequate form and this software offers experience to migrate to more sophisticated software as Matlab and Mathematica. Wolfram Alpha shows step by step resolution for problems and results are presented in different ways [32].

The previous research for Wolfram Alpha analyzed in this section used methodologies different to the one used for the study presented in this paper. Only the following two studies used a quasi-experimental methodology; however, to different math areas. Wolfram Alpha and HD calculator apps were evaluated in a differential equations course where pre and post tests were conducted for control and experimental groups. Data analysis revealed academic achievement improvement, positive perception towards the course and anxiety reduction [33]. For pre-service teachers, Wolfram Alpha was used to help to improve performance in pre-

calculus problems. Pre and post tests analysis showed an improvement as errors were reduced [34]. While there are many studies under quasi-experimental methodology for GeoGebra, there are few works with this methodology for Wolfram Alpha and there is no prior study with this methodology for analytic geometry using Wolfram Alpha.

IV. MATERIALS USED FOR THIS STUDY

Wolfram Alpha can be used for analytic geometry or coordinate geometry to determine algebraic equations, compute geometric properties and plot many geo-metric figures. The examples presented in Wolfram Alpha web site correspond to the following groups: lines, planes, conic sections, intercepts and quadrants (Fig. 2).

Examples for Coordinate Geometry

Lines
 Compute the equation and draw the graph of a line.
 Specify a line through two points:
 line through (1,2) and (2,1) =
 Specify a line by slope and intercept:
 line, slope=1/5, y-intercept=3 =

Planes
 Compute the equation and draw the graph of a plane in 3D.
 Specify a plane through three points:
 plane through (0, 1, 0), (5, 6, 7), and (6, 7, 8) =

Conic Sections
 Identify and compute the properties of circles, ellipses, parabolas and hyperbolas.
 Plot a conic section and identify its type:
 $x^2 - 2y^2 = 1$ =
 Find a circle passing through three points:
 circle through (0,0), (1,0), (0,1) =
 Compute properties of a parabola:
 parabola with focus (3,4) and vertex (-4,5) =

Quadrants
 Identify the quadrant in which a given 2D point lies.
 Identify the quadrant that contains a given point:
 What quadrant is (1,2) in? =
 is (-2, 2) in quadrant 4? =

Fig. 2: Analytic or coordinate geometry examples presented by Wolfram Alpha

This study used the content for lines and the following conic sections: circle, parabola, ellipse and hyperbola. Examples of the content used is presented only for parabola geometry. Fig. 3 shows input and output for the query parabola followed by an equation. Despite the query is short the output is extended as it presents the graph and the main properties of parabola.

parabola $x^2 - 6x - 8y - 7 = 0$

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Input interpretation
 parabola Cartesian equation $x^2 - 6x - 8y - 7 = 0$

Visual representation
 [Graph of a parabola opening upwards]

Properties

focus	(3, 0)
vertex	(3, -2)
semi-axis length	2
focal parameter	4
eccentricity	1
directrix	$y = -4$

Fig. 3: Parabola equation problem solved by Wolfram Alpha.

The main tool used for this course was Wolfram Alpha; however, other Wolfram content was also used for explaining theoretical concepts to the experimental group like Wolfram Demonstration Project which has a collection of interactive material.

V. METHOD

A. Research Method and Data Collection

This study used a quasi-experimental methodology (Fig. 4). It is called quasi because participants are not selected randomly. This method had been used for other educational studies described in the section of related work [23-26, 33-34]. Students were divided into two groups, control and experimental groups. Groups were created randomly from students of a pre-calculus course. Control and experimental groups were taught through Zoom as part of distance education due to Covid pandemic. Experimental group learnt with the aid of Wolfram Alpha while control group without this tool. The same pre and post tests were implemented for both groups to assess their academic achievement. Students from experimental group answered Likert-scale questionnaires to assess their perception of using Wolfram Alpha. Questionnaires were available through Google Forms.

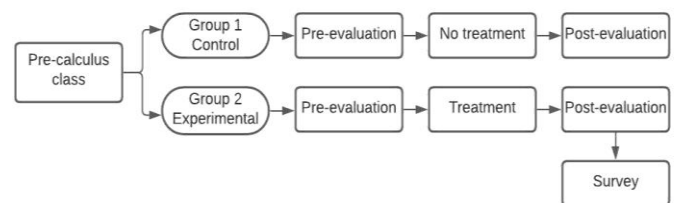


Fig. 4: Methodology used for this study

This study was conducted for a pre-calculus course during online education due to Covid outbreak. This course took place at Universidad Estatal Península de Santa Elena from Ecuador. Each group had a similar number of participants. Control group had 35 students and experimental group had 36 students. Female participants accounted for almost 17% in both groups. Most of previous studies performed for analytic geometry used GeoGebra software. This research chose Wolfram Alpha tool rather than GeoGebra software because Wolfram Alpha does not require to know programming language, offers step by step solutions for queries, presents detailed outcome for short instructions which can be written in natural language or structured math format syntax and displays a good visual representation. Wolfram Alpha can be used at universities and high schools. Partial step-by-step solution is available for free version and full step-by-step solution for paid version. Students had access to full paid version. Syllabus was updated and teaching materials prepared. Even tough writing instructions in Wolfram Alpha is not difficult, as students' native language is Spanish, it was necessary to introduce some commands in English. Control and experimental groups were taught by the same lecturer. The analytic geometry's topics included in this study were line, circle, parabola, ellipse and hyperbola. Students accessed the course through their computers or mobile devices at home. Wolfram Alpha was accessed through a web browser. Pre and post tests had the

same difficulty level and included multiple choice questions and problems. Students taught with Wolfram Alpha were not allowed to use this tool during examinations.

B. Objectives and Research Questions

One of the objectives of this study is to assess the effectiveness of using the web resource Wolfram Alpha for teaching analytic geometry as part of a pre-calculus course taught online during the pandemic. Another objective is to evaluate students’ perceptions of using this tool and whether they recommend its further use or not.

This work addressed these research questions:

- Do students who learnt analytic geometry with the aid Wolfram Alpha tool have better academic achievement than the ones who learnt without this tool?
- What are student’s perceptions of using Wolfram Alpha web resource during an online course due to pandemic?

C. Data Analysis

Pre and post tests’ scores were analyzed using descriptive statistics. Normality test was performed so t-test can be applied. Two-sample t-test was applied to evaluate the difference between means and the effectiveness of the treatment. For surveys’ answers, Likert scale was translated to the following numerical scale: 0 = strongly disagree, 1 = disagree, 2 = neutral, 3 = agree, 4 = strongly agree. Questionnaires’ results were analyzed with descriptive statistics. Minitab v18 Statistical Software was used. Study results were compared to previous research cited in the related work section.

VI. RESULTS AND DISCUSSION

A two-sample t-test was performed to pre test scores for both groups to determine if there is a significance difference between means with an alpha value of 0.05 (Table 1).

Null hypothesis (H_0) = no significant difference between the means of pre-test scores for experimental and control groups ($p > 0.05$).

Alternative hypothesis (H_a) = significant difference between the means of pre-test scores for experimental and control groups ($p < 0.05$)

Group	No of students	Mean	S. D.	t-value	P
Control	35	1.43	1.24	1.15	0.252
Experimental	36	1.11	1.06		

Table 1: Pre-Evaluation’s T-Test For Control And Experimental Groups

The means for both groups have similar values. As p value is 0.252, which is greater than 0.05, the null hypothesis is accepted, which suggests that there is not a significant difference between means of control and experimental groups. This implies that control and Wolfram Alpha taught groups are homogenous and have similar academic achievement before treatment. For post-test scores a two-sample t-test was performed to evaluate the effect of using Wolfram Alpha.

Null hypothesis (H_0) = no significant difference between the means of post-test scores for experimental and control groups ($p > 0.05$).

Alternative hypothesis (H_a) = significant difference between the means of post-test scores for experimental and control groups ($p < 0.05$).

Group	No of students	Mean	S. D.	t-value	P
Control	35	5.09	2.96	-3.66	0.000
Experimental	36	7.58	2.78		

Table 2: Post-evaluation’s t-test for control and experimental groups

The obtained p value is 0.000 (Table 2) which is less than 0.05; therefore, the null hypothesis is rejected, and the alternative hypothesis accepted; consequently, there is a significant difference between means of control and experimental groups. This result means that students who learnt analytic geometry with the aid of Wolfram Alpha achieved significantly better than the ones who were taught with traditional methodology, supporting the fact that this tool contributes to improve understanding and learning of mathematics. This finding is coherent with related previous research using GeoGebra and Wolfram Alpha tools [23-26, 33-34].

Surveys applied to students will aid to analyze in deep the effect of Wolfram Alpha and what students think about this web resource. In general, questionnaires results (Table 4) show that students have positive perception towards Wolfram Alpha which is aligned with previous Wolfram Alpha research on other math topics [15, 30, 32-33]. The question with the least mean is the one which relates to cellphone or tablet users. This may relate to the fact students working on these devices had to access the tool through a web browser as the mobile application had an extra cost. The question with the highest mean relates that student strongly support that Wolfram Alpha should continue to be used for analytic geometry teaching in precalculus. Students felt confident that this tool helped them during distance learning during pandemic. In face-to-face education, teachers were physically in the classroom to clarify their doubts during the whole class aiding students to do exercises; however, in distance learning zoom sessions cannot last so long to avoid students to get tired, distracted or bored. Wolfram Alpha contributed to understanding and problem solving during online learning. This web resource helps to improve visual representation of equations and to distinguish between conic types. For a conic, even though equation is well written, sometimes it can be difficult to determine correctly all its properties and its graph. Students found this tool useful to verify their results and avoid making mistakes. Step-by-step solution was particularly useful for line problems. The use of Wolfram Alpha with other Wolfram resources as Wolfram Demonstration Project and Wolfram Math World allowed the teacher to introduce ICT in precalculus classes offering interactive and interesting activities to develop their learning process. Students who used this tool answered positively towards permanent learning; however, further research

should be done around this topic like delayed post-test like the ones performed by Bakar [25].

Item	Min.	Max.	Mean	S. D.
I think Wolfram Alpha helped me to learn analytic geometry	2	4	3,677	0,541
I believe Wolfram Alpha allowed me to achieve permanent learning.	2	4	3,335	0,608
I think that Wolfram Alpha adds interaction to geometry classes	2	4	3,516	0,677
I liked using Wolfram Alpha for this course	2	4	3,613	0,558
I support that Wolfram Alpha should continue to be utilized for geometry online classes	3	4	3,807	0,402
I support that Wolfram Alpha should be used in face-to-face courses when feasible	2	4	3,645	0,608
I think Wolfram Alpha has helped me to shift to an online course during the pandemic	2	4	3,581	0,564
Wolfram used on a mobile device offers a similar experience than computer	1	4	3,152	0,614

Table Results of questionnaires applied to experimental group after treatment

0 = strongly disagree, 1 = disagree, 2 = neutral, 3 = agree, 4 = strongly agree

VII. CONCLUSION

Most of previous research performed in analytic geometry field corresponds to GeoGebra software, finding innovative the use of the web resource Wolfram Alpha. This tool offers the advantages of good visual representation, extended results, step-be-step solution and does not require to know programming language. One disadvantage is that it requires internet connection.

After performing a quasi-experiment with pre and post tests, it could be determined that students taught with the aid

of Wolfram Alpha had significant better academic achievement than the ones taught without it when learning analytic geometry in a pre-calculus course. Students' perceptions about this web resource are positive and suggest that this tool should be implemented in regular courses because aids student to improve visual representation, understanding of theoretical concepts and step-by-step problem solution. Wolfram Alpha is available through a web browser and allowed students to performed interactive activities. This tool provided support to distance learning as students could obtain immediate feedback for many problems. Wolfram Alpha allowed TICs actively use during math learning.

Further research should be performed to determine if Wolfram Alpha allows permanent learning using delayed post-tests. Other research should relate how Wolfram Alpha helps students to use more complicated software like Wolfram Mathematica.

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