Non Localized Traveling Wave Solutions to the (2 + 1)-D generalized Breaking Soliton Equation with the Assist of Riemann Equation

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Abstract:- With the help of the Riemann equation, the $\exp(-\emptyset(\xi))$-expansion method is used to develop innovative explicit and precise solutions as well as solitary wave solutions for the (2 + 1)-D generalized breaking soliton equation. We can acquire exact explicit kink single kink, and periodic kink solutions with the help of Maple and the $\exp(-\emptyset(\xi))$-expansion approach. By assigning special values to the parameters, solitary wave solutions can be generated from exact solutions. Furthermore, we may infer that our preferred method is powerful, simple, and easy to use, and that it provides far more trustworthy novel exact answers for mathematical physics and engineering treatments.

Keywords:- *The (2 + 1)-D Generalized Breaking Soliton Equation; Riemann Equation; $exp(-\emptyset(\xi))$-Expansion Method; Exact Solution, Mathematical Physics.*

I. INTRODUCTION

Nonlinear phenomena can be found in a wide range of theoretical scientific fields, including fluid dynamics, optical fiber communication systems, plasma physics, and solid-state physics, among others. Due to their numerous arrivals in diverse submissions in various domains, such as mathematical physics, engineering, signal processing, control theory, biology, and so on, nonlinear fractional partial differential equations (NFPDEs) have been the concentration of many educations. A large deal of recent research has contributed to providing the innovative accurate and explicit solutions to NFPDEs. Numerous powerful schemes have been projected to obtain the approximate, exact and explicit solutions of these nonlinear equations, such as, Hirota bilinear technique [1], Bäcklund transformation scheme [2], inverse scattering transform [3] , extended tanh-function (mETF) method [4-6], homotopy analysis scheme [7-10], homotopy perturbation technique [11-13], Adomian decomposition system [14, 15], fractional sub-equation procedure [16], Lagrange characteristic scheme [17], $(\boldsymbol{G'}/\boldsymbol{G})$-expansion technique [18, 19], Sardar sub-equation methods [20], $(\frac{G'}{G},\frac{1}{G})$ expansion method [21], multiple exp-function method [22, 23], Frobenius decomposition technique [24], local fractional variation iteration method [25], local fractional differential equations approach [26, 27], multiple $(\boldsymbol{G'}/\boldsymbol{G})$-expansion approach [28], Riccati equation method mutual with the $(\boldsymbol{G'}/\boldsymbol{G})$-expansion technique [29], cantor-type cylindrical coordinate system [30], modified simple equation method [31], ractional complex transform method [32], $\exp(-\emptyset(\xi))$-expansion method [33-36], and so on.

In this study we have mainly engrossed on (2 + 1)-D generalized breaking soliton equation. Here we have converted the equation into the Riemann equation of the following form.

The (2 + 1)-D generalized breaking soliton equation [37] is given by

$$u_t + \alpha u_{xxx} + \beta u_{xxy} + \gamma u u_{xy} + \delta u u_y + \varepsilon u_x {\delta_x}^{-1} u_y = 0 \quad (1)$$

The Riemann wave's (2 + 1)-D interaction is interpreted (1), in which overlapping solutions have been produced for the case $\alpha = 0$.. The spectral parameter is named after the well-known breaking behavior that is characteristic of such equations. The spectral value is otherwise handled as a multivalued function. As a result, these equations' solution types would be multivalued. Our studied equation is studied by previous authored before. Recently Xu, Gui-qiong, and Abdul-Majid Wazwaz [38], Sachin, et al.[39], Yan, Xue-Wei, et al [40] and so on used this equation as extracting the abundant soliton solutions. As far our knowledge there is no extracting about Riemann equation through (2 + 1)-D generalized breaking soliton equation. In this study our main goal is to solve Riemann equation which is a computational from of (2 + 1)-D generalized breaking soliton equation. Another important to know that to solve this equation we have utilized a mathematical method $\exp(-\emptyset(\xi))$- expansion method [41] which is not imposed before regarding to solve our preferred equation.

This is how the article is written: The $\exp(-\emptyset(\xi))$-expansion approach was explored in section 2 of this paper. We use this strategy to solve the nonlinear evolution equations mentioned earlier in section 3. Graphical depiction, results, and discussion are all included in section four. Conclusions are presented in Section 5.

II. CONSTRUCTION OF THE $\exp(-\emptyset(\xi))$-EXPANSION METHOD:

The $\exp(-\emptyset(\xi))$-expansion method will be discussed word by term in this section. Consider the following example of a nonlinear partial differential equation:

$$\Re(U, U_{xx}, U_{xz}, U_{xy}, U_{xtt}, \dots \dots) = 0 \quad (2)$$

$\Re$ is a polynomial of U, its different type partial derivatives, in which the nonlinear terms and the highest order derivatives are involved, and

$U = U(x, y, z, t)$ is an unfamiliar function.

Step-1.
Now we'll look at a transformation variable that will combine all of the independent variables into a single variable, such as

$$U(x,t) = u(\xi), \xi = kx + ly + mz \pm Vt \quad (3)$$

By fulfilling this variable Eq. (3) permits us reducing Eq. (2) in an ODE for $U(x,t) = u(\xi)$ $P(u, u', u'', u''', \dots \dots)$ (4)

Step-2.
Assume that the solution of the ODE Eq. (4) can be stated as follows using a polynomial in $\exp(-\emptyset(\xi))$

$$u = \sum_{i=0}^{m} a_i \exp(-\emptyset(\xi))^{\,i}, \quad (5)$$

where the derivative of $\emptyset(\xi)$ satisfies the ODE in the following form

$$\emptyset'(\xi) = \exp(-\emptyset(\xi)) + \mu \exp(\emptyset(\xi)) + \lambda \quad (6)$$

then the solutions of ODE Eq. (6) are

Situation I:
Hyperbolic function solution (when $\lambda^2 - 4\mu > 0, \mu \neq 0$):

$$\emptyset(\xi) = \ln\left(\frac{-\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+C)\right)-\lambda}{2\mu}\right)$$

and $$\emptyset(\xi) = \ln\left(\frac{-\sqrt{\lambda^2-4\mu}\coth\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+C)\right)-\lambda}{2\mu}\right)$$

Situation II:
Trigonometric function solution (when $\lambda^2 - 4\mu < 0, \mu \neq 0$)):

$$\emptyset(\xi) = \ln\left(\frac{\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+C)\right)-\lambda}{2\mu}\right)$$

and $$\emptyset(\xi) = \ln\left(\frac{\sqrt{4\mu-\lambda^2}\cot\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+C)\right)-\lambda}{2\mu}\right)$$

Situation III:
Exponential function solution (when $\lambda^2 - 4\mu > 0, \mu = 0$):

$$\emptyset(\xi) = -\ln\left(\frac{\lambda}{\exp(\lambda(\xi+C))-1}\right)$$

Situation IV:
Rational function solution (when $\lambda^2 - 4\mu = 0, \mu \neq 0, \lambda \neq 0$):

$$\emptyset(\xi) = \ln\left(-\frac{2(\lambda(\xi+C)+2)}{\lambda^2(\xi+C)}\right)$$

Situation V:
Other solution (when $\lambda^2 - 4\mu = 0, \mu =, \lambda = 0$):

$$\emptyset(\xi) = \ln(\xi + C)$$

Where $a_i, V, \lambda; i = 0,1, \dots, m$ and μ are constants to be determined later. The homogeneous balance between the highest order derivatives and nonlinear terms appearing in ODE can be used to calculate the positive integer m in (3).

Step-3.
We execute a polynomial form of $\exp(-\emptyset(\xi))$ by replacing Eq. (5) into Eq.(4) and utilizing the ODE (6) to aggregate all orders of $\exp(-\emptyset(\xi))$ together. The algebraic system for $a_i, V, \lambda; i = 0,1, \dots, m$ and μ is obtained by equating each coefficient of this polynomial to zero.

Step-4.
Assuming that the constants $a_i, V, \lambda; i = 0,1, \dots, m$ and μ can be obtained by solving the algebraic system, and that the general solutions of the auxiliary ODE (6) are well known to us, we may substitute $a_i, V, \lambda; i = 0,1, \dots, m$ and μ the general solutions of Eq.(5) into Eq (6). As a result, we are able to find exact and explicit traveling wave solutions to nonlinear partial differential equations (2)

III. USE OF THE PROPOSED METHOD

In this part, we use the Riemann equation to obtain the exact solution of the (2 + 1)-D generalized breaking soliton problem using our proposed $\exp(-\emptyset(\xi))$-expansion approach. The generalized breaking soliton equation of (2+1)-D in this case is of the form

$$u_t + \alpha u_{xxx} + \beta u_{xxy} + \gamma u u_{xy} + \delta u u_y + \varepsilon u_x \delta_x^{\;-1} u_y = 0 \quad (7)$$

Where α, β, δ are nonzero constants. In this situation, we turn the problem into a handy Riemann equation by creating overlapping solutions for the case $\alpha = 0$. If we put $\varepsilon u_x \delta_x^{\;-1} u_y = 0$ and free parameters are changed in unit we get the Riemann equation is of the form

$$u_t + nu_{xxy} + luu_y + \gamma uu_{xy} + su_x u_y = 0 \quad (8)$$

where $u_y = u_x$. The travelling wave equation we take as the form

$$u = u(x,t), \xi = gx - \mu t, u = u(\xi), u(x,t) = u(\xi) \quad (9)$$

The following ordinary differential equation is obtained by using the travelling wave equation Eq. (9) and integrating Eq. (8) with regard to ξ.

$$2mnr^2u'' + m(l+s)u^2 - 2\mu u = 0 \quad (10)$$

We find $N = 2$ when we analyze the homogeneous balance between the highest order derivative u'' and the nonlinear factor u^2. As a result, we can employ the auxiliary solution in the following form using our suggested method.

$$u(\xi) = A_0 + A_1 e^{-\emptyset(\xi)} + A_2\left(e^{-\emptyset(\xi)}\right)^2 \quad (11)$$

Where A_0, A_1 and A_2 are arbitrary constant to be resolute such that $A_1 = 0$ while λ, μ are arbitrary constants.

We get the value of u'' by differentiating Eq. (11) and utilizing Eq. (6).

Now placing the value of $u\ and\ u''$ in Eq. (10) and coefficient of $e^{i\varphi(\xi)}, i = 0, \pm1, \pm2$ .... to zero, we get

$mlA_0{}^2 + msA_0{}^2 + 2\mu A_0$
$4m\mu^2nr^2A_1{}^2 + 2lmA_0A_1 + 2msA_0A_1 + 2\mu A_1$
$2lmA_0A_2 + lmA_1{}^2 + 2msA_0A_2 + msA_1{}^2 + 2\mu A_2$
$4\lambda m\mu nr^2A_1 + 2lmA_1A_2 + 2msA_1A_2$
$lmA_2{}^2 + msA_2{}^2$

Resolving this set of polynomial by using maple we get following solutions set

Set-1:

$$\mu = -\tfrac{1}{2}lmA_0 - \tfrac{1}{2}msA_0, \quad A_0 = \frac{\sqrt{lm\mu}}{2}\cdot(r+s), \quad A_2 = A_2, A_1 = 1,$$

Where μ and λ are arbitrary constants.

Now replacing the values of l, m, A_0, A_1, A_2 into Eq. (11) we get

Incident-I: (when $\lambda^2 - 4\mu > 0, \mu \neq 0$)) we get following hyperbolic solution

Family-1

$$u_{1(\xi)} = A_0 + \frac{1}{6}\frac{\sqrt{-2lm}\,(r+s)\sqrt{6}}{\tanh\left(\sqrt{6}\,(\xi+C)\right)} + \frac{2}{3}\frac{A_2}{\tanh\left(\sqrt{6}\,(\xi+C)\right)^2}$$

Where, $\xi = gx - \left(-\tfrac{1}{2}lmA_0 - \tfrac{1}{2}msA_0\right)$ and C is an arbitrary constant.

Family-2

$$u_{2(\xi)} = A_0 + \frac{A_1}{\sqrt{-\frac{\lambda}{\mu}}\coth\left(\sqrt{-\lambda\mu}\,(\xi+C)\right)} - \frac{A_2\mu}{\lambda\coth\left(\sqrt{-\lambda\mu}\,(\xi+C)\right)^2}$$

Incident-II: (when $\lambda^2 - 4\mu > 0, \mu \neq 0$) get following trigonometric solution

Family-2

$$u_{3(\xi)} = A_0 + \frac{A_1}{\sqrt{\frac{\lambda}{\mu}}\tan\left(\sqrt{\lambda\mu}\,(\xi+C)\right)} + \frac{A_2\mu}{\lambda\tan\left(\sqrt{\lambda\mu}\,(\xi+C)\right)^2}$$

$$u_{4(\xi)} = A_0 - \frac{1}{6}\frac{\sqrt{2}\sqrt{lm}\,(r+s)\sqrt{6}}{\cot\left(\sqrt{6}\,(\xi+C)\right)} + \frac{2}{3}\frac{A_2}{\cot\left(\sqrt{6}\,(\xi+C)\right)^2}$$

Where, $\xi = gx - \left(-\tfrac{1}{2}lmA_0 - \tfrac{1}{2}msA_0\right)$ and C is an arbitrary constant.

Incident-III: (when $\lambda^2 - 4\mu > 0, \mu = 0, \lambda \neq 0$) we get following exponential solution

Family-3

$$u_{4(\xi)} = A_0 + \frac{1}{2}\sqrt{2}\sqrt{lm}\,(r+s)\,(-2\xi - 2C) + A_2\,(-2\xi - 2C)^2$$

Where, $\xi = gx - \left(-\tfrac{1}{2}lmA_0 - \tfrac{1}{2}msA_0\right)$

And C is an arbitrary constant.

Incident IV & Incident V:

When $\lambda^2 - 4\mu = 0$ the executing value of A_0 is undefined. So the solution cannot be obtained. For this purpose incident IV is rejected.

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Likewise when $\lambda^2 - 4\mu = 0, \mu =, \lambda = 0$ the accomplishing value of A_0, A_1 are undefined. As a result, no solution can be found. So **incident V** is also excluded.

IV. OUTCOMES AND DELIBERATIONS

The physical elucidation of the Riemann equation's established exact traveling wave solutions will be covered in this part. In the section 'physical explanation,' the acquired traveling-wave solutions of the unique 3D time-fractional WBBM equations are described in three-dimensional 3D surface plots. MATLAB is used to construct the charts for u(x,t) at various time intervals with various values of (0,1). The plots clearly depict a range of solutions, such as the kink solution, solitary kink shape solution, and periodic wave solutions, which are all dependent on the selection of various free parameters with the necessary physical explanation. A three-dimensional line plot compares several wave constituents or displays the degree of variation across time. Wave points are connected by a line and constructed in a series with equally spaced breaks to emphasize the relationships between them. The 3D elegance of the graphic adds to its visual appeal. A solitary wave, or soliton, is a self-reinforcing wave packet that maintains its shape while travelling at a constant amplitude and velocity, according to mathematical physics. Soliton are physical system solutions to a class of nonlinear dispersive partial differential equations that are weakly nonlinear. Figure 1 depicts the key physical structures of these kink-type solutions, including their paths, segment shifts after impact, and dissociation into separate single kink soliton (1-5). The frequency and amplitude of the wave were modified as the parameter was improved, and the kink solution profile became single kink.

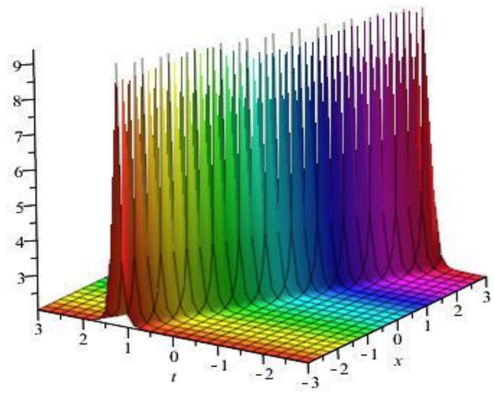

Fig. 1. Depicts the singular kink solution for the function solution $u_{1(\xi)}$ for the parameters $\lambda = 3, \mu = -2, c = 0.5, l = 1, s = 1, m = 1.9, g = 1.5,\ r = 1, A_0 = 1$, and $A_2 = 1$.

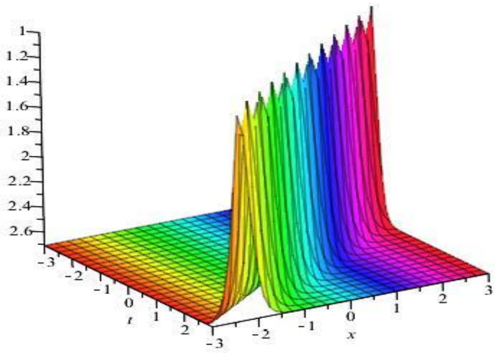

Fig.2. Denotes the bright kink solution for the function solution $u_{2(\xi)}$ for the parameters $\lambda = 3, \mu = -2, c = -2.5, l = 1.8, s = 1, r = 1, m = 1.9, g = 1.5,\ A_0 = 1$, and $A_2 = 1$.

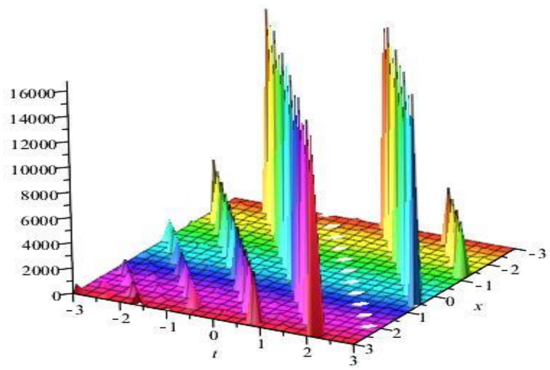

Fig3. Signifies the periodic kink solution for the function solution $u_{3(\xi)}$ for the parameters $\lambda = 3, \mu = 2, c = 2.5, l = 1.8, s = 1, r = 1, m = 1.9, g = 1.5$, $A_0 = 1$, and $A_2 = 1$

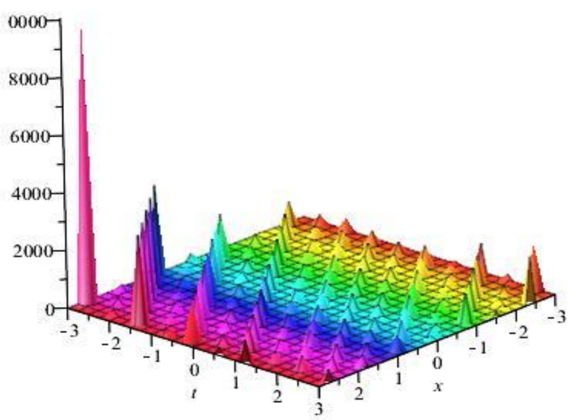

Fig. 4. Characterizes the bright periodic kink solution for the function solution $u_{4(\xi)}$ for the parameters $\lambda = 3, \mu = -2, c = -2.5, l = 1.8, s = 1, r = 1, = 1.9, g = 1.5,\ A_0 = 1$, and $A_2 = 1$

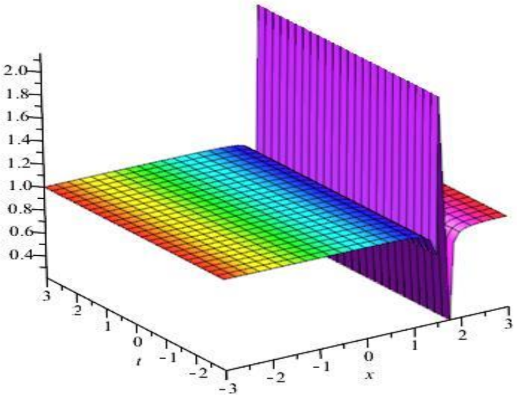

Fig. 5. Symbolizes the singular kink solution for the function solution $u_{5(\xi)}$ for the parameters $\lambda = 3, \mu = 0, c = -2.5, l = 1.8, s = 10, r = 10, A_0 = 1.5, \text{and } A_2 = 1$

V. CONCLUSIONS

In this paper, the Riemann equation is used to examine accurate traveling-wave solutions of the (2 + 1)-dimensional generalized breaking soliton problem using the $\exp(-\emptyset(\xi))$-expansion method. Using the companionable wave transform, the equations are reduced to several ODEs. The ODE's consequence form is then used to exchange the expected solutions. The coefficients of like power of $\exp(-\emptyset(\xi))$ are compared to zero to determine the SAE. The relationships between the parameters are shown by solving this system. Unwavering explicitly are certain physical and composite solutions made up of tangent, cotangent, cosecant, hyperbolic tangent, hyperbolic cotangent, and hyperbolic cosecant functions. To understand the impact of b, a graphical representation of certain solutions is represented in some finite fields using Maple. As a result, we strongly recommend that the findings of this study be made public.

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