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The Use of Scheffe's Model for the Optimization of Compressive Strength of Polypropylene Fibre Reinforced Concrete (PFRC)

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Abstract:- This research work is aimedat using an optimization model based on Scheffe's Second Degree Polynomial (5,2) to optimize the compressive strength of Polypropylene Fibre Reinforced Concrete (PFRC).In this study, Scheffe's Second Degree Polynomial (5,2) model developed by Nwachukwu and others (2017) for five component mixture will be used to optimize the mix proportion that will produce the maximum strength of PFRC. Using Scheffe's Simplex method, the compressive strength of PFRC was determined for different mix ratios. Control experiments were also carried out and the compressive strength determined. After the tests have been conducted, the adequacy of the model was tested using Student's t-test. The test statistics found the model adequate for predicting the compressive strength of PFRC when the mix ratio is known. Optimum compressive strength for the Scheffe's(5,2) model was obtained as 25.23N/mm^2. Since structural concrete elements are generally made with concrete having a compressive strength of 20 to 35 MPa (or 20 to35N/mm^2), it then means that optimized PFRC based on Scheffe's model can produce the required compressive strength needed in major construction projects such as bridges and light-weight structures. Stakeholders in the construction industry are therefore advised to use the optimized PFRC, mainly for its economic and safety advantages.

Keywords:- PFRC, Scheffe's(5,2) Polynomial Model, Optimization, Compressive strength ,Regression**.**

I. INTRODUCTION

Due to the rigorous, time consuming nature with several trial mixes before a desired quality of mixture is attained in empirical method, concrete mix design is proved to be uneconomical and laborious. Owing to this problem, optimization process is usually sought for. An optimization problem is one requiring the determination of the optimal (maximum or minimum) value of a given function, called the objective function, subject to a set of stated restrictions, or constraints placed on the variables concerned. Every optimization problem requires an objective which might be to maximize profit or benefit, to minimize cost or to minimize the use of material resources. Optimization of the concrete mixture design is a process of search for a mixture for which the sum of the costs of the ingredients is lowest, yet satisfying the required performance of concrete, such as workability, strength and durability. A typical example of optimization model is Scheffe's Polynomial Models. In this study, Scheffe's Second Degree Polynomial for five components mixtures (namely cement, fine aggregate, coarse aggregate, water and polypropylene fibre) will be on focus.

In general, concrete is a very important material widely used in construction since ancient time. Concrete is of no doubt an important building material. According to Neville(1990), concrete plays a crucial part in all building structures owing to its numerous advantages which ranges from low built in fire resistance, high compressive strength to low maintenance. At the same time, it also has a major disadvantage which is that concrete is inherently a brittle material. Also, concrete is known for its problem associated with its low tensile strength compared to its compressive strength. As a result of this, many new technologies of concrete and some modern concrete specification approach were introduced. One of the technologies introduced for concrete was the addition of steel bars to reinforce its tension zone. This enables concrete gain an amount of tensile strength and thus reducing its brittle nature. Over the years the reinforcement (usually steel bars) has been replaced with other materials like fibre (glass fibre, polypropylene fibre, nylon fibre, steel fibre , etc) to further increase both its tensile strength and compressive strength and also, produce light weighted reinforced concrete unlike when reinforced with steel bars. Concrete's compressive strength is one of the most useful properties of concrete and in most structural applications, concrete primarily resists compressive stress.

Polypropylene Fibre Reinforced Concrete (PFRC) is concrete mixture where the conventionally steel reinforcement in concrete production is replaced with polypropylene fibre. Polypropylene fibre is a kind of linear polymer synthetic fibre obtained from propylene polymerization. It is a light fibre, its density (0.91 gm/cm^3) being the lowest of all synthetic fibres. It is manufactured from propylene gas in the presence of a catalyst such as titanium chloride..In addition, polypropylene fibre (PF) is a by- product of oil refining process. PF are composed of crystalline and non- crystalline regions as shown in Figure 1. It has excellent chemical resistance to acids and alkalis and high abrasion resistance. .PF. is easy to process and inexpensive compared to other synthetic fibres. PF has some advantages which include light weight, high strength, high toughness and corrosion resistance. Thus, because of its superior performance characteristics and comparatively low-cost, PF finds extensive use as construction material in asphalt manufacturing, industrial pavements, and highly resistant concrete production.

The present study therefore focuses on the use of scheffe's second degree polynomial model to optimize the compressive strength of PFRC. Already, Bayasi and Zeng (1993) and Patel and others (2012) have investigated the properties of PFRC .In recent years, many researchers have used Scheffe's method to carry out one form of optimization project or the other. To buttress this point, Nwakonobi and Osadebe (2008) used Scheffe's model to optimize the mix proportion of Clay- Rice Husk Cement Mixture for Animal Building. Ezeh and Ibearugbulem (2009) applied Scheffe's model to optimize the compressive cube strength of River Stone Aggregate Concrete. Scheffe's model was used by Ezeh and others (2010a) to optimize the compressive strength of cement- sawdust Ash Sandcrete Block. Again Ezeh and others (2010b) optimized the aggregate composition of laterite/ sand hollow block using Scheffe's simplex method. The work of Ibearugbulem (2006) and Okere(2006) were also based on the use of Scheffe' mathematical model in the optimization of compressive strength of Perwinkle Shell-Granite Aggregate Concrete and optimization of the Modulus of Rupture of Concrete respectively. Obam (2009) developed a mathematical model for the optimization of strength of concrete using shear modulus of Rice Husk Ash as a case study. The work of Obam (2006) was based on four component mixtures, that is Scheffe's(4,2) and Scheffe's(4,3). Nwachukwu and others (2017) developed and employed Scheffe's Second Degree Polynomial model to optimize the compressive strength of Glass Fibre Reinforced Concrete (GFRC). That is to say, no work has been done on the use of Scheffe's method to optimize the compressive strength of PFRC .Henceforth, the need for this research work.

Fig. 1: Polypropylene Fibre

II. DEVELOPMENT OF THE OPTIMIZATION MODEL USING SCHEFFE'S SECOND DEGREE POLYNOMIAL

According to Aggarwal (2002), a simplex lattice is a structural representation of lines joining the atoms of a mixture, and these atoms are constituent components of the mixture. For PFRC mixture, the constituent elements are the water, cement, fine aggregate (sand), coarse aggregate and polypropylene fibre. Thus, a simplex of five-component mixture is a four-dimensional solid. See Nwachukwu and others (2017) According to Obam (2009), mixture components are subject to the constraint that the sum of all the components must be equal to 1. That is:

$$X_1 + X_2 + X_3 + \dots + X_q = 1 \; ; \quad \Rightarrow \sum_{i=1}^{q} X_i = 1 \qquad \textbf{(1)}$$

where $X_i \geq 0$ and $i = 1, 2, 3\dots q$, and q = the number of mixtures

A. THE SIMPLEX LATTICE DESIGN

The (q, m) simplex lattice design are characterized by the symmetric arrangements of points within the experimental region and a well chosen polynomial equation to represent the response surface over the entire simplex region(Aggarwal, 2002). The (q, m) simplex lattice design given by Scheffe, according to Nwakonobi and Osadebe (2008) contains $^{q+m-1}C_m$ points where each components proportion takes (m+1) equally spaced values $X_i = 0, \frac{1}{m}, \frac{2}{m}, \frac{3}{m}, \dots, 1; \quad i = 1, 2, \dots, q$ ranging between 0 and 1 and all possible mixture with these component proportions are used, and m is scheffe's polynomial degee, which in this present study is 2.

For example a (3, 2) lattice consists of $^{3+2-1}C_2$ i.e. $^{4}C_2$ = 6 points. Each X_i can take m+1 = 3 possible values; that is $x = 0, \frac{1}{2}, 1$with which the possible design points are: $(1,0,0), (0,1,0), (0,0,1), \left(\frac{1}{2},\frac{1}{2},0\right), \left(0,\frac{1}{2},\frac{1}{2}\right), \left(\frac{1}{2},0,\frac{1}{2}\right)$.

According to Obam (2009), a Scheffe's polynomial function of degree, m in the q variable X_1, X_2, X_3,X_4 …X_q is given in form of:

$$Y = b_0 + \sum bi\ xi + \sum bijxj + \sum bijxjxk + + \sum bi^{j}{}_2 + \dots i_n xi_2 xi_n \qquad \textbf{(2)}$$

where ($1 \leq i \leq q$, $1 \leq i \leq j \leq k \leq q$, $1 \leq i_1 \leq i_2 \leq \dots \leq i_n \leq q$ respectively) , b = constant coefficients and Y is the response(the response is a polynomial function of pseudo component of the mix) which represents the property under study, which ,in this case is the compressive strength.

This research work is based on the Scheffe's(5, 2) simplex hence the actual form of Eqn. (2) will be developed for Scheffe's(5, 2) lattice subsequently.

B. RELATIONSHIP BETWEEN PSEUDO AND ACTUAL COMPONENTS.

In Scheffe's mix design, the relationship between the pseudo components and the actual components is given as:

$$Z = A * X \qquad \textbf{(3)}$$

where Z is the actual component; X is the pseudo component and A is the coefficient of the relationship

Re-arranging the equation

$$X = A^{-1} * Z \qquad \textbf{(4)}$$

In this research work a five component concrete mix constituents, cement, river sand as fine aggregate, granite as coarse aggregate, water/cement (w/c) ratio and polypropylene fibre will be on focus.

C. FORMULATION OF REGRESSION EQUATION FOR SCHEFFE'S (5, 2) LATTICE

The regression equation by Scheffe(1958), otherwise known as response is given in Eqn.(2) .Hence, for Scheffe's (5,2) simplex lattice, the regression equation for five component mixtures has been derived from Eqn.(2) by Nwachukwu and others (2017) and is given as follows:

$$Y = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_{11}X_1^2 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{15}X_1X_5 + b_{22}X_2^2 + b_{23}X_2X_3 + b_{24}X_2X_4 + b_{25}X_2X_5 + b_{33}X_3^3 + b_{34}X_3X_4 + b_{35}X_3X_5 + b_{44}X_4^4 + b_{45}X_4X_5 + b_{55}X_5^5 \quad \textbf{(5)}$$

$$= ß_1X_1 + ß_2X_2 + ß_3X_3 + ß_4X_4 + ß_5X_5 + ß_{12}X_1X_2 + ß_{13}X_1X_3 + ß_{14}X_1X_4 + ß_{15}X_1X_5 + ß_{23}X_2X_3 + ß_{24}X_2X_4 + ß_{25}X_2X_5 + ß_{34}X_3X_4 + ß_{35}X_3X_5 + ß_{45}X_4X_5 \quad \textbf{(6)}$$

Where,

$ß_1 = b_0 + b_1 + b_{11}$; $ß_2 = b_0 + b_2 + b_{22}$; $ß_3 = b_0 + b_3 + b_{33}$; $ß_4 = b_0 + b_4 + b_{44}$; $ß_5 = b_0 + b_5 + b_{55}$;

$ß_{12} = b_{12} – b_{11} – b_{22}$; $ß_{13} = b_{13} – b_{11} – b_{33}$; $ß_{14} = b_{14} – b_{11} – b_{44}$; $ß_{15} = b_{15} – b_{11} – b_{55}$; $ß_{23} = b_{23} – b_{22} – b_{33}$;

$ß_{24} = b_{24} – b_{22} – b_{44}$; $ß_{25} = b_{25} – b_{22} – b_{55}$; $ß_{34} = b_{34} – b_{35} – b_{44}$; $ß_{35} = b_{35} – b_{33} – b_{55}$;

$ß_{45} = b_{45} – b_{44} – b_{55}$. **(7)**

D. DETERMINATION OF THE COEFFICIENTS OF THESCHEFFE'S (5, 2) POLYNOMIAL

The procedure for the determination of the coefficient of Scheffe's (5,2) regression model has been explained by Nwachukwu and others (2017). Under is the mixture design model for optimization of PFRC, after evaluation of the coefficients.

$$Y = X_1(2X_1 – 1)Y_1 + X_2(2X_2 – 1)Y_2 + X_3(2X_3 – 1)Y_3 + X_4(2X_4 – 1)Y_4 + X_5(2X_5 – 1)Y_5 + 4Y_{12}X_1X_2 + 4Y_{13}X_1X_3 + 4Y_{14}X_1X_4 + 4Y_{15}X_1X_5 + 4Y_{23}X_2X_3 + 4Y_{24}X_2X_4 + 4Y_{25}X_2X_5 + 4Y_{34}X_3X_4 + 4Y_{35}X_3X_5 + 4Y_{45}X_4X_5 \quad \textbf{(8)}$$

Eqn. (8) is the second degree based mix design model for the optimization of a concrete mix that comprises five components, such as PFRC.Y_1 ,Y_2 ……. $Y_{45}$are determined through laboratory test.

E. ACTUAL AND PSEUDO MIX RATIO

The requirement of simplex lattice design that$\sum_{i=1}^{q} Xi = 1$ make it impossible to use the conventional mix ratios such as 1:2:4, 1:3:6, etc., at a given water/cement ratio for the actual mix ratio. This necessitates the transformation of the actual components (ingredients) proportions to meet the above criterion. Such transformed ratios, $x_1^{(i)}$, $x_2^{(i)}$, $x_3^{(i)}$, for the ith experimental points are called pseudo – components (or coded components). Based on experience and previous knowledge from literature, the following arbitrary prescribed mix proportions were chosen for the five points/vertices.

A_1 (0.67:1: 1.7: 2:0.5); A_2 (0.56:1:1.6:1.8:0.8); A_3 (0.5:1:1.2:1.7:1); A_4 (0.7:1:1:1.8:1.2) and A_5 (0.75:1:1.3:1.2:1.5), which represent water/cement ratio, cement, fine aggregate, coarse aggregate and polypropylene fibre.

For the pseudo mix ratio, we have the following corresponding mix ratios at the vertixes: A_1(1:0:0:0:0), A_2(0:1:0:0: 0), A_3(0:0:1:0:0), A_4(0:0:0:1:0), and A_5(0:0:0:0:1)

For the transformation of the actual component, z to pseudo component, x, and vice versa ,Eqns.(3)and (4) are used. Substituting the mix ratios from point A1 into Eqn. (3) gives:

$$\begin{Bmatrix} 0.67 \\ 1 \\ 1.7 \\ 2 \\ 0.5 \end{Bmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} \end{pmatrix} \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad \textbf{(9)}$$

Transforming the R.H matrix and solving

$$0.67 \equiv A_{11}(1) + A_{12}(0) + A_{13}(0) + A_{14}(0) + A_{15}(0)$$

$\Rightarrow$ $0.67 = A_{11}$ i.e. $A_{11} = 0.67$

Thus

$$1 = A_{21}(1) + A_{22}(0) + A_{23}(0) + A_{24}(0) + A_{25}(0)$$

$\Rightarrow$ $A_{21} = 1$

$$1.7 = A_{31}(1) + A_{32}(0) + A_{33}(0) + A_{34}(0) + A_{35}(0)$$

$\Rightarrow$ $A_{31} = 1.7$

Similarly;

$$2 = A_{41}(1) + A_{42}(0) + A_{43}(0) + A_{44}(0) + A_{45}(0)$$

Hence $A_{41} = 2$

Finally,

$$0.5 = A_{51}(1) + A_{52}(0)\ A_{53}(0)\ A_{54}(0)\ A_{55}(0)$$

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Therefore $A_{51} = 0.5$

The same goes for point 2 through point 5 and the results are depicted in Eqn. (10)
Thus we have

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1 & 1 & 1 & 1 & 1\ X_2 \\ 1.7 & 1.6 & 1.2 & 1 & 1.3 \\ 2 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1 & 1.2 & 1.5 \end{pmatrix} \begin{Bmatrix} X_1 \\ \\ X_3 \\ X_4 \\ X_5 \end{Bmatrix} \quad \textbf{(10)}$$

$$\begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{Bmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1 & 1 & 1 & 1 & 1 \\ 1.7 & 1.6 & 1.2 & 1 & 1.3 \\ 2 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1 & 1.2 & 1.5 \end{pmatrix}^{-1} \begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} \quad \textbf{(11)}$$

Thus

$$\begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{Bmatrix} = \begin{pmatrix} 3.99 & 10.37 & -2.14 & -3.05 & -4.62 \\ -4.88 & -21.46 & 5.40 & 5.95 & 7.31 \\ -1.78 & 17.83 & -3.49 & -4.20 & -4.62 \\ 1.04 & -9.24 & 0.37 & 3.28 & 2.69 \\ 1.63 & 3.49 & -0.13 & -1.98 & -0.77 \end{pmatrix} \begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3. \\ Z_4 \\ Z_5 \end{Bmatrix} \quad \textbf{(12)}$$

Considering the mix ratios at the midpoints, we have

A_{12} (0.5, 0.5, 0, 0, 0); A_{13} (0.5, 0, 0.5, 0, 0); A_{14} (0.5, 0, 0, 0.5, 0); A_{15} (0.5, 0, 0, 0, 0.5); A_{23} (0, 0.5, 0.5, 0,0); A_{24} (0, 0.5, 0, 0.5, 0); A_{25} (0, 0.5, 0, 0, 0.5); A_{34} (0, 0, 0.5, 0.5, 0); A_{35} (0, 0, 0.5, 0, 0.5) and A_{45} (0, 0, 0, 0.5, 0.5)

Substituting these pseudo mix ratios in turn into Eqn. (10) will give the corresponding actual mix ratio
For point A12

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1 & 1 & 1 & 1 & 1\ 0.5 \\ 1.7 & 1.6 & 1.2 & 1 & 1.3 \\ 2 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1 & 1.2 & 1.5 \end{pmatrix} \begin{pmatrix} 0.5 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{Bmatrix} 0.62 \\ \\ 1.65 \\ 1.90 \\ 0.65 \end{Bmatrix} \quad \textbf{(13)}$$

Hence comparing

$Z_1 = 0.62$, $Z_2 = 1$, $Z_3 = 1.65$, $Z_4 = 1.9$, $Z_5 = 0.65$

The rest are shown in Table 1

Hence to generate the regression coefficients, fifteen experimental tests will be carried out and the corresponding mix ratio are as depicted in Table 1 below

Points	Water/cement ratio	Cement	Fine Aggregate	Coarse Aggregate	Polypropylene fibre	Response
1	0.67	1	1.7	2	0.5	Y_1
2	0.56	1	1.6	1.8	0.8	Y_2
3	0.5	1	1.2	1.7	1	Y_3
4	0.7	1	1	1.8	1.2	Y_4
5	0.75	1	1.3	1.2	1.5	Y_5
12	0.62	1	1.65	1.9	0.65	Y_{12}
13	0.59	1	1.45	1.85	0.75	Y_{13}
14	0.69	1	1.35	1.9	0.85	Y_{14}
15	0.71	1	1.5	1.6	1	Y_{15}
23	0.53	1	1.4	1.75	0.9	Y_{23}
24	0.63	1	1.3	1.8	1	Y_{24}

25	0.66	1	1.45	1.5	1.15	Y_{25}
34	0.6	1	1.1	1.75	1.1	Y_{34}
35	0.63	1	1.25	1.45	1.25	Y_{35}
45	0.73	1	1.15	1.5	1.5	Y_{45}

Table 1: Actual mix ratios for theof Scheffe's (5, 2) lattice

F. CONTROL POINTS

For the purpose of this research, fifteen different controls were predicted which according to Scheffe, their summation should not be more than one

C_1 = (0.25, 0.25, 0.25, 0.25, 0), C_2 = (0.25, 0.25, 0.25, 0, 0.25), C_3 = (0.25, 0.25, 0, 0.25, 0.25), C_4 = (0.25, 0, 0.25, 0.25, 0.25), C_5 = (0, 0.25, 0.25, 0.25, 0.25), C_{12} = (0.20, 0.20, 0.20, 0.20, 0.20), C_{13} = (0.30, 0.30, 0.30, 0.10, 0), C_{14} = (0.30, 0.30, 0.30, 0, 0.10), C_{15} = (0.30, 0.30, 0, 0.30, 0.1), C_{23} = (0.30, 0, 0.30, 0.30, 0.1), C_{24} = (0, 0.30, 0.30, 0.30, 0.10), C_{25} = (0.10, 0.30, 0.30, 0.30, 0), C_{34} = (0.30, 0.10, 0.30, 0.30, 0), C_{35} = (0.30, 0.30, 0.10, 0.30, 0), C_{45} = (0.10, 0.20, 0.30, 0.40, 0),

Substituting into Eqn.(10) , we obtain the values of the actual mixes as follows:

Control 1 C_1

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1 & 1 & 1 & 1 & 1 \\ 1.7 & 1.6 & 1.2 & 1 & 1.3 \\ 2 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1 & 1.2 & 1.5 \end{pmatrix} \begin{pmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \\ 0 \end{pmatrix} = \begin{Bmatrix} 0.61 \\ 1 \\ 1.38 \\ 1.83 \\ 0.5 \end{Bmatrix} \quad \textbf{(14)}$$

The rest are depicted in Table 2

Points	Pseudo					Actual				
	water	cement	Fine agg	Coarse agg	Polypropylene fibre	water	cement	Fine agg	Coarse agg	Polypropylenefibre
C_1	0.25	0.25	0.25	0.25	0	0.61	1	1.38	1.83	0.5
C_2	0.25	0.25	0.25	0	0.25	0.62	1	1.45	1.68	0.8
C_3	0.25	0.25	0	0.25	0.25	0.67	1	1.4	1.7	1
C_4	0.25	0	0.25	0.25	0.25	0.66	1	1.3	1.68	1.2
C_5	0	0.25	0.25	0.25	0.25	0.63	1	1.28	1.63	1.5
C_{12}	0.2	0.2	0.2	0.2	0.2	0.64	1	1.36	1.7	0.65
C_{13}	0.3	0.3	0.3	0.1	0	0.59	1	1.45	1.83	0.75
C_{14}	0.3	0.3	0.3	0	0.1	0.59	1	1.48	1.77	0.85
C_{15}	0.3	0.3	0	0.3	0.1	0.65	1	1.42	1.8	1
C_{23}	0.3	0	0.3	0.3	0.1	0.64	1	1.3	1.77	0.9
C_{24}	0	0.3	0.3	0.3	0.1	0.6	1	1.27	1.71	1
C_{25}	0.1	0.3	0.3	0.3	0	0.6	1	1.31	1.79	1.15
C_{34}	0.3	0.1	0.3	0.3	0	0.62	1	1.33	1.83	1.1
C_{35}	0.3	0.3	0.1	0.3	0	0.63	1	1.41	1.85	1.25
C_{45}	0.1	0.2	0.3	0.4	0	0.61	1	1.25	1.79	1.35

Table 2: Actual (Z_i) and pseudo (X_i) component of Scheffe's (5, 2) simplex lattice

III. MATERIALS AND METHODS

A. MATERIALS

The materials investigated are the mixture of cement, water, fine and coarse aggregate and polypropylene fibre. The cement is Dangote cement, a brand of Ordinary Portland Cement, conforming to British Standard Institution BS 12 (1978). The fine aggregate, whose size ranges from 0.05 - 4.5mm was procured from the local river. Crushed granite of 20mm size downgraded to 4.75mm obtained from a local stone market was used in the experimental investigation .Also, Polypropylene Fibre shown in Figure 1 was used in the experimental investigation and water drawn from the clean water source.

B. METHOD

a) SPECIMEN PREPARATION / BATCHING/ CURING

The specimens for the compressive strength were concrete cubes. They were cast in steel mould measuring 150mm*150mm*150mm. The mould and its base were damped together during concrete casting to prevent leakage of mortar. Thin engine oil was applied to the inner surface of the moulds to make for easy removal of the cubes. Batching of all the constituent material was done by weight using a weighing balance of 50kg capacity based on the adapted mix ratios and water cement ratios. A total number of 30 mix ratios were to be used to produce 90 prototype concrete cube. Fifteen (15) out of the 30 mix ratios were as control

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mix ratios to produce 45 cubes for the conformation of the adequacy of the mixture design given by the Eqn. (8).. Curing commenced 24hours after moulding. The specimens were removed from the moulds and were placed in clean water for curing. After 28days of curing the specimens were taken out of the curing tank.

b) COMPRESSIVE STRENGTH TEST

Testing was conducted immediately after the specimen was removed from the curing process and dried. Smooth surface metal plate (serving as base plate) was placed at the bottom and top of each of the specimen cube so as to ensure uniform distribution of load for accurate crushing. Three samples were crushed for each mix ratio. The compressive strength was then calculated using the formula below:

$$\text{Compressive Strength} = \frac{\text{Average failure Load (N) } P}{\text{Cross- sectional Area (mm}^2\text{) } A} \quad \textbf{(15)}$$

IV. RESULTS AND DISCUSSION

A. Compressive Strength Test Results Based On Scheffe's (5,2) Simplex Lattice

a) Experimental Test Results

The result of compressive strength test basedon Eqn. (15) are shown in Table 3

Points	Experiment no	Response Y_i, N/mm^2	Response symbol	$\sum Y_i$	Average response Y, N/mm^2
1	1A 1B 1C	5.82 5.95 7.55	Y_1	19.32	6.44
2	2A 2B 2C	7.99 9.34 9.19	Y_2	26.52	8.84
3	3A 3B 3C	8.72 9.86 10.37	Y_3	28.95	9.65
4	4A 4B 4C	5.74 6.79 6.52	Y_4	19.05	6.35
5	5A 5B 5C	20.58 22.88 21.64	Y_5	35.1	22.7
12	6A 6B 6C	6.47 7.43 7.58	Y_{12}	21.48	7.16
13	7A 7B 7C	6.7 7.04 8.49	Y_{13}	22.23	7.41
14	8A 8B 8C	5.47 5.98 6.79	Y_{14}	18.24	6.08
15	9A 9B 9C	7.32 7.96 9.11	Y_{15}	24.39	8.13
23	10A 10B 10C	8.31 9.54 9.84	Y_{23}	27.69	9.23
24	11A 11B 11C	6.71 7.36 8.31	Y_{24}	22.38	7.46
25	12A 12B 12C	8.82 11.12 9.46	Y_{25}	29.4	9.8
34	13A 13B 13C	6.95 8.76 7.45	Y_{34}	23.16	7.72
35	14A 14B 14C	9.21 11.61 9.87	Y_{35}	30.69	10.23
	15A	7.61			

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45	15B 15C	9.6 8.16	Y_{45}	25.37	8.46

Table 3: Compressive Strength Test Results Based on Eqn.(15)

Thus substituting the values of Y_1, Y_2, …. Y_{45} into Eqn. (8) yields:

$$Y = 6.44X_1(2X_1 - 1) + 8.84X_2(2X_2 - 1) + 9.65X_3(2X_3 - 1) + 6.35X_4(2X_4 - 1) + 22.7X_5(2X_5 - 1) + 4(7.16)X_1X_2 + 4(7.41)X_1X_3 + 4(6.08)X_1X_4 + 4(8.13)X_1X_5 + 4(9.23)X_2X_3 + 4(7.46)X_2X_4 + 4(9.8)X_2X_5 + 4(7.72)X_3X_4 + 4(10.23)X_3X_5 + 4(8.46)X_4X_5 \quad \textbf{(16)}$$

b) Experimental (Control) Test Results
The response (compressive strength) of control points from experimental tests is shown in Table 4

Points	Experiment no	Response N/mm^2	Z_1	Z_2	Z_3	Z_4	Z_5	Average response	
C1	1A 1B 1C	9.38 11.83 10.06	0.61	1	1.38	1.83	0.5	10.42	10.42
C2	2A 2B 2C	8.14 10.26 8.72	0.62	1	1.45	1.68	0.8	9.04	9.04
C3	3A 3B 3C	8.77 8.32 4.9	0.67	1	1.4	1.7	1	7.33	7.33
C4	4A 4B 4C	9.44 8.96 5.27	0.66	1	1.3	1.68	1.2	7.89	7.89
C5	5A 5B 5C	19.33 19.54 18.56	0.63	1	1.28	1.63	1.5	19.81	12.81
C12	6A 6B 6C	12.89 12.22 7.19	0.64	1	1.36	1.7	0.65	10.77	10.77
C13	7A 7B 7C	9.1 8.63 5.08	0.59	1	1.45	1.83	0.75	7.6	7.6
C14	8A 8B 8C	9.7 7.94 6.67	0.59	1	1.48	1.77	0.85	8.1	8.1
C15	9A 9B 9C	8.44 6.91 5.8	0.65	1	1.42	1.8	1	7.05	7.05
C23	10A 10B 10C	8.68 7.11 5.97	0.64	1	1.3	1.77	0.9	7.25	7.25
C24	11A 11B 11C	9.62 7.88 6.62	0.6	1	1.27	1.71	1	8.04	8.04
C25	12A 12B 12C	9.53 7.8 6.55	0.6	1	1.31	1.79	1.15	7.96	7.96
C34	13A 13B 13C	9.74 7.98 6.7	0.62	1	1.33	1.83	1.1	8.14	8.14
C35	14A 14B 14C	12.62 10.33 8.67	0.63	1	1.41	1.85	1.25	10.54	10.54
C45	15A 15B 15C	13.19 10.8 9.07	0.61	1	1.25	1.79	1.35	11.02	11.02

Table 4: Response of Control Points from Experimental Tests (5, 2) Simplex Lattice

c) SCHEFFE'S (5,2) SIMPLEX MODEL RESULTS

1) RESPONSE OF EXPERIMENTAL POINTS FROM SCHEFFE'S (5, 2) SIMPLEX MODEL RESULTS

By substituting the pseudo mix ratio points of the initial experiment A_1, A_2, A_3, A_4, A_5, A_{12}, A_{13}, A_{14}, A_{15}, A_{23}, A_{24}, A_{25}, A_{34}, A_{35}, and A_{45} of Table 1 into Eqn. (16), we obtain the second model response as shown in Table 5 below.

points	X_1	X_2	X_3	X_4	X_5	Response N/mm^2
1	1	0	0	0	0	6.47
2	0	1	0	0	0	8.88
3	0	0	1	0	0	9.69
4	0	0	0	1	0	6.39
5	0	0	0	0	1	25.23
12	0.5	0.5	0	0	0	7.14
13	0.5	0	0.5	0	0	17.38
14	0.5	0	0	0.5	0	6.08
15	0.5	0	0	0	0.5	8.17
23	0	0.5	0.5	0	0	9.27
24	0	0.5	0	0.5	0	7.5
25	0	0.5	0	0	0.5	9.76
34	0	0	0.5	0.5	0	7.76
35	0	0	0.5	0	0.5	10.19
45	0	0	0	0.5	0.5	8.44

Table 5: Response of Experimental Points from Model (5, 2) Simplex lattice

2) RESPONSE OF CONTROL POINTS FROM SCHEFFE'S (5,2) SIMPLEX MODEL RESULTS

By substituting the pseudo mix ratio into points c_1, c_2, c_3, c_4, c_5, c_{12}, c_{13}, c_{14}, c_{15}, c_{23}, c_{24}, c_{25}, c_{34}, c_{35}, and c_{45} of Table 2 into Eqn.(16) , we obtain the second order model response as shown in Table 6

Points	X_1	X_2	X_3	X_4	X_5	Response, N/mm^2
C_1	0.25	0.25	0.25	0.25	0	9.38
C_2	0.25	0.25	0.25	0	0.25	8.43
C_3	0.25	0.25	0	0.25	0.25	7.65
C_4	0.25	0	0.25	0.25	0.25	8.86
C_5	0	0.25	0.25	0.25	0.25	20.1
C_{12}	0.2	0.2	0.2	0.2	0.2	9.71
C_{13}	0.3	0.3	0.3	0.1	0	7.5
C_{14}	0.3	0.3	0.3	0	0.1	8.16
C_{15}	0.3	0.3	0	0.3	0.1	7.82
C_{23}	0.3	0	0.3	0.3	0.1	7.1
C_{24}	0	0.3	0.3	0.3	0.1	8.08
C_{25}	0.1	0.3	0.3	0.3	0	10.05
C_{34}	0.3	0.1	0.3	0.3	0	9.11
C_{35}	0.3	0.3	0.1	0.3	0	12.94
C_{45}	0.1	0.2	0.3	0.4	0	13.34

Table 6: Response of Control points from Model (5, 2) Simplex Lattice

d) SUMMARY OF RESPONSES FROMSCHEFFE'S (5,2) SIMPLEX

Table 7 shows the summary of responses from Scheffe's (5, 2) simplex

S/No	Experimental Test Results	Scheffe Model Results	Control Points	Experimental Test Results	Scheffe Model Results
1	6.44	6.47	C_1	10.42	9.38
2	8.84	8.88	C_2	9.04	8.43
3	9.65	9.69	C_3	7.33	7.65
4	6.35	6.39	C_4	7.89	8.86
5	22.70	25.23	C_5	19.81	20.1
12	7.16	7.14	C_{12}	10.77	9.71
13	7.41	7.38	C_{13}	7.6	7.5
14	6.08	6.08	C_{14}	8.1	8.16
15	8.13	8.17	C_{15}	7.05	7.82
23	9.23	9.27	C_{23}	7.25	7.1

24	7.46	7.5	C_{24}	8.04	8.08
25	9.8	9.76	C_{25}	7.96	10.05
34	7.72	7.76	C_{34}	8.14	9.11
35	10.23	10.19	C_{35}	10.54	12.94
45	8.46	8.44	C_{45}	11.02	13.34

Table 7: Summary of Responses of Scheffe's (5, 2) Simplex

e) TEST OF THE ADEQUACY OF THE MODEL USING STUDENT'S – T - TEST BETWEEN LAB RESULT ANDSCHEFFE'S (5, 2) SIMPLEX LATTICE RESULT

Here we are to determine if there is any significant difference between the lab responses (results) given in Table 4 and model responses given in Table 6. Table 8 begins the procedure.

Parameter	Lab result	Model result
$\sum x_i$(sum of responses)	133.96	143.23
$\sum (x_i)^2$	1238.79	1279.99
$\sum (x_i)^2 - \frac{(\sum xi)2}{15}$	1238.79 – 1196.35 = 42.44	1447.44 – 1367.66 = 79.78
Mean	$X_1 = 8.93$	$X_2 = 9.55$
variance	$S_1^2 = 42.44/14 = 3.032$	$S_2^2 = 79.78/14 = 5.698$

Table 8: Students't – test between Lab result and Model result for Scheffe's (5, 2) Simplex Lattice

Since the sample sizes to lab results and model results are not equal, then the combined unbiased estimate of variance is given by

$$S^2 = \frac{(N_1 - 1)\, S_1^2 + (N_2 - 1)\, S_2^2}{N_1 + N_2 - 2}$$

Where N_1 (for experimental result) = 15 and N_2 (for second ordermodel result) = 15

$\Rightarrow$

$$S^2 = \frac{(15 - 1)*3.032 + (15 - 1)*5.698}{15 + 15 - 2} = 4.365$$

$\Rightarrow$ S = 2.089

• Hypothesis

Let μ_1 and μ_2 represent the true mean of the results (responses) for lab and model respectively. Then the null and model alternative hypothesis are given as

H_0: $\mu_1 = \mu_2$ and there is no difference in the sample mean s, if any it is merely due to chance

H_1: $\mu_1 \neq \mu_2$ and there is a significant difference in the sample means of responses from the lab and model results..

$$\text{Then, } t = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\left(\frac{1}{N1} + \frac{1}{N2}\right)}}$$

$$= \frac{5.68 - 3.032}{2.089\sqrt{\left(\frac{1}{15} + \frac{1}{15}\right)}} = 2.648/5.721 = 0.463$$

$\Rightarrow$ **t = 0.463**

from the students' t – distribution table, Assuming 1% level significance.

Degree of freedom v = $N_1 + N_2 - k$

V = 15 + 15 – 2 = 28

Then $t_{0.005}$,28 = 2.76 > t = 0.463

Thus the Null hypothesis ($\mu_1 = \mu_2$) is accepted. That is, there is no significant difference between the experimental results and model results.Thus, the model is adequate for predicting the compressive strength of PFRC.

B. DISCUSSION OF RESULTS

Using Scheffe's (5,2) simplex model the values of the compressive strength were obtained. The model gave highest compressive strength of 25.23 Nmm^{-2} corresponding to mix ratio of 0.75:1:1:3:1.2:1.5 for water, cement, fine and coarse aggregate and polypropylene fibre respectively. The lowest strength was found to be 6.08Nmm^{-2} corresponding to mix ratio of 0.69:1:1.35:1.9:0.85. The maximum strength value from themodel wasgreater than the minimum value specified by the American Concrete Institute for the compressive strength of good concrete.Using the model, compressive strength of all points in the simplex can be derived.

V. CONCLUSION

Scheffe's second degree polynomial (5,2) was used to formulate a model for predicting the compressive strength of PFRC cubes. This model could predict the compressive strength of the PFRC concrete cubes if the mix ratios are known and vice versa. The strengths predicted by the models are in good agreement with the corresponding experimentally observed results.The optimum attainable compressive strength predicted by the Scheffe's (5,2) model at the 28th day was 25.23N/mm^{2}. . This meets the minimum standard requirement stipulated by American Concrete Institute (ACI) of 20N/mm^2 for the compressive strength.. With the model, any desired strength of Polypropylene Fibre Reinforced Concrete, given any mix proportions can be easily evaluated.Thus the problem of having to go through a vigorous mix- design procedure for a desiring strength has been reduced by utilizing this model.

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