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Effect of Preservation Technology on a Supply Chain Inventory Model with Time Varying Holding Cost, Price and Stock Dependent Demand under Replenishment Policy

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Abstract:- We look at a supply chain inventory model with time-varying holding costs, price and stock-dependent demand under a replenishment policy in this work. Both the length of non-deterioration time and the rate of deterioration are affected by preservation technology. We investigate two scenarios in this model: Shortages occur after or before the non-deterioration phase. In this paper, we show that for any given price and preservation investment policies, there exists a global replenishment policy. A numerical example is provided to explain the development model. In terms of important parameters, the optimum solution is also subjected to a sensitivity analysis.

Keywords:- *Time varying holding cost, price/stock dependent demand, replenishment rate, preservation technology, inflation.*

I. INTRODUCTION

In this paper we study a joint pricing, replenishment and preservation technology investment problem for non-instantaneous deteriorating items. Preservation technology affects both the length of non-deterioration period and deterioration rate. Shortages are allowed and partially backlogged. We use price-dependent and stock dependent demand, time-varying deterioration and waiting-time-dependent backlog rates in a general framework to formulate the model with time dependent holding cost. We consider two cases: shortages happen after or before the non-deterioration period. We analytically show the existence and uniqueness of the optimal replenishment schedule, price or preservation investment for any given two of them in two cases. We also prove that there exists a global replenishment policy for any given pricing and preservation investment policies. We then provide an iterative algorithm to search for the optimal solution.

Deterioration is a common phenomenon in inventory management, especially for food industry. About 20% of food never reaches consumers' table because of spoilage. Safeway grocery store stated that 63% of supermarket disposed waste in US comes from food industry and on average each employee throws away 3,000 pounds annually.

All food products undergo certain degrees of deterioration due to physical, chemical and microbiological changes. The deterioration/spoilage of food products, such as fruit and vegetables, is no accident but a natural process. The item decomposes from its harvested moment, and maintains its desired quality attributes for a period called "shelf life". At the end of its shelf life, the item deteriorates to a quality point that is below the standard set by the retailer, or the item is not even edible to people. We refer to the time period that no items in a batch need to be disposed as "non-deterioration period".

In Classical inventory problems, it is assumed that products have an infinite shelf life, while the most of items lose their initial values over time and for some of them this occurs faster than usual which is called deterioration. The criticise articles by Raafat (1991), Shah & Shah (2000), Aggoun and Benkhenouf (2001), Sett et al. (2012), on deteriorating items for inventory system throw light on the part of deterioration. Chung and Barron (2013) established the simplified solution. Procedure for deteriorating items under stock dependent demand and two-level trade credit in the supply chain management.

On the other hand to reduce deterioration, use preservation technology, Hsu et al. (2010) proposed a deteriorating inventory model with constant demand rate and exponential decay which the retailer is allowed to invest the preservation technology to reduce the deterioration rate. Dye and Hsieh (2012) presented an extended model of Hsu et al. (2010) by assuming the preservation technology cost is a function of the length of replenishment cycle. Yang et al. (2015) examined an optimal dynamic decision-making problem for a retailer selling a single deteriorating product, the demand rate of which varies simultaneously with time and the length of credit period that is offered to the customers. The deterioration rate was time dependent and can be reduced by an investment in preservation technology. Tayal et al. (2016) developed an integrated inventory policy, when the retailer invests on the preservation technology to reduce the product rate of deterioration and to investigated the model under conditions of permissible delay in payment and allowable shortages.

Several researchers like Giri and Bardhan (2015), Saha (2017), and Sheikh et al. (2017) derived their models considering price dependent demand. Mashud et al. (2018) considered a non-instantaneous inventory model having different deterioration rates with price as well as dependent demand. An inventory model for deteriorating items with

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price-dependent demand under two-level trade credit policy considered by Rameswari, et al. (2018). Biswas & Giri (2019) derived an integrated model for deteriorating items with partial backlogging and price dependent market demand. An inventory model for deteriorating item with preservation technology in time-dependent demand with trade credit facility and partial backlogging was developed by Shaikh et al. (2019). Saha & Sen (2019) Presented an inventory model with selling price and time-dependent demand, constant holding cost and time dependent deterioration. Singh and Rana (2020) considered an effect of inflation and variable holding cost on life time inventory model with multi-variable demand and lost sales.

In the presented work, we have presumed that the inventory model is described variable deterioration, variable holding cost, price/stock dependent demand rate and partially backlogged shortages. Moreover, in this model inflation taken and the replenishment rate is considered to be infinite.

II. ASSUMPTION AND NOTATION

This model is developed under the given assumptions and notations.

A. *Assumptions:*

The assumptions used in this manuscript are as follows:

1. The demand rate of items is stock and price dependent .i.e. (D = a – b×P – c× Q(t))
   Where a , b, c are non-zero constants.
2. The time horizon is infinite.
3. The Rate of deterioration is constant .
4. The holding cost varies with time.
5. Shortages are partially backlogged and are fulfilled at the beginning of the next cycle.
6. Replenishment rate is considered to be infinite.
7. Inflation is considered in this Model.
8. Lead time is considered to be negligible.

B. *Notations:*

The summary of notations:

Q^o	Initial Inventory level.
T_0	Original non-deterioration period without preservation technology investment.
T_d	Non-deterioration period with preservation technology investment.
M	Proportion of reduced deterioration rate with preservation technology Investment.
u	purchasing cost per unit.
p	retail price per unit.
D	variable demand rate.
P	coefficient of price dependent demand rate.
Q(t)	Inventory level at time "t"
s	Backlogged demand during shortage period.
λ_1	Length of inventory holding period with s and $\lambda_1 > 0$
λ	Preservation technology investment per unit time.
A	Ordering cost for whole inventory.
T	Length of replenishment cycle.
H	Holding cost for whole inventory.
O	opportunity cost per unit lost sale.
r	Rate of inflation 0<r<1.
β	Backlogged coefficient 0<β<1.
d	Deterioration cost per unit per unit time.
θ	Deterioration rate.
h_1 ,h_2	Non zero constants used in holding cost.
TP_1	Total profit for one replenishment cycle. ($\lambda_1 \geq T_d$)
TP_2	Total profit for one replenishment cycle. ($\lambda_1 \leq T_d$)
AP_1	Average profit per unit time. ($\lambda_1 \geq T_d$)
AP_2	Average profit for per unit time. ($\lambda_1 \leq T_d$)

III. MATHEMATICAL FORMULATION

In case 1, At initial state i.e. t = 0 the inventory level is Q^0 units of item. During t = 0 to T_d, deterioration is not considered and preservation technology is used. Deterioration starts during t = T_d to λ_1. After time t = λ_1 shortage occurs.

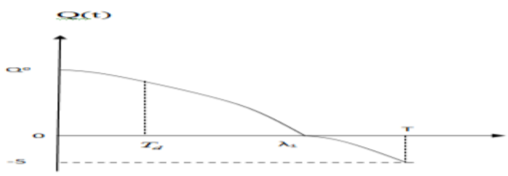

Fig.1: Inventory time graph for Case 1: $\lambda_1 \geq$ Td

In case 2, At initial state t = 0 the inventory is Q0 units of item. Deterioration is not considering in this case. At time t = T_d shortage occurs.

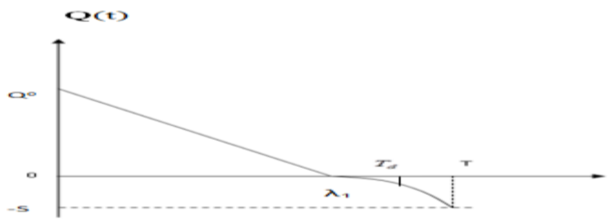

Fig. 2: Inventory time graph for Case 2: $\lambda_1 \leq$ Td

$$\frac{dQ_1(t)}{dt} = -D \qquad for(0 \le t \le T_d), \qquad (1a)$$

$$\frac{dQ_1(t)}{dt} = -(a - bP(s) + cQ_1(t)) \qquad (2a)$$

$$Q_1(t) = (a-bP)[(\lambda_1 - T_d) + \frac{c(\lambda_1^2 - T_d^2)}{2} + \frac{\theta(1-m)(\lambda_1^3 + T_d^3)}{6} -$$

$$\frac{T_d(\lambda_1^2 + T_d^2)}{2}]e^{-ct+\frac{\theta(1-m)T_d^2}{2}}$$

$$\frac{dQ_2(t)}{dt} = -D - \theta(1-m)(t - T_d)Q_2(t), \ for(T_d \le t \le \lambda_1) \qquad (3a)$$

$$Q_2(t) = (a-bP)((\lambda_1 - t) + \frac{c(\lambda_1^2 - t^2)}{2} + \theta(1-m)((\frac{\lambda_1^3 + t^3}{6}) -$$

$$T_d(\frac{\lambda_1^2 + t^2}{2})))e^{-ct-\theta(1-m)(\frac{t^2}{2} - tT_d)} \qquad (4a)$$

$$\frac{dQ_3(t)}{dt} = -\beta D, \ for(\lambda_1 \le t \le T) \qquad (5a)$$

$$Q_3(t) = \frac{(a-bP)\beta c(\lambda_1 - t)}{c} \qquad (6a)$$

$$With\ initial\ conditions\ , \mathrm{Q}_1(0) = \mathrm{Q}^0, \mathrm{Q}_1(T_d) = \mathrm{Q}_2(T_d),$$

$$\mathrm{Q}_2(\lambda_1) = \mathrm{Q}_3(\lambda_1), \mathrm{Q}_3(\mathrm{T}) = 0$$

Case 1: ($\lambda_1 \ge T_d$) Differential equation for inventory level is given by

$$\frac{dQ_1(t)}{dt} = -D .... (1b)\ for(0 \le t \le \lambda_1)$$

$$Q_1(t) = \frac{(a-bP)(e^{c(\lambda_1 - t)} - 1)}{c} \qquad (1b)$$

$$\frac{dQ_2(t)}{dt} = -\beta D .... (2b)\ for(\lambda_1 \le t \le T)$$

$$Q_2(t) = \frac{\beta(a-bP)(e^{c(\lambda_1 - t)} - 1)}{c} \qquad (2b)$$

Case 2: ($\lambda_1 \le T_d$) Differential equation for inventory level is given by

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With initial conditions $Q_1(0) = Q^0$, $Q_2(\lambda_1) = 0$,

$Q_2(T) = -S_1$

$$Q^0 = \frac{(a-bP)(e^{c\lambda_1}-1)}{c}$$

IV. COSTS FOR THE INVENTORY PROBLEM

Case:1 ($\lambda_1 \geq T_d$) When time λ_1 is greater than or equal to the time T_d. The costs are as follows:

A. Sales revenue cost:

$$S.R.C. = \int_0^{\lambda_1} D(t)e^{-rt}dt\,.$$

Total revenue cost for the inventory model for case 1 is given as follows:

$$\begin{aligned}
S.R.C. &= (a-bP)\lambda_1 + \frac{c(a-bP)}{c+r}((\lambda_1 - T_d) + c(\lambda_1^2 - T_d)) + \theta(1-m)(\frac{\lambda_1 + T_d^2}{6}) \\
&-T_d(\frac{\lambda_1^2 + T_d^2}{2})(1 - \frac{\theta(1-m)T_d^2}{2})((c+r)T_d) + c(a-bP)((\frac{\lambda_1^2}{2} + \frac{(c-2T_d)}{3})\lambda_1^3 \\
&+\frac{\theta(1-m)\lambda_1^4}{3} - T_d(\lambda_1 + c\lambda_1^2 + \frac{\theta(1-m)\lambda_1^3}{6} - T_d^2(\frac{\lambda_1^2 - 1}{2}) + \frac{T_d^3 c}{6} - \frac{T_d^4}{6}(\theta(1-m)-1)) \\
&+((c+r-\theta(1-m)T_d)(\frac{\lambda_1^3}{6} + \lambda_1^4\frac{(c-3T_d)}{8}) + \frac{7\theta(1-m)\lambda_1^5}{60} - T_d^2(\frac{\lambda_1}{2} + \frac{c\lambda_1^2}{4}) \\
&+T_d^3\frac{(4+3\lambda_1^2)}{12} + \frac{T_d^4 c}{8} + T_d^5\frac{(15-4\theta(1-m))}{120} + (\theta(1-m)(\frac{\lambda_1^4}{24} + \frac{\lambda_1^5(c+4T_d)}{30} \\
&+\frac{\theta(1-m)\lambda_1^2}{24} - T_d^3\frac{(2\lambda_1 + c\lambda_1^2)}{12} - \frac{\theta(1-m)\lambda_1^3}{36} - \frac{\lambda_1^2 T_d^4}{12} + \frac{cT_d^5}{20} - \frac{T_d^6(18+5\theta(1-m))}{360}
\end{aligned}$$

B. Fixed ordering : 'A'.

C. Purchasing cost:

$$P.C. = u(Q^0 + Q_3)$$

$$\text{P.C.} = u((a-bP)(\lambda_1 - T_d) + \frac{c(\lambda_1^2 - T_d^2)}{2} + \frac{\theta(1-m)(T_d^3 + \lambda_1^3)}{6}$$
$$-\frac{T_d(T_d^2 + \lambda_1^2)}{2} + (a-bP)\beta(\lambda_1 - t))$$

D. Preservation technology investment:

The cost for preservation technology investment in this model is given by '$\chi\ \lambda_1$'.

E. Holding cost:

$$H.C.=\int_0^{\lambda_1}(h_1+th_2)Q(t)e^{-rt}dt$$

$$H.C.=h_1(\frac{(a-bP)}{(c+r)}((\lambda_1-T_d)+c(\lambda_1^2-T_d))+\theta(1-m)(\frac{\lambda_1+T_d^2}{6})-T_d(\frac{\lambda_1^2+T_d^2}{2})$$

$$(1-\frac{\theta(1-m)T_d^2}{2})((c+r)T_d)+(a-bP)((\frac{\lambda_1^2}{2}+\frac{(c-2T_d)}{3})\lambda_1^3+$$

$$\frac{\theta(1-m)\lambda_1^4}{3}-T_d(\lambda_1+c\lambda_1^2+\frac{\theta(1-m)\lambda_1^3}{6}-T_d^2(\frac{\lambda_1^2-1}{2})+\frac{T_d^3c}{6}-\frac{T_d^4}{6}(\theta(1-m)-1))$$

$$+((c+r-\theta(1-m)T_d)(\frac{\lambda_1^3}{6}+\lambda_1^4\frac{(c-3T_d)}{8})+\frac{7\theta(1-m)\lambda_1^5}{60}-T_d^2(\frac{\lambda_1}{2}+\frac{c\lambda_1^2}{4})$$

$$+T_d^3\frac{(4+3\lambda_1^2)}{12}+\frac{T_d^4c}{8}+T_d^5\frac{(15-4\theta(1-m))}{120}+(\theta(1-m)(\frac{\lambda_1^4}{24}+\frac{\lambda_1^5(c+4T_d)}{30}+\frac{\theta(1-m)\lambda_1^2}{24}$$

$$-T_d^3\frac{(2\lambda_1+c\lambda_1^2)}{12}+\frac{\theta(1-m)\lambda_1^3}{36}-\frac{\lambda_1^2T_d^4}{12}+\frac{cT_d^5}{20}-\frac{T_d^6(18+5\theta(1-m))}{360})$$

$$+h_2((a-bP)((\lambda_1-T_d)+(c(\lambda_1^2-T_d^2))+\frac{\theta(1-m)(\lambda_1^3+T_d^3)}{6}-\frac{T_d(\lambda_1^2+T_d^2)}{2}$$

$$(\frac{T_d^2}{2}-\frac{(c+r)T_d^3}{3}-\frac{\theta(1-m)T_d^4}{4})+(a-bP)((\frac{\lambda_1^3}{6}+\frac{c\lambda_1^4}{8}+\frac{7\theta(1-m)\lambda_1^5}{60}-\frac{3T_d\lambda_1^4}{8}$$

$$-\frac{(3\lambda_1T_d^2-2T_d^3)}{6}-\frac{c(2\lambda_1^2T_d^2-T_d^4)}{8}-\theta(1-m)(\frac{\lambda_1^3T_d^2}{12}+\frac{T_d^5}{30})+\frac{T_d(2\lambda_1^2T_d^2+T_d^4)}{8})-$$

$$(r+c+\theta(1-m)T_d)(\frac{\lambda_1^4}{12}+\frac{c\lambda_1^5}{15}+\frac{\theta(1-m)\lambda_1^6}{12})-\frac{4T_d\lambda_1^5}{15}-\frac{(4\lambda_1T_d^3-3T_d^4)}{12}-\frac{c(5\lambda_1^2T_d^3-3T_d^5)}{30}$$

$$-\frac{\theta(1-m)(2\lambda_1^3T_d^3+T_d^6)}{36}+\frac{T_d(5\lambda_1^2T_d^3+3T_d^5)}{30})+\theta(1-m)(\frac{\lambda_1^5}{40}+\frac{c\lambda_1^6}{48}+\frac{2\theta(1-m)\lambda_1^7}{21}-$$

$$\frac{5T_d\lambda_1^6}{48}-\frac{(5\lambda_1T_d^4-4T_d^5)}{40}-\frac{c(3\lambda_1^2T_d^4-2T_d^6)}{24}-\frac{\theta(1-m)(7\lambda_1^3T_d^4+T_d^7)}{84}+T_d\frac{(3T_d^4\lambda_1^2+2T_d^6)}{48}))$$

F. Shortage cost:

$$S.C.=-s\int_{\lambda_1}^{T}Q_3(t)e^{-rt}dt$$

$$S.C.=-s\beta(a-bP)(\lambda_1(T+\frac{rT^2}{2})+\lambda_1^2\frac{(\lambda_1r-1)}{2}-\frac{r\lambda_1^3}{3}-\frac{T^2}{2}+\frac{rT^3}{3}))$$

G. Deterioration cost:

$$D.C.=d\int_{T_d}^{\lambda_1}\theta(t-T_d)Q_2(t)e^{-rt}dt$$

$$D.C.=d\theta((a-bP)((\frac{\lambda_1^3}{6}+\frac{c\lambda_1^4}{8}+\frac{7\theta(1-m)\lambda_1^5}{60}-\frac{3T_d\lambda_1^4}{8}-\frac{(3\lambda_1T_d^2-2T_d^3)}{6}-\frac{c(2\lambda_1^2T_d^2-T_d^4)}{8}$$
$$-\theta(1-m)(\frac{\lambda_1^3T_d^2}{12}+\frac{T_d^5}{30})+\frac{T_d(2\lambda_1^2T_d^2+T_d^4)}{8})-(r+c+\theta(1-m)T_d)(\frac{\lambda_1^4}{12}+\frac{c\lambda_1^5}{15}+\frac{\theta(1-m)\lambda_1^6}{12})$$
$$-\frac{4T_d\lambda_1^5}{15}-\frac{(4\lambda_1T_d^3-3T_d^4)}{12}-\frac{c(5\lambda_1^2T_d^3-3T_d^5)}{30}-\frac{\theta(1-m)(2\lambda_1^3T_d^3+T_d^6)}{36}+\frac{T_d(5\lambda_1^2T_d^3+3T_d^5)}{30})$$
$$+\theta(1-m)(\frac{\lambda_1^5}{40}+\frac{c\lambda_1^6}{48}+\frac{2\theta(1-m)\lambda_1^7}{21}-\frac{5T_d\lambda_1^6}{48}-\frac{(5\lambda_1T_d^4-4T_d^5)}{40}-\frac{c(3\lambda_1^2T_d^4-2T_d^6)}{24}-$$
$$\frac{\theta(1-m)(7\lambda_1^3T_d^4+T_d^7)}{84}+T_d\frac{(3T_d^4\lambda_1^2+2T_d^6)}{48})-T_d+(a-bP)((\frac{\lambda_1^2}{2}+\frac{(c-2T_d)}{3})\lambda_1^3+$$
$$((c+r-\theta(1-m)T_d)(\frac{\lambda_1^3}{6}+\lambda_1^4\frac{(c-3T_d)}{8})+\frac{7\theta(1-m)\lambda_1^5}{60}-T_d^2(\frac{\lambda_1}{2}+\frac{c\lambda_1^2}{4})\frac{\theta(1-m)\lambda_1^4}{3}$$
$$-T_d(\lambda_1+c\lambda_1^2+\frac{\theta(1-m)\lambda_1^3}{6}-T_d^2(\frac{\lambda_1^2-1}{2})+\frac{T_d^3c}{6}-\frac{T_d^4}{6}(\theta(1-m)-1))+T_d^3\frac{(4+3\lambda_1^2)}{12}+\frac{T_d^4c}{8}$$
$$+T_d^5\frac{(15-4\theta(1-m))}{120}+(\theta(1-m)(\frac{\lambda_1^4}{24}+\frac{\lambda_1^5(c+4T_d)}{30}+\frac{\theta(1-m)\lambda_1^2}{24}-T_d^3\frac{(2\lambda_1+c\lambda_1^2)}{12}$$
$$\frac{\theta(1-m)\lambda_1^3}{36})-\frac{\lambda_1^2T_d^4}{12}+\frac{cT_d^5}{20}-\frac{T_d^6(18+5\theta(1-m))}{360}))$$

H. Opportunity Cost:

$$O.P.=o\int_{\lambda_1}^{T}(1-\beta)D(t)dt$$

$$=o(1-\beta)(a-bP)(\frac{r(T-\lambda_1)}{r}+c\beta(\lambda_1(T+\frac{rT^2}{2})+\lambda_1^2\frac{(\lambda_1r-1)}{2}$$
$$-\frac{r\lambda_1^3}{3}-\frac{T^2}{2}+\frac{rT^3}{3})$$

Case 2: ($\lambda_1\le T_d$) When time λ_1 is less than or equal to the time T_d. The costs are as follows:

a) Sales revenue cost:

$$S.R.C.=\int_{0}^{\lambda_1}D(t)e^{-rt}dt$$

$$S.R.C.=\frac{(a-bP)(e^{c\lambda_1}-e^{-r\lambda_1})}{(c+r)}$$

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b) Fixed ordering:

This cost is fixed and let this cost is 'A'.

c) Holding cost :

$$H.C. = \int_0^{\lambda_1} (h_1 + th_2)Q(t)e^{-rt}dt$$

$$H.C. = h_1 \frac{(a-bP)(e^{c\lambda_1} - e^{-r\lambda_1})}{(c+r)} + h_2(a-bP)(\frac{(e^{-r\lambda_1} - 1)}{(c+r)^2} - \lambda_1 \frac{e^{-r\lambda_1}}{(c+r)})$$

d) Purchasing cost:

$$P.C. = uQ_1$$

$$P.C. = u\frac{(a-bP)}{c}(e^{c(\lambda_1 - t)} - 1)$$

e) Preservation technology investment:

The cost for preservation technology investment in this model is given by '$\chi \lambda_1$'.

f) Shortage cost:

The shortage cost for this model is given as follows

$$S.C. = -s\int_{\lambda_1}^{T} Q_2(t)e^{-rt}dt$$

$$S.C. == -s\beta \frac{(a-bp)}{c}(\frac{(e^{-r\lambda_1} - e^{c\lambda_1 - (c+r)T})}{(c+r)} + \frac{(e^{-r\lambda_1} - e^{-r\lambda_1})}{r})$$

g) Opportunity Cost:

$$O.C. = o\int_{\lambda_1}^{T} (1-\beta)D(t)e^{-rt}dt$$

$$= o(1-\beta)(a-bP)(\frac{(e^{-r\lambda_1} - e^{-rT})}{r} + \frac{(e^{-r\lambda_1} - e^{c\lambda_1 - (c+r)T})}{(c+r)} + (e^{-rT} - e^{-r\lambda_1}))$$

V. TOTAL INVENTORY COSTS FOR THE INVENTORY PROBLEM

Total cost for inventory problem in both case are given as follows:

A. Total cost (TP_1) for case 1:

Seles revenue cost - Fixed ordering cost - Purchasing cost- Holding cost - Preservation technology investment cost - Deterioration cost - Shortage cost - Opportunity cost.

$$\mathrm{T.P}_1(\lambda_1,T,P,\chi)=[\{(a-bP)\lambda_1+\frac{c(a-bP)}{c+r}((\lambda_1-T_d)+c(\lambda_1^2-T_d))+\theta(1-m)$$

$$(\frac{\lambda_1+T_d^2}{6})\frac{\theta(1-m)\lambda_1^4}{3}-T_d(\lambda_1+c\lambda_1^2+\frac{\theta(1-m)\lambda_1^3}{6}-T_d^2(\frac{\lambda_1^2-1}{2})+\frac{T_d^3c}{6}-\frac{T_d^4}{6}$$

$$(\theta(1-m)-1))-T_d(\frac{\lambda_1^2+T_d^2}{2})(1-\frac{\theta(1-m)T_d^2}{2})((c+r)T_d)+c(a-bP)((\frac{\lambda_1^2}{2}+$$

$$\frac{(c-2T_d)}{3})\lambda_1^3+((c+r-\theta(1-m)T_d)(\frac{\lambda_1^3}{6}+\lambda_1^4\frac{(c-3T_d)}{8})+\frac{7\theta(1-m)\lambda_1^5}{60}-T_d^2$$

$$(\frac{\lambda_1}{2}+\frac{c\lambda_1^2}{4})+T_d^3\frac{(4+3\lambda_1^2)}{12}+\frac{T_d^4c}{8}+T_d^5\frac{(15-4\theta(1-m))}{120}+(\theta(1-m)(\frac{\lambda_1^4}{24}+$$

$$\frac{\lambda_1^5(c+4T_d)}{30}+\frac{\theta(1-m)\lambda_1^2}{24}-T_d^3\frac{(2\lambda_1+c\lambda_1^2)}{12}-\frac{\theta(1-m)\lambda_1^3}{36})-\frac{\lambda_1^2T_d^4}{12}+\frac{cT_d^5}{20}-$$

$$\frac{T_d^6(18+5\theta(1-m))}{360}-A-\{u((a-bP)(\lambda_1-T_d)+\frac{c(\lambda_1^2-T_d^2)}{2}+\frac{\theta(1-m)(T_d^3+\lambda_1^3)}{6}$$

$$-\frac{T_d(T_d^2+\lambda_1^2)}{2}+(a-bP)\beta(\lambda_1-t))\}\frac{\lambda_1^3}{6}+\lambda_1^4\frac{(c-3T_d)}{8})+\frac{7\theta(1-m)\lambda_1^5}{60}-T_d^2$$

$$(\frac{\lambda_1}{2}+\frac{c\lambda_1^2}{4})+h_1(\frac{(a-bP)}{(c+r)}((\lambda_1-T_d)+c(\lambda_1^2-T_d))+\theta(1-m)(\frac{\lambda_1+T_d^2}{6})-T_d$$

$$(\frac{\lambda_1^2+T_d^2}{2})(1-\frac{\theta(1-m)T_d^2}{2})+((c+r)T_d)+(a-bP)((\frac{\lambda_1^2}{2}+\frac{(c-2T_d)}{3})\lambda_1^3-$$

$$\frac{\theta(1-m)\lambda_1^4}{3}-T_d(\lambda_1+c\lambda_1^2+\frac{\theta(1-m)\lambda_1^3}{6}-T_d^2(\frac{\lambda_1^2-1}{2})+\frac{T_d^3c}{6}-\frac{T_d^4}{6}$$

$$(\theta(1-m)-1))+((c+r-\theta(1-m)T_d)(\frac{\lambda_1^3}{6}+\lambda_1^4\frac{(c-3T_d)}{8})+\frac{7\theta(1-m)\lambda_1^5}{60}$$

$$-T_d^2(\frac{\lambda_1}{2}+\frac{c\lambda_1^2}{4})+T_d^3\frac{(4+3\lambda_1^2)}{12}+\frac{T_d^4c}{8}+T_d^5\frac{(15-4\theta(1-m))}{120}$$

$$+(\theta(1-m)(\frac{\lambda_1^4}{24}+\frac{\lambda_1^5(c+4T_d)}{30}+\frac{\theta(1-m)\lambda_1^2}{24}-T_d^3\frac{(2\lambda_1+c\lambda_1^2)}{12}-\frac{\theta(1-m)\lambda_1^3}{36}-\frac{\lambda_1^2T_d^4}{12}+$$

$$\frac{cT_d^5}{20}-\frac{T_d^6(18+5\theta(1-m))}{360}+h_2((a-bP)((\lambda_1-T_d)+(c(\lambda_1^2-T_d^2))+\frac{\theta(1-m)(\lambda_1^3+T_d^3)}{6}-$$

$$\frac{T_d(\lambda_1^2+T_d^2)}{2}(\frac{T_d^2}{2}-\frac{(c+r)T_d^3}{3}-\frac{\theta(1-m)T_d^4}{4})+(a-bP)((\frac{\lambda_1^3}{6}+\frac{c\lambda_1^4}{8}+\frac{7\theta(1-m)\lambda_1^5}{60}-\frac{3T_d\lambda_1^4}{8}-$$

$$\frac{(3\lambda_1T_d^2-2T_d^3)}{6}-\frac{c(2\lambda_1^2T_d^2-T_d^4)}{8}-\theta(1-m)(\frac{\lambda_1^3T_d^2}{12}+\frac{T_d^5}{30})+\frac{T_d(2\lambda_1^2T_d^2+T_d^4)}{8})-(r+c+\theta(1-m)T_d)$$

$$(\frac{\lambda_1^4}{12}+\frac{c\lambda_1^5}{15}+\frac{\theta(1-m)\lambda_1^6}{12}-\frac{4T_d\lambda_1^5}{15}-\frac{(4\lambda_1T_d^3-3T_d^4)}{12}-\frac{c(5\lambda_1^2T_d^3-3T_d^5)}{30}-\frac{\theta(1-m)(2\lambda_1^3T_d^3+T_d^6)}{36}+$$

$$\frac{T_d(5\lambda_1^2T_d^3+3T_d^5)}{30})+\theta(1-m)(\frac{\lambda_1^5}{40}+\frac{c\lambda_1^6}{48}+\frac{2\theta(1-m)\lambda_1^7}{21}-\frac{5T_d\lambda_1^6}{48}-\frac{(5\lambda_1T_d^4-4T_d^5)}{40}$$

$$-\frac{c(3\lambda_1^2T_d^4-2T_d^6)}{24}-\frac{\theta(1-m)(7\lambda_1^3T_d^4+T_d^7)}{84}+T_d\frac{(3T_d^4\lambda_1^2+2T_d^6)}{48}))\}-\lambda_1\chi$$

$$d\theta((a-bP)((\frac{\lambda_1^3}{6}+\frac{c\lambda_1^4}{8}+\frac{7\theta(1-m)\lambda_1^5}{60}-\frac{3T_d\lambda_1^4}{8}-\frac{(3\lambda_1T_d^2-2T_d^3)}{6}-\frac{c(2\lambda_1^2T_d^2-T_d^4)}{8})$$

$$-\theta(1-m)(\frac{\lambda_1^3T_d^2}{12}+\frac{T_d^5}{30})+\frac{T_d(2\lambda_1^2T_d^2+T_d^4)}{8})-(r+c+\theta(1-m)T_d)(\frac{\lambda_1^4}{12}+\frac{c\lambda_1^5}{15}+\frac{\theta(1-m)\lambda_1^6}{12}$$

$$-\frac{4T_d\lambda_1^5}{15}-\frac{(4\lambda_1T_d^3-3T_d^4)}{12}-\frac{c(5\lambda_1^2T_d^3-3T_d^5)}{30}-\frac{\theta(1-m)(2\lambda_1^3T_d^3+T_d^6)}{36}+\frac{T_d(5\lambda_1^2T_d^3+3T_d^5)}{30})$$

$$+\theta(1-m)(\frac{\lambda_1^5}{40}+\frac{c\lambda_1^6}{48}+\frac{2\theta(1-m)\lambda_1^7}{21}-\frac{5T_d\lambda_1^6}{48}-\frac{(5\lambda_1T_d^4-4T_d^5)}{40}-\frac{c(3\lambda_1^2T_d^4-2T_d^6)}{24}-$$

$$\frac{\theta(1-m)(7\lambda_1^3T_d^4+T_d^7)}{84}+T_d\frac{(3T_d^4\lambda_1^2+2T_d^6)}{48})-T_d+(a-bP)((\frac{\lambda_1^2}{2}+\frac{(c-2T_d)}{3})\lambda_1^3$$

$$+((c+r-\theta(1-m)T_d)(\frac{\lambda_1^3}{6}+\lambda_1^4\frac{(c-3T_d)}{8})+\frac{7\theta(1-m)\lambda_1^5}{60}-T_d^2(\frac{\lambda_1}{2}+\frac{c\lambda_1^2}{4})$$

$$+T_d^3\frac{(4+3\lambda_1^2)}{12}+\frac{T_d^4c}{8}+T_d^5\frac{(15-4\theta(1-m))}{120}+(\theta(1-m)(\frac{\lambda_1^4}{24}+\frac{\lambda_1^5(c+4T_d)}{30}+\frac{\theta(1-m)\lambda_1^2}{24}$$

$$-T_d^3\frac{(2\lambda_1+c\lambda_1^2)}{12}\frac{\theta(1-m)\lambda_1^3}{36})-\frac{\lambda_1^2T_d^4}{12}+\frac{cT_d^5}{20}-\frac{T_d^6(18+5\theta(1-m)}{360}))\}-\{s\beta(a-bP)(\lambda_1(T+\frac{rT^2}{2})$$

$$+\lambda_1^2\frac{(\lambda_1r-1)}{2}-\frac{r\lambda_1^3}{3}-\frac{T^2}{2}+\frac{rT^3}{3}))\}\frac{\theta(1-m)\lambda_1^3}{36})-\frac{\lambda_1^2T_d^4}{12}+\frac{cT_d^5}{20}-\frac{T_d^6(18+5\theta(1-m)}{360}))\}$$

$$-\{o(1-\beta)(a-bP)(\frac{r(T-\lambda_1)}{r}+c\beta(\lambda_1(T+\frac{rT^2}{2})+\lambda_1^2\frac{(\lambda_1r-1)}{2}-\frac{r\lambda_1^3}{3}-\frac{T^2}{2}+\frac{rT^3}{3})\}]$$

B. Total cost (TP_2) for case 2:

Seles revenue cost - Fixed ordering cost - Purchasing cost- Holding cost - Preservation technology investment cost - Shortage cost - Opportunity cost.

$$\mathrm{TP}_2(\lambda_1,T,P,\chi)=[\{\frac{(a-bP)(e^{c\lambda_1}-e^{-r\lambda_1})}{(c+r)}\}-A-\{h_1\frac{(a-bP)(e^{c\lambda_1}-e^{-r\lambda_1})}{(c+r)}+h_2(a-bP)$$
$$(\frac{(e^{-r\lambda_1}-1)}{(c+r)^2}-\lambda_1\frac{e^{-r\lambda_1}}{(c+r)})\}-\{u\frac{(a-bP)}{c}(e^{c(\lambda_1-t)}-1)\}-\lambda_1\chi-\{o(1-\beta)(a-bP)$$
$$(\frac{e^{-r\lambda_1}-e^{c\lambda_1-(c+r)T}}{c+r})\}-\{-s\beta\frac{(a-bp)}{c}(\frac{(e^{-r\lambda_1}-e^{c\lambda_1-(c+r)T})}{(c+r)}+\frac{(e^{-r\lambda_1}-e^{-r\lambda_1})}{r})\}]$$

VI. TOTAL AVERAGE COST FOR THE INVENTORY PROBLEM

The total average cost for inventory problem is given by dividing the total inventory cost to the total time horizon 'T'.

$$\mathrm{A.P_1.}(\lambda_1,T,P,\chi)=\frac{1}{T}[\{(a-bP)\lambda_1+\frac{c(a-bP)}{c+r}((\lambda_1-T_d)+c(\lambda_1^2-T_d))+\theta(1-m)$$
$$(\frac{\lambda_1+T_d^2}{6})\frac{\theta(1-m)\lambda_1^4}{3}-T_d(\lambda_1+c\lambda_1^2+\frac{\theta(1-m)\lambda_1^3}{6}-T_d^2(\frac{\lambda_1^2-1}{2})+\frac{T_d^3c}{6}-\frac{T_d^4}{6}$$
$$(\theta(1-m)-1))-T_d(\frac{\lambda_1^2+T_d^2}{2})(1-\frac{\theta(1-m)T_d^2}{2})((c+r)T_d)+c(a-bP)((\frac{\lambda_1^2}{2}+$$
$$\frac{(c-2T_d)}{3})\lambda_1^3+((c+r-\theta(1-m)T_d)(\frac{\lambda_1^3}{6}+\lambda_1^4\frac{(c-3T_d)}{8})+\frac{7\theta(1-m)\lambda_1^5}{60}-T_d^2$$
$$(\frac{\lambda_1}{2}+\frac{c\lambda_1^2}{4})+T_d^3\frac{(4+3\lambda_1^2)}{12}+\frac{T_d^4c}{8}+T_d^5\frac{(15-4\theta(1-m))}{120}+(\theta(1-m)(\frac{\lambda_1^4}{24}+$$
$$\frac{\lambda_1^5(c+4T_d)}{30}+\frac{\theta(1-m)\lambda_1^2}{24}-T_d^3\frac{(2\lambda_1+c\lambda_1^2)}{12}-\frac{\theta(1-m)\lambda_1^3}{36})-\frac{\lambda_1^2T_d^4}{12}+\frac{cT_d^5}{20}-$$
$$\frac{T_d^6(18+5\theta(1-m))}{360}-A-\{u((a-bP)(\lambda_1-T_d)+\frac{c(\lambda_1^2-T_d^2)}{2}+\frac{\theta(1-m)(T_d^3+\lambda_1^3)}{6}$$
$$-\frac{T_d(T_d^2+\lambda_1^2)}{2}+(a-bP)\beta(\lambda_1-t))\}\frac{\lambda_1^3}{6}+\lambda_1^4\frac{(c-3T_d)}{8})+\frac{7\theta(1-m)\lambda_1^5}{60}-T_d^2$$
$$(\frac{\lambda_1}{2}+\frac{c\lambda_1^2}{4})+h_1(\frac{(a-bP)}{(c+r)}((\lambda_1-T_d)+c(\lambda_1^2-T_d))+\theta(1-m)(\frac{\lambda_1+T_d^2}{6})-T_d$$
$$(\frac{\lambda_1^2+T_d^2}{2})(1-\frac{\theta(1-m)T_d^2}{2})+((c+r)T_d)+(a-bP)((\frac{\lambda_1^2}{2}+\frac{(c-2T_d)}{3})\lambda_1^3-$$
$$\frac{\theta(1-m)\lambda_1^4}{3}-T_d(\lambda_1+c\lambda_1^2+\frac{\theta(1-m)\lambda_1^3}{6}-T_d^2(\frac{\lambda_1^2-1}{2})+\frac{T_d^3c}{6}-\frac{T_d^4}{6}$$
$$(\theta(1-m)-1))+((c+r-\theta(1-m)T_d)(\frac{\lambda_1^3}{6}+\lambda_1^4\frac{(c-3T_d)}{8})+\frac{7\theta(1-m)\lambda_1^5}{60}$$
$$-T_d^2(\frac{\lambda_1}{2}+\frac{c\lambda_1^2}{4})+T_d^3\frac{(4+3\lambda_1^2)}{12}+\frac{T_d^4c}{8}+T_d^5\frac{(15-4\theta(1-m))}{120}$$

$$+(\theta(1-m)(\frac{\lambda_1^4}{24}+\frac{\lambda_1^5(c+4T_d)}{30}+\frac{\theta(1-m)\lambda_1^2}{24}-T_d^3\frac{(2\lambda_1+c\lambda_1^2)}{12}-\frac{\theta(1-m)\lambda_1^3}{36}-\frac{\lambda_1^2T_d^4}{12}+$$

$$\frac{cT_d^5}{20}-\frac{T_d^6(18+5\theta(1-m))}{360}+h_2((a-bP)((\lambda_1-T_d)+(c(\lambda_1^2-T_d^2))+\frac{\theta(1-m)(\lambda_1^3+T_d^3)}{6}-$$

$$\frac{T_d(\lambda_1^2+T_d^2)}{2}(\frac{T_d^2}{2}-\frac{(c+r)T_d^3}{3}-\frac{\theta(1-m)T_d^4}{4})+(a-bP)((\frac{\lambda_1^3}{6}+\frac{c\lambda_1^4}{8}+\frac{7\theta(1-m)\lambda_1^5}{60}-\frac{3T_d\lambda_1^4}{8}-$$

$$\frac{(3\lambda_1T_d^2-2T_d^3)}{6}-\frac{c(2\lambda_1^2T_d^2-T_d^4)}{8}-\theta(1-m)(\frac{\lambda_1^3T_d^2}{12}+\frac{T_d^5}{30})+\frac{T_d(2\lambda_1^2T_d^2+T_d^4)}{8})-(r+c+\theta(1-m)T_d)$$

$$(\frac{\lambda_1^4}{12}+\frac{c\lambda_1^5}{15}+\frac{\theta(1-m)\lambda_1^6}{12}-\frac{4T_d\lambda_1^5}{15}-\frac{(4\lambda_1T_d^3-3T_d^4)}{12}-\frac{c(5\lambda_1^2T_d^3-3T_d^5)}{30}-\frac{\theta(1-m)(2\lambda_1^3T_d^3+T_d^6)}{36}+$$

$$\frac{T_d(5\lambda_1^2T_d^3+3T_d^5)}{30})+\theta(1-m)(\frac{\lambda_1^5}{40}+\frac{c\lambda_1^6}{48}+\frac{2\theta(1-m)\lambda_1^7}{21}-\frac{5T_d\lambda_1^6}{48}-\frac{(5\lambda_1T_d^4-4T_d^5)}{40}$$

$$-\frac{c(3\lambda_1^2T_d^4-2T_d^6)}{24}-\frac{\theta(1-m)(7\lambda_1^3T_d^4+T_d^7)}{84}+T_d\frac{(3T_d^4\lambda_1^2+2T_d^6)}{48})))\}-\lambda_1\chi$$

$$d\theta((a-bP)((\frac{\lambda_1^3}{6}+\frac{c\lambda_1^4}{8}+\frac{7\theta(1-m)\lambda_1^5}{60}-\frac{3T_d\lambda_1^4}{8}-\frac{(3\lambda_1T_d^2-2T_d^3)}{6}-\frac{c(2\lambda_1^2T_d^2-T_d^4)}{8})$$

$$-\theta(1-m)(\frac{\lambda_1^3T_d^2}{12}+\frac{T_d^5}{30})+\frac{T_d(2\lambda_1^2T_d^2+T_d^4)}{8})-(r+c+\theta(1-m)T_d)(\frac{\lambda_1^4}{12}+\frac{c\lambda_1^5}{15}+\frac{\theta(1-m)\lambda_1^6}{12}$$

$$-\frac{4T_d\lambda_1^5}{15}-\frac{(4\lambda_1T_d^3-3T_d^4)}{12}-\frac{c(5\lambda_1^2T_d^3-3T_d^5)}{30}-\frac{\theta(1-m)(2\lambda_1^3T_d^3+T_d^6)}{36}+\frac{T_d(5\lambda_1^2T_d^3+3T_d^5)}{30})$$

$$+\theta(1-m)(\frac{\lambda_1^5}{40}+\frac{c\lambda_1^6}{48}+\frac{2\theta(1-m)\lambda_1^7}{21}-\frac{5T_d\lambda_1^6}{48}-\frac{(5\lambda_1T_d^4-4T_d^5)}{40}-\frac{c(3\lambda_1^2T_d^4-2T_d^6)}{24}-$$

$$\frac{\theta(1-m)(7\lambda_1^3T_d^4+T_d^7)}{84}+T_d\frac{(3T_d^4\lambda_1^2+2T_d^6)}{48})-T_d+(a-bP)((\frac{\lambda_1^2}{2}+\frac{(c-2T_d)}{3})\lambda_1^3$$

$$+((c+r-\theta(1-m)T_d)(\frac{\lambda_1^3}{6}+\lambda_1^4\frac{(c-3T_d)}{8})+\frac{7\theta(1-m)\lambda_1^5}{60}-T_d^2(\frac{\lambda_1}{2}+\frac{c\lambda_1^2}{4})$$

$$+T_d^3\frac{(4+3\lambda_1^2)}{12}+\frac{T_d^4c}{8}+T_d^5\frac{(15-4\theta(1-m))}{120}+(\theta(1-m)(\frac{\lambda_1^4}{24}+\frac{\lambda_1^5(c+4T_d)}{30}+\frac{\theta(1-m)\lambda_1^2}{24}$$

$$-T_d^3\frac{(2\lambda_1+c\lambda_1^2)}{12}\frac{\theta(1-m)\lambda_1^3}{36})-\frac{\lambda_1^2T_d^4}{12}+\frac{cT_d^5}{20}-\frac{T_d^6(18+5\theta(1-m)}{360})))\}-\{s\beta(a-bP)(\lambda_1(T+\frac{rT^2}{2})$$

$$+\lambda_1^2\frac{(\lambda_1r-1)}{2}-\frac{r\lambda_1^3}{3}-\frac{T^2}{2}+\frac{rT^3}{3}))\}\frac{\theta(1-m)\lambda_1^3}{36})-\frac{\lambda_1^2T_d^4}{12}+\frac{cT_d^5}{20}-\frac{T_d^6(18+5\theta(1-m)}{360}))\}$$

$$-\{o(1-\beta)(a-bP)(\frac{r(T-\lambda_1)}{r}+c\beta(\lambda_1(T+\frac{rT^2}{2})+\lambda_1^2\frac{(\lambda_1r-1)}{2}-\frac{r\lambda_1^3}{3}-\frac{T^2}{2}+\frac{rT^3}{3})\}]$$

$$\mathrm{A.P_2}.(\lambda_1,T,P,\chi)=\frac{1}{T}[\{\frac{(a-bP)(e^{c\lambda_1}-e^{-r\lambda_1})}{(c+r)}\}-A-\{h_1\frac{(a-bP)(e^{c\lambda_1}-e^{-r\lambda_1})}{(c+r)}+h_2(a-bP)$$
$$(\frac{(e^{-r\lambda_1}-1)}{(c+r)^2}-\lambda_1\frac{e^{-r\lambda_1}}{(c+r)})\}-\{u\frac{(a-bP)}{c}(e^{c(\lambda_1-t)}-1)\}-\lambda_1\chi-\{o(1-\beta)(a-bP)$$
$$(\frac{e^{-r\lambda_1}-e^{c\lambda_1-(c+r)T}}{c+r})\}-\{-s\beta\frac{(a-bp)}{c}(\frac{(e^{-r\lambda_1}-e^{c\lambda_1-(c+r)T})}{(c+r)}+\frac{(e^{-r\lambda_1}-e^{-r\lambda_1})}{r})\}]$$

VII. NUMERICAL EXAMPLE

The above given result are illustrated through the numerical examples. To illustrate the model we consider the following input data.

Same example for case 2 is given as:

Let a=70, b=1.8, c=0.25, s=30, o=0.55, u=0.42, A=350, r=0.45, β=0.095, d=0.45, θ=0.4,

T_o=1.8, h_1=1.1, h_2=1.92.

Answer: Applying the solution process of the given last section for case 1, we find the following results: T=2, P=45, TP_1=1362.99, λ_1=1.

Let a=70, b=1.8, c=0.25, s=30, o=0.55, u=0.42, A=350, r=0.45, β=0.095, d=0.45, θ=0.4, T_o=1.8, h_1=1.1, h_2=1.92.

Answer: Applying the solution process of the given last section for case 2, we get the following results, T=3, P=95, TP_2=66.4215, λ_1=0.5.

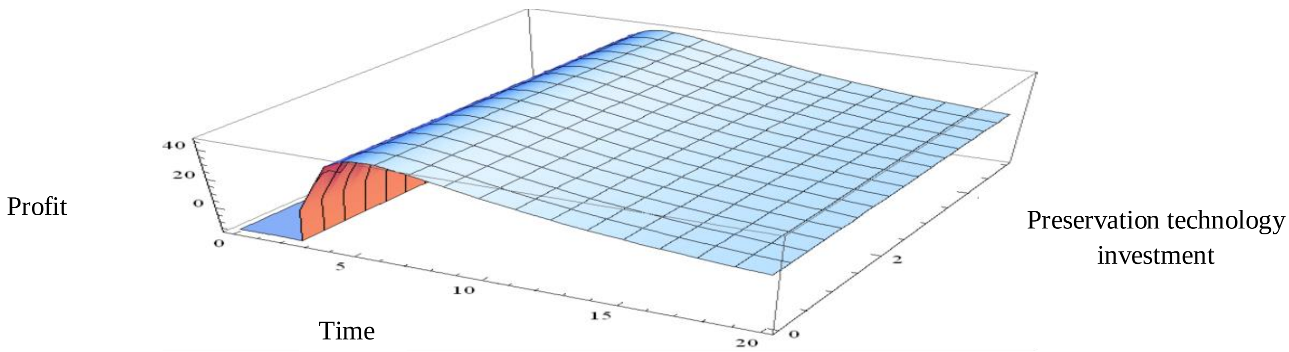

Fig. 3: Behavior of optimum cost function

VIII. SENSITIVITY ANALYSIS

h_1	T	P	λ_1	χ	AP_1
1.320	2.0001	45.0013	0.9998	4.9998	1475.39
1.210	2.0001	45.0010	0.9997	4.9999	1419.19
1.100	2.0015	45.0011	0.9999	4.9999	1362.99
0.990	2.0001	45.0010	0.9999	4.9999	1306.78
0.880	2.0000	45.0000	0.9996	4.9998	1250.57

Table 1: Sensitivity analysis for parameter 'h_1':

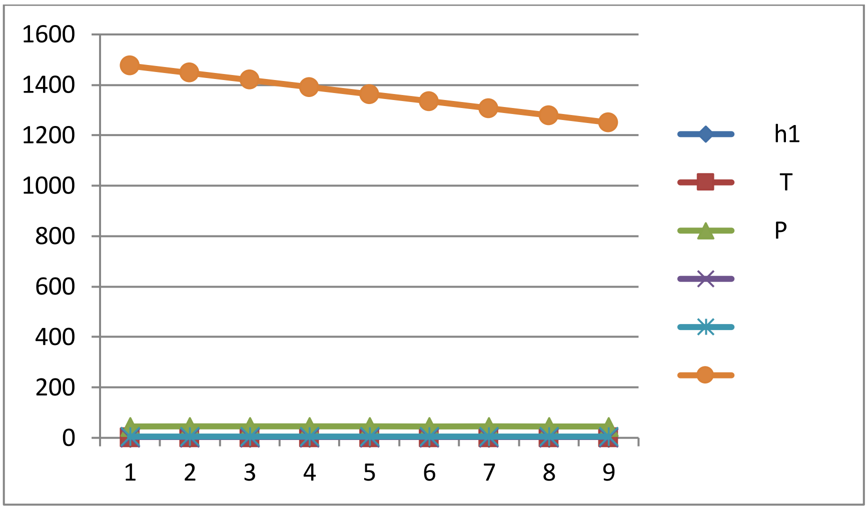

Fig. 4: Variation in T.A.C. (AP_1) with Variation in 'h_1'

h_2	T	P	λ_1	χ	AP_1
2.304	2.0001	45.0008	0.9996	4.9998	1460.00
2.112	2.0001	45.0006	0.9999	4.9999	1411.49
1.920	2.0002	45.0008	0.9999	4.9996	1362.99
1.728	2.0004	45.0009	0.9999	4.9998	1314.48
1.536	2.0002	45.0009	0.9998	4.9997	1265.95

Table 2: Sensitivity analysis for parameter 'h_2'

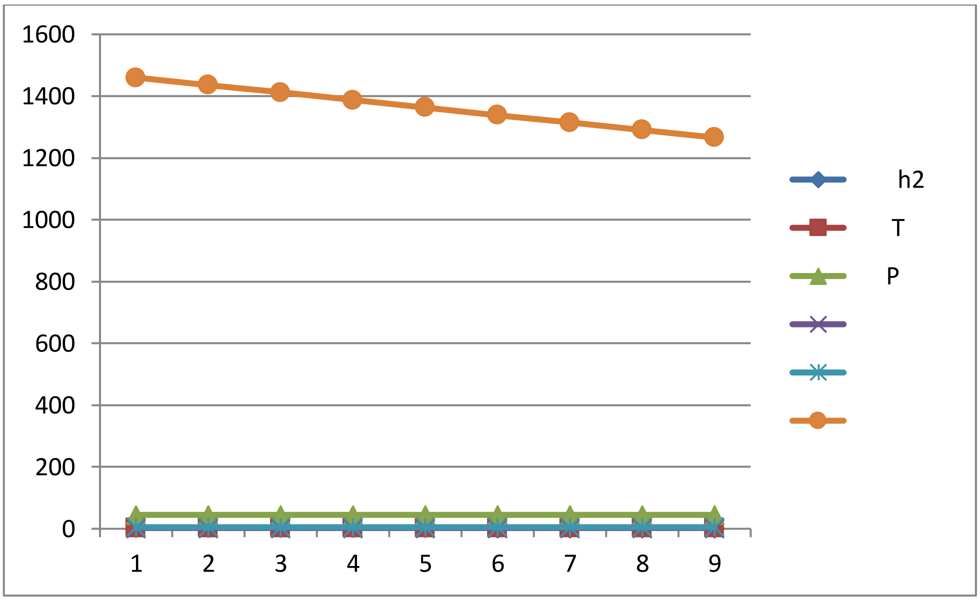

Fig. 5: Variation in T.A.C. (AP_1) with Variation in 'h_2'

β	T	P	λ_1	χ	AP_1
0.11400	2.0010	45.0009	0.9998	4.9998	1464.44
0.10450	2.0009	45.0007	0.9997	4.9999	1413.62
0.09500	2.0010	45.0008	0.9996	4.9999	1362.98
0.08550	2.0018	45.0009	0.9997	4.9999	1312.54
0.07600	2.0009	45.0009	0.9998	4.9998	1262.29

Table 3: Sensitivity analysis for parameter 'β'

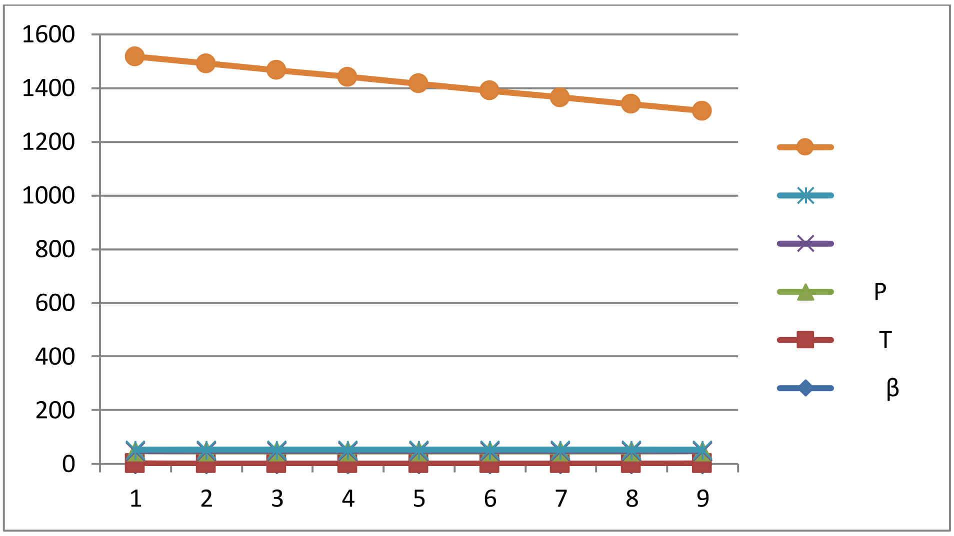

Fig 6: Variation in T.A.C. (AP_1) with Variation in ‘β’

r	T	P	λ_1	χ	AP_1
0.5400	2.0001	45.0010	0.9998	4.9999	1381.20
0.4950	2.0001	45.0010	0.9997	4.9999	1372.09
0.4500	2.0000	45.0020	0.9997	4.9999	1362.98
0.4050	2.0001	45.0020	0.9999	4.9999	1353.88
0.3600	2.0000	45.0030	0.9999	4.9996	1344.77

Table 4: Sensitivity analysis for parameter ‘r’

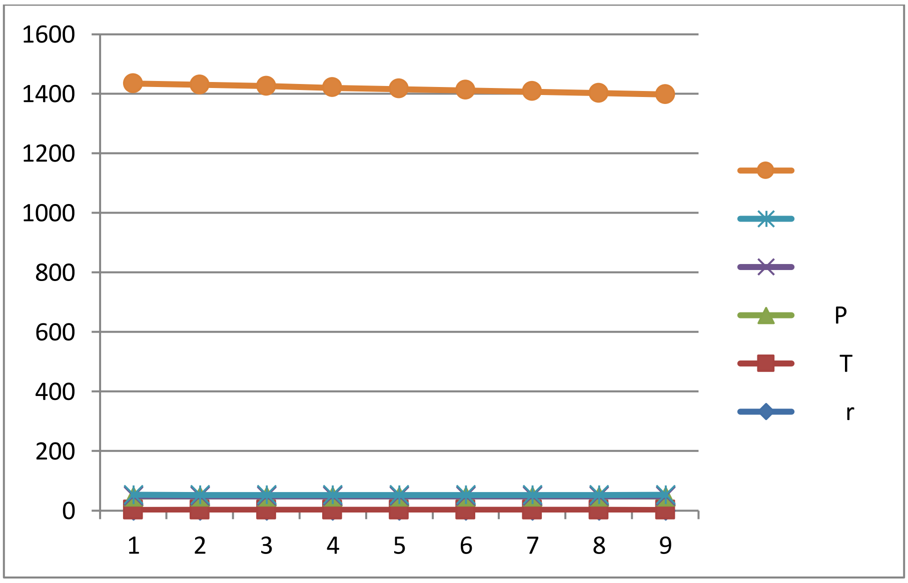

Fig. 7: Variation in T.A.C. (AP_1) with Variation in ‘r’

a	T	P	λ_1	χ	AP_1
264	2.0001	45.0010	0.9998	4.9997	1850.10
242	2.0000	45.0010	0.9997	4.9999	1606.54
220	2.0003	45.0030	0.9999	4.9998	1362.98
198	2.0000	45.0010	0.9998	4.9998	1119.42
176	2.0001	45.0020	0.9997	4.9998	875.86

Table 5: Sensitivity analysis for parameter ‘a’

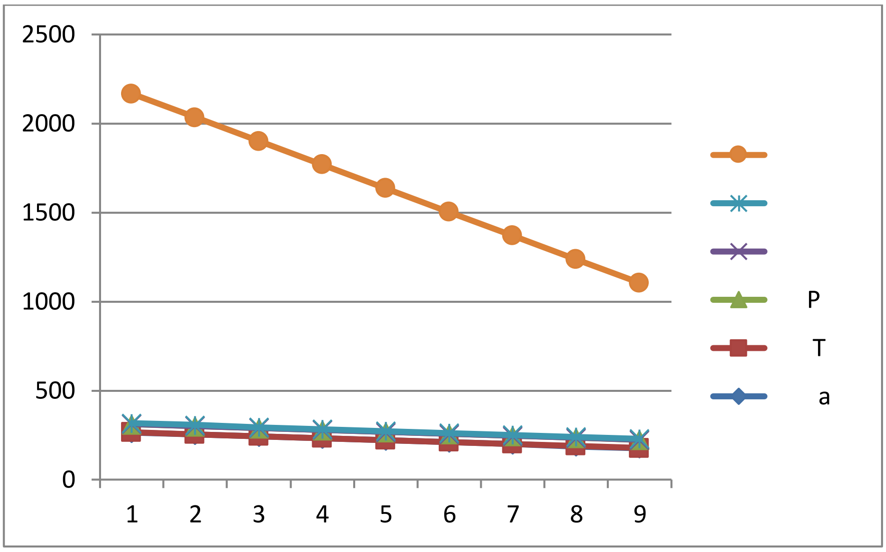

Fig. 8: Variation in T.A.C. (AP_1) with Variation in ‘a’

b	T	P	λ_1	χ	AP_1
2.16	2.0001	45.0002	0.9998	4.9996	1183.64
1.98	2.0001	45.0001	0.9996	4.9997	1273.31
1.80	2.0003	45.0003	0.9996	4.9999	1362.98
1.62	2.0000	45.0002	0.9995	4.9999	1452.60
1.44	2.0003	45.0001	0.9998	4.9999	1542.34

Table 6: Sensitivity analysis for parameter ‘b’

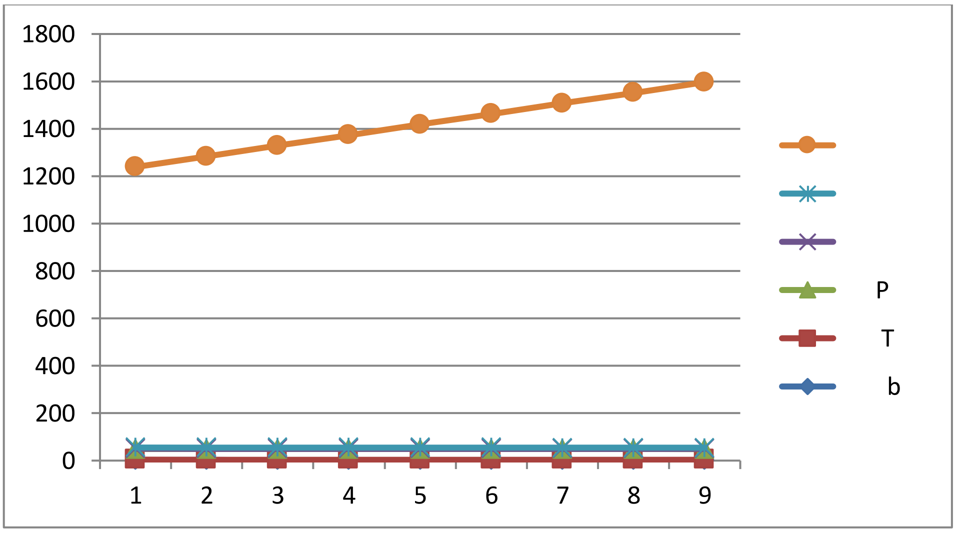

Fig. 9: Variation in T.A.C. (AP_1) with Variation in ‘b’

c	T	P	λ_1	χ	AP_1
0.2890	2.0001	45.0020	0.9998	4.9995	1323.93
0.2750	2.0002	45.0010	0.9999	4.9994	1337.99
0.2500	2.0000	45.0020	0.9996	4.9999	1362.99
0.2250	2.0000	45.0040	0.9997	4.9995	1387.85
0.2000	2.0002	45.0030	0.9998	4.9998	1412.56

Table 7: Sensitivity analysis for parameter ‘c’

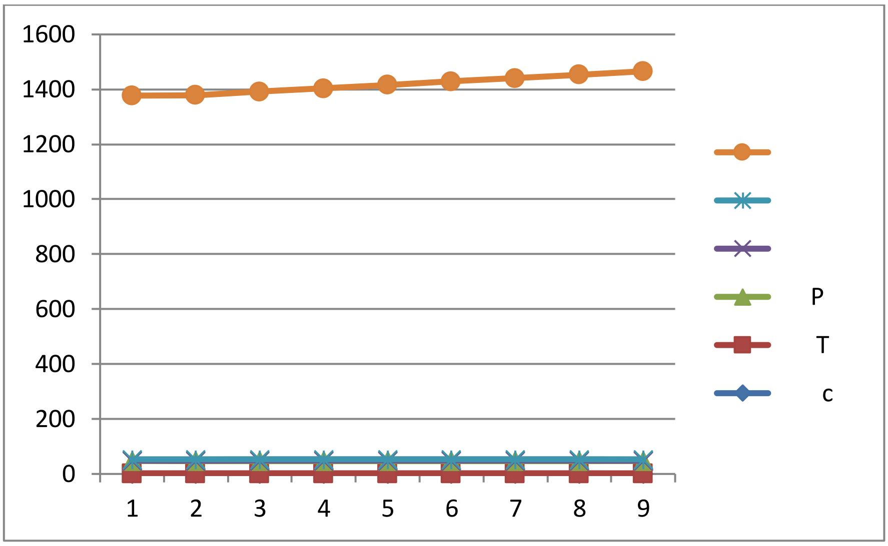

Fig.10: Variation in T.A.C. (AP_1) with Variation in 'c'

θ	T	P	λ_1	χ	AP_1
0.48	2.0001	45.0011	0.9998	4.9999	1371.24
0.44	2.0002	45.0010	0.9998	4.9999	1367.19
0.40	2.0000	45.0010	0.9996	4.9999	1362.99
0.36	2.0000	45.0014	0.9997	4.9999	1358.66
0.32	2.0002	45.0013	0.9998	4.9998	1354.21

Table 8: Sensitivity analysis for parameter 'θ'

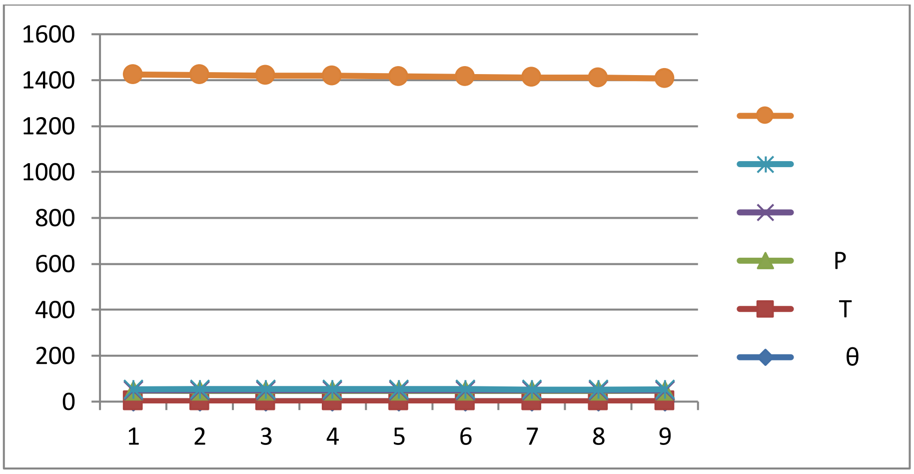

Fig. 11: Variation in T.A.C. (AP_1) with Variation in 'θ'

A	T	P	λ_1	χ	AP_1
418.80	2.0001	45.0010	0.9997	4.9996	1328.59
384.40	2.0002	45.0010	0.9998	4.9997	1345.79
350.00	2.0000	45.0018	0.9997	4.9998	1362.98
316.60	2.0002	45.0021	0.9999	4.9996	1379.69
292.20	2.0001	45.0018	0.9998	4.9997	1391.89

Table 9: Sensitivity analysis for parameter 'A'

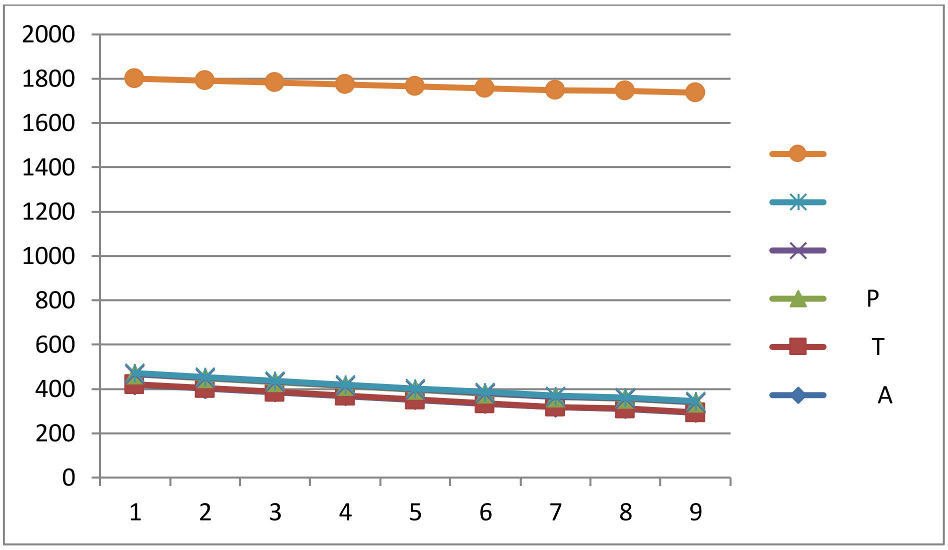

Fig. 12: Variation in T.A.C. (AP_1) with Variation in 'A'

d	T	P	λ_1	χ	AP_1
0.5400	2.0001	45.0030	0.9996	4.9995	1381.20
0.4950	2.0002	45.0020	0.9995	4.9994	1372.09
0.4500	2.0003	45.0030	0.9998	4.9996	1362.98
0.4050	2.0000	45.0020	0.9998	4.9995	1353.88
0.3600	2.0002	45.0025	0.9997	4.9996	1344.77

Table 10- Sensitivity analysis for parameter 'd'

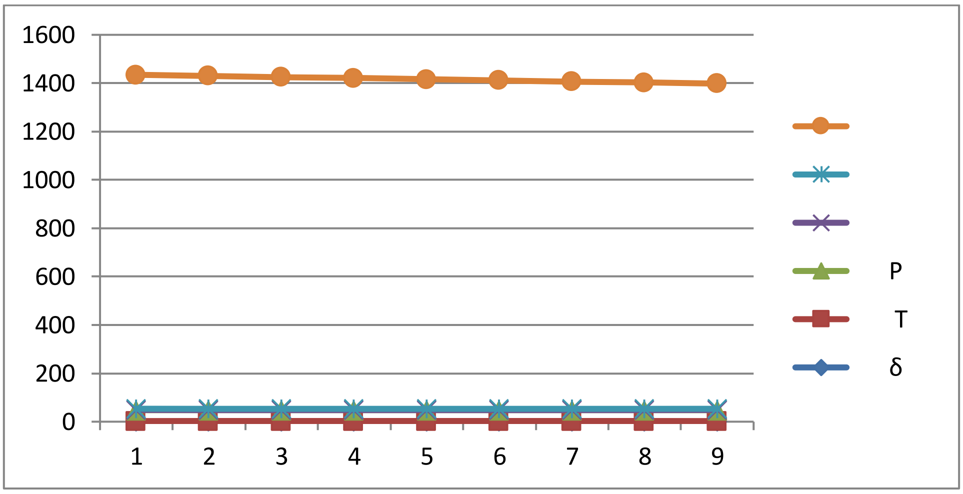

Fig. 13: Variation in T.A.C. (AP_1) with Variation in 'd'

IX. OBSERVATION OF THE TABLES

- It is clearly visible from the table (1) that with the increment in holding cost parameter (h_1), the replenishment cycle (T), price-dependent demand rate (P), holding period (λ_1) and preservation coefficient (χ_2) remains approximately constant while total average cost of the system shows the similarly effect.
- From the table (2), we remarked that as the values of holding cost parameter (h_2) decreases, the values of replenishment cycle (T), price dependent demand rate (P), holding period (λ_1) and preservation coefficient (χ) remains pretty nearly unchanged and the value of total average cost (T.A.C.) of the system decreases.
- We detected from table (3) that as the values of backlogged coefficient (β) decreases, the values of replenishment cycle (T), price dependent demand rate (P), holding period (λ_1) and preservation coefficient (χ) remains approximately unchanged, while the total average cost of the system decreases.
- Table (4) shows that with the hike in inflation rate (r) results nearly unchanged for replenishment cycle (T), price dependent demand rate (P), holding period (λ_1) and preservation coefficient (χ) while total average cost remains hike continuously.

- Table (5) states that the values of replenishment cycle (T), price dependent demand rate (P), holding period (λ_1) and preservation coefficient (χ) are very slightly decreases, to the decreases value of parameter (a). Parameter (a) and T.A.C. have proportional relation between them, it means that if parameter (a) increases, then T.A.C. increases and if parameter (a) decreases, then T.A.C. decreases.
- We discerned from table (6) that the total average cost for the system has inversely proportional relation to the demand parameter 'b'. In this table, replenishment cycle (T), price dependent demand rate (P), holding period (λ_1) and preservation coefficient (χ) have roughly stable values, for the each variation of demand rate parameter 'b'.
- From the table (7), we perceived that the total average cost (T.A.C.) is negative sensitive to the change in demand rate parameter (C), Replenishment cycle (T), price-dependent demand rate (P), holding period (λ_1) and preservation coefficient (χ) have generally unchanged values for the expansion of demand rate parameter (C).
- With the help of table (8), it is cleared that the total average cost for the system is positive sensitive to the change in the deterioration rate (θ). Replenishment cycle (T), price dependent demand rate (P), holding period (λ_1) and preservation coefficient (χ) remains pretty nearly stable for the increment of deterioration rate 'θ' in this table.
- Table (9) lists the variation in ordering cost parameter 'A'. It observed from this table that with the increment in 'A', the values of replenishment cycle (T), price dependent demand rate (P), holding period (λ_1) and preservation coefficient (χ) decrements barely and the total average cost of the system also decrement. That is, change in 'A' will cause negative change in (AP_1).
- We distinguished that as soon as the values of deterioration cost (d) decreases, then the value of total average cost increases, and other parameters like replenishment cycle (T), price-dependent demand rate (P), holding period (λ_1) and preservation coefficient (χ) remains almost constant in table (10).

X. CONCLUSION

In this work, we look at a joint pricing, ordering, and preservation technology investment dilemma for a store with a degrading inventory system that is not instantaneous. To avoid deterioration related losses, the retailer invests in preservation technology. We previously indicated that preservation technology can both reduce the pace of deterioration and extend the time between deterioration and non-deterioration. We develop a mathematical model that includes a price and stock dependent demand rate, a time varying deterioration rate, and partially backlogged shortages. The ideal solutions attributes are described. When the deterioration rate, ordering cost, or purchasing cost are low, or when the holding cost is high, our numerical examples show that it is best for the retailer not to invest in preservation technology. The impact of preservation technology investment on operational policy with varied parameters is the primary topic of this paper. Even if the pricing decision is taken into account in the model, the focus on the pricing policy is limited because it is believed to be constant for items whose quality remains consistent throughout the replenishment cycle.

REFERENCES

[1.] Aggoun, L., Benkherouf, L. & Tadj, L. (2001) : On a stochastic inventory model with deteriorating items. Int. *Journal of Mathematics and Mathematical Sciences* 25(3), pp. 197-203.

[2.] Biswas, S. & Giri, B.C. (2019) : A Single – Vendor Single- buyer integrated model for deteriorating items with partial backlogging and price-dependent market demand. *International Journal of Applied Engineering Research*, ISSN 0973-4562, Vol. 14, No. 1 (2019), pp. 254-261.

[3.] Chung, K.J. & Barron, L.E.C. (2013) : The simplified solution procedure for deteriorating items under stock-dependent demand and two-level trade credit in the supply chain management. *Applied Mathematical Modelling* 37(7), pp. 4653-4660.

[4.] Dye, C.Y. & Hsieh, T.P. (2012) : An Optimal replenishment policy for deteriorating items with effective investment in preservation technology. *European Journal of Operational Research* 218(1), pp. 106-112.

[5.] Giri, B.C. & Bardhan, S. (2015) : A vendor- buyer JELS model with stock-dependent demand and consigned inventory under buyer's space constraint. *Operational Research : An International Journal,* 15, pp. 79-93.

[6.] Hsu, P.H., Wee, H.M. & Teng, H.M. (2010) : Preservation technology investment for deteriorating inventory. *International Journal of Production Economics* 124(2), pp. 388-394.

[7.] Mashud, A., Khan, M., Uddin, M. & Islam, M. (2018) : A non-instantaneous inventory model having different deterioration rates with stock and price-dependent demand under partially backlogged shortages. *Uncertain Supply Chain Management, 6 (1),* pp. 49-64.

[8.] Raafat, F. (1991) : Survey of literature on continuously deteriorating inventory models. *Journal of the operational Research Society* 42(1), pp. 27-37.

[9.] Rameswari, M. & Uthayakumar, R. (2018) : An integrated inventory model for deteriorating items with price-dependent demand under two-level trade credit policy. *International Journal of Systems Science : Operations & Logistics* 5 (3), pp. 253-267.

[10.] Saha, S. & Sen, N. (2019) : An inventory model for deteriorating items with time and price dependent demand and shortages under the effect of inflation. *Int. J. of Mathematics in Operational Research,* Vol. 14, No. 3, pp. 377-388.

[11.] Saha, S. (2017) : A two-echelon supply chain model for deteriorating product with time-dependent demand, demand-dependent Production rate and Shortage. *IOSR Journal of Engineering,* 8, pp. 33-38.

[12.] Sett, B.K., Sarkar, B. & Goswami, A. (2012) : A two-warehouse inventory model with increasing demand and time varying deterioration. *Scientia Iranica* 19(6), pp. 1969-1977.

[13.] Shaikh, A.A., Panda, G.C., Sahu, S. & Das, A.K. (2019) : EOQ model for deteriorating item with preservation technology in time dependent demand with partial backlogging and trade credit. *International Journal of Logistics Systems and management* 32 (1), pp. 1-24.

[14.] Sheikh, S.R. & Patel, R. (2017) : Deteriorating items production inventory model with different deterioration rates under stock and price-dependent demand. *International Journal of Statistics and Systems,* 12(3), pp. 607-618.

[15.] Singh, S.R. & Rana, K. (2020) : Effect of inflation and variable holding cost on life time inventory model with multi variable demand and lost sales. *International Journal of Recent Technology and Engineering (IJRTE),* ISSN 2277-3878, Volume-8, Issue-5, pp. 5513-5519.

[16.] Tayal, S., Singh, S.R. & Sharma, R., (2016) : An integrated Production inventory model for perishable products with trade credit period and investment in preservation technology. *International Journal of Mathematics in operational Research,* 8(2), pp. 137-163.

[17.] Yang, C.T., Dye, C.Y. & Ding, J.F. (2015) : Optimal dynamic trade credit and preservation technology allocation for a deteriorating inventory model. *Computers & Industrial Engineering* 87, pp. 356-369.