General Integral Transform for the Solution of Models in Health Sciences

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Abstract:- Recently many researchers introduced lot of integral transforms. Integral transforms play important role for solving differential equations, integral equations, Integro differential equations, difference equations and systems. General integral transform is introduced by Hossein Jafari in 2021. We have models in health sciences which contains a system of differential equations with boundary conditions. In this paper we solve two models. First model is, two compartment model for drug absorption and circulation through gastrointestinal tract and blood. Second model is, Model for the intravenous drug administration. We use general integral transform introduced by Jafari for solving the models in health sciences and bio-technology and obtain useful results.

Keywords:- *Models in health sciences, General integral transform, Two compartment model, intravenous drug administration model.*

I. INTRODUCTION

Recently, integral transforms are one of the most useful and simple mathematical technique for obtaining the solutions of advance problems occurred in many fields like science, Engineering, technology, commerce and economics. To provide exact solution of problem without lengthy calculations is the important feature of integral transforms.

Due to this important feature of the integral transforms many researchers are attracted to this field and are engaged in introducing various integral transforms. Recently, Kushare and Patil [3] introduced new integral transform called as Kushare transform for solving differential equations in time domain. Further, Savita Khakale and Dinkar Patil [4] introduced Soham transform in November 2021. As researchers are interested in introducing the new integral transforms at the same time they are also interested in applying the transforms to various fields, various equations in different domain. In January 2022, Sanap and Patil [5] used Kushare transform for obtaining the solution of the problems on Newton's law of Cooling.

In April 2022 D. P. Patil, et al [6] solved the problems on growth and decay by using Kushare transform. D.P. Patil [7] also used Sawi transform in Bessel functions. Further, Patil [8] evaluate improper integrals by using Sawi transform of error functions. Laplace transforms and Shehu transforms are used in chemical sciences by Patil [9]. Dinkar Patil [10] used Sawi transform and its convolution theorem for solving wave equation. Using Mahgoub transform, parabolic boundary value problems are solved by D .P. Patil [11].

D .P. Patil [12] used double Laplace and double Sumudu transforms to obtain the solution of wave equation. Further Dr. Patil [13] also obtained dualities between double integral transforms. Kandalkar, Gatkal and Patil [14] solved the system of differential equations using Kushare transform. D. P. Patil [15] solved boundary value problems of the system of ordinary differential equations by using Aboodh and Mahgoub transforms. Double Mahgoub transformed is used by Patil [16] to solve parabolic boundary value problems.

Laplac, Sumudu , Aboodh , Elazki and Mahagoub transforms are compared and used it for solving boundary value problems by Dinkar Patil [17]. D. P. Patil et al [18] solved Volterra Integral equations of first kind by using Emad-Sara transform. Futher Patil with Tile and Shinde [19] used Anuj transform and solved Volterra integral equations for first kind. Rathi sisters and D. P. Patil [20] solved system of differential equations by using Soham transform. Vispute, Jadhav and Patil [21] used Emad Sara transform for solving telegraph equation. Kandalkar, Zankar and Patil [22] evaluate the improper integrals by using general integral transform of error function. Dinkar Patil, Prerana Thakare and Prajakta Patil[23] obtained the solution of parabolic boundary value problems by using double general integral transform. Dinkar Patil used Emad-Falih transform for solving problems based on Newton's law of cooling [24]. D. P. Patil et al [25] used Soham transform to obtain the solution of Newton's law of cooling. Dinkar Patil et al [26] used HY integral transform for handling growth and Decay problems, D. P. Patil et al used HY transform for Newton's law of cooling [27]. D. P. Patil et al [28] used Emad-Falih transform for general solution of telegraph equation. Dinkar Patil et al [29] introduced double kushare transform. Recently, D. P. Patil et al [30] solved population growth and decay problems by using Emad Sara transform. Alenzi transform is used in population growth and decay problems by patil et al [31]. Thete, et al [32] used Emad Falih transform for handling growth and decay problems. Nikam, Patil et al [33] used , Kushare transform of error functions in evaluating improper integrals. Wagh sisters and Patil used Kushare [34] and Soham [35] transform in chemical Sciences. Malpani, Shinde and Patil [36] used Convolution theorem for Kushare transform and applications in convolution type Volterra integral equations of first kind. Raundal and Patil [37] used double general integral transform for solving boundary value problems in partial differential equations. Rahane, Derle and Patil [38] eveloped generalized double rangaig integral transform. Kushare transform is used for solving Volterra Integro-Differential equations of first kind by Shinde, et al [39].

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Kandekar et al [40] used new general integral equation to solve Abel's integral equations. Pardeshi, Shaikh and Patil[41] used Kharrat Toma transform for solving population growth and decay problems.

II. PRELIMINARY

A. Definition of general Integral Transform[1]:

Let f(t) be integrable function defined for $t \geq 0$, $p(v) \neq 0$ and q(v) are positive real functions, we define the general integral transform T(s) of f(t) by formula

$$T\{f(t);s\} = T(s) = p(s)\int_0^\infty f(t)e^{-q(s)}\,dt \qquad (1)$$

B. Linearity property of general integral transform[1]:

If new integral transform of two functions function $f_1(t)$ and $f_2(t)$ are $T_1(v)$ and $T_2(v)$ respectively, then new integral transform of$[c_1 f_1(t) + c_2 f_2(t)]$ is given by $c_1 T_1(s) + c_2 T_2(s)$. Where c_1 and c_2 are arbitrary constants.

C. Derivative property general integral transform[1]:

Let $f(t)$ is differentiable and $p(v)$ and $q(v)$ are positive real functions, then

- $T\{f^{'}(t);s\} = q(s)\,T(s) - p(s)f(0),$
- $T\{f^{''}(t);s\} = q^2(s)T\{f(t);s\} - q(s)p(s)f(0) - p(s)f^{'}(0),$
- $T\{f^n(t);s\} = q^n(s)T\{f(t);s\} - p(s)\sum_{k=0}^{n-1} q^{n-1-k}(s)f^k(0).$

D. Convolution Theorem form general integral transform[1]:

Let $f_1(t)$ and $f_2(t)$ have new integral transform $F_1(v)$ and $F_2(v)$. Then the new integral transform of the Convolution of f_1 and f_2 is

$$f_1 * f_2 = \int_0^\infty f_1(t)f_1(t-T)dT = \frac{1}{p(s)}F_1(s).F_2(s)$$

E. General Integral Transform of some frequently Encountered Functions[1]

Functions $\boldsymbol{f(t) = T^{-1}\{T(s)\}}$	New Integral transform $\boldsymbol{T(s) = T\{f(t);s\}}$
1	$\frac{p(s)}{q(s)}$
T	$\frac{p(s)}{(q(s))^2}$
t^α	$\frac{\Gamma[\alpha+1]p(s)}{(q(s))^{\alpha+1}}, \quad \alpha > 0$
$sint$	$\frac{p(s)}{(q(s))^2+1}$
$sinat$	$\frac{ap(s)}{a^2+(q(s))^2}$
$cost$	$\frac{q(s)p(s)}{[q(s)]^2+1}$
e^t	$\frac{p(s)}{q(s)-1}, \quad q(v) > 1,$
$tH(t-1)$	$\frac{e^{-q(s)}(q(s)+1)p(s)}{(q(s))^2}$
$f'(t)$	$q(s)T(s) - p(s)f(0)$

Table 1 : Integral Transform of some functions

F. Inverse general Integral Transform:

If $T(s)$ is new integral transform of the function $f(t)$, then $f(t)$ is said to be inverse new integral transform of $T(s)$.

$$f(t) = T^{-1}\{T(s)\}$$

G. Linearity property of inverse New integral transform:

If$T^{-1}\{T_1(s)\} = f_1(t)$ and $T^{-1}\{T_2(s)\} = f_2(t)$. Then

$$T^{-1}\{c_1T_1(s) + c_2T_2(s)\} = c_1T^{-1}\{T_1(s)\} + c_2T^{-1}\{T_2(s)\}$$

$$T^{-1}\{c_1T_1(s) + c_2T_2(s)\} = c_1f_1(t) + c_2f_2(t).$$

Where c_1 and c_2 are arbitrary constants.

III. USE OF TRANSFORM IN MATHEMATICAL MODELS FROM HEALTH SCIENCES

Khanday et al introduced following two models in 2017[2]. They solve these models by using Eigen value method and Laplace transform. While developing models, the relation between drug intake and concentration of drug at the target site through the various compartments in biological process is considered.

We solve these models by using General integral transform introduced by Jafari in 2021.

A. MATHEMATICAL MODEL 1:

"A two compartment model for drug absorption and circulation through gastrointestinal tract and blood."[2]

The general form of the two compartment model describing the rate of change in oral drug administration is given as,

$\frac{dc_1(t)}{dt} = -k_1 c_1(t);$ $\quad c_1(0) = c_0$ ………(1)

$\frac{dc_2(t)}{dt} = k_1 c_1(t) - k_e c_2(t);$ $\quad c_2(0) = 0$ ……….(2)

Here $c_1(t)$: concentration of drug in stomach or GI tract,

$c_2(t)$: Concentration of drug in bloodstream compartments,

c_0: Initial concentration of blood,

k_1: Rate constants from one compartment to another,

$k_e(> 0)$: Clearance constant

Equation (1) and (2) implies,

$\frac{dc_1(t)}{dt} + k_1 c_1(t) = 0$ ………(3)

& $\frac{dc_2(t)}{dt} - k_1 c_1(t) + k_e c_2(t) = 0$ ………(4)

Apply New General Integral Transform on equation (3),

$T\left[\frac{dc_1}{dt}\right] + k_1 T(c_1) = 0$ ……… (5)

We have,

$$T[f'(t)] = q(s)T(s) - p(s)f(0)$$

$$\Rightarrow T\left[\frac{dc_1}{dt}\right] = q(s)T(c_1) - p(S)c_1(0)$$

$equation(3) \Rightarrow$

$$q(s)T(c_1) - p(s)c_1(0) + k_1 T(c_1) = 0$$

$$(q(s) + k_1)T(c_1) = p(s)c_1(0)$$

$$(q(s) + k_1)T(c_1) = p(s)c_0$$

$T(c_1) = \frac{p(s)c_0}{q(s)+k_1}$……….(6)

Using inverse transform, we get

$$c_1(t) = c_0 e^{-kt} \qquad \left\{\because T(e^{-\alpha t}) = \frac{p(s)}{q(s)+\alpha}\right\}$$

Now, apply New General Integral Transform on equation (4), we get

$$T\left[\frac{dc_2}{dt}\right] - k_1 T(c_1) + k_e T(c_2) = 0$$

$$\Rightarrow q(s)T(c_2) - p(s)c_2(0) - k_1 T(c_1) + k_e T(c_2) = 0$$

$$\Rightarrow (q(s) + k_e)T(c_2) = k_1 \frac{p(s)c_0}{q(s) + k_1} \qquad \{\because from(4)\ \&\ c_2(0) = 0\}$$

$$\therefore T(c_2) = \frac{k_1 c_0 p(s)}{(q(s) + k_e)(q(s) + k_1)}$$

Now, $\frac{p(s)}{(q(s)+k_e)(q(s)+k_1)} = \frac{A}{(q(s)+k_e)} + \frac{B}{(q(s)+k_1)}$

$\Rightarrow p(s) = A(q(s) + k_1) + B(q(s) + k_e)$ …….. (7)

Put $q(s) = -k_1$ in equation (7)

Equation (7) implies,

$$p(s) = A(0) + B(-k_1 + k_e)$$

$$\therefore B = \frac{p(s)}{k_e - k_1}$$

Put $q(s) = -k_e$ in equation (7)

Equation (7) implies,

$$p(s) = A(-k_e + k_1) + B(0)$$

$$\therefore A = \frac{p(s)}{-k_e + k_1} = -\frac{p(s)}{k_e - k_1}$$

$$\therefore T(c_2) = k_1 c_0 \left[\frac{p(s)}{(k_e - k_1)(q(s) + k_1)} - \frac{p(s)}{(k_e - k_1)(q(s) + k_e)}\right]$$

$$\therefore T(c_2) = \frac{k_1 c_0}{k_e - k_1}\left[\frac{p(s)}{(q(s) + k_1)} - \frac{p(s)}{(q(s) + k_e)}\right]$$

Taking inverse transform, we get

$$\therefore c_2(t) = \frac{k_1 c_0}{k_e - k_1}\left[T^{-1}\left(\frac{p(s)}{(q(s) + k_1)}\right) - T^{-1}\left(\frac{p(s)}{(q(s) + k_e)}\right)\right]$$

$$\therefore c_2(t) = \frac{k_1 c_0}{k_e - k_1}\left[e^{-k_1 t} - e^{-k_e t}\right] \quad ; k_1 \neq k_e$$

$$\therefore c_2(t) = \frac{k_1 c_0}{k_1 - k_e}\left[e^{-k_e t} - e^{-k_1 t}\right] \quad ; k_1 \neq k_e$$

Thus we have obtained $c_1(t)$: concentration of drug in stomach or GI tract, $c_2(t)$: Concentration of drug in bloodstream compartments,

B. MATHEMATICAL MODEL 2:

"Model for the intravenous drug administration."[2]

In these models, only two compartments viz. Blood and tissue were identified as the main exchangers. The first one is the blood stream into which the drug is injected and the second one is the tissue where the drug has therapeutic effect.

Then mathematical form of two compartment model describing the drug administration is,

$\frac{dc_b(t)}{dt} = -(k_b + k_e)c_b + k_t c_t; \quad c_b(0) = c_0$ ....(1)

$\frac{dc_t(t)}{dt} = k_b c_b - k_t c_t; \quad c_t(0) = 0$ ....(2)

Where c_b: concentration of drug in the compartments of blood,

c_t:concentration of drug in the compartments of tissue,

c_0:initial concentration of drug through intravenous route of the body,

k_b:rate of blood takes a part of drug onto tissue,

k_e:clearance rate,

Equation (1) and (2) implies,

$\frac{dc_b(t)}{dt} + (k_b + k_e)c_b - k_t c_t = 0$ ....(3)

$\frac{dc_t(t)}{dt} - k_b c_b - k_t c_t = 0$ ....(4)

Apply New General Integral Transform on equation (3),

$$T\left[\frac{dc_b(t)}{dt}\right] + (k_b + k_e)T(c_b) - k_t T(c_t) = 0$$

$$q(s)T(c_b) - p(s)c_b(0) + (k_b + k_e)T(c_b) - k_t T(c_t) = 0$$

$(q(s) + k_b + k_e)T(c_b) = p(s)c_0 + k_t T(c_t)$ .... (5) $\{\because c_b(0) = c_0\}$

Now, Apply New General Integral Transform on equation (4),

$$T\left[\frac{dc_t}{dt}\right] - k_b T(c_b) + k_t T(c_t) = 0$$

$$q(s)T(c_t) - p(s)c_t(0) - k_b T(c_b) + k_t T(c_t) = 0$$

$(q(s) + k_t)T(c_t) = p(s)(0) + k_b T(c_b)$ $\{\because c_t(0) = 0\}$

$$(q(s) + k_t)T(c_t) = k_b T(c_b)$$

$T(c_t) = \frac{k_b T(c_b)}{q(s)+k_t}$ .... (6)

Put equation (6) in equation (5), we get

$$(q(s) + k_b + k_e)T(c_b) = p(s)c_0 + k_t\left(\frac{k_b T(c_b)}{q(s) + k_t}\right)$$

$$(q(s) + k_b + k_e)T(c_b) - \frac{k_t k_b}{q(s) + k_t}T(c_b) = p(s)c_0$$

$$\left[q(s) + k_b + k_e - \frac{k_t k_b}{q(s) + k_t}\right]T(c_b) = p(s)c_0$$

$$\left[\frac{q(s)^2 + q(s)k_t + q(s)k_b + k_b k_t + q(s)k_e + k_e k_t - k_t k_b}{q(s) + k_t}\right]T(c_b) = c_0 p(s)$$

$\left[\frac{q(s)^2 + q(s)(k_t + k_b + k_e) + k_e k_t}{q(s) + k_t}\right]T(c_b) = c_0 p(s)$ .... (7)

Consider, $q(s)^2 + q(s)(k_t + k_b + k_e) + k_e k_t$

$$q(s) = \frac{-(k_t + k_b + k_e) \pm \sqrt{(k_t + k_b + k_e)^2 - 4k_e k_t}}{2}$$

$$\Rightarrow q(s) = -\xi_1 \ \& \ q(s) = -\xi_2$$

where,

$$\xi_1 = \frac{+(k_t + k_b + k_e) - \sqrt{(k_t + k_b + k_e)^2 - 4k_e k_t}}{2}$$

$$\xi_2 = \frac{+(k_t + k_b + k_e) + \sqrt{(k_t + k_b + k_e)^2 - 4k_e k_t}}{2}$$

$$\therefore q(s) + \xi_1 = 0 \ \& \ q(s) + \xi_2 = 0$$

$$q(s)^2 + q(s)(k_t + k_b + k_e) + k_e k_t = (q(s) + \xi_1)(q(s) + \xi_2)$$

Equation (7) becomes,

$$\left[\frac{(q(s) + \xi_1)(q(s) + \xi_2)}{q(s) + k_t}\right]T(c_b) = c_0 p(s)$$

$T(c_b) = c_0 p(s)\left[\frac{q(s)+k_t}{(q(s)+\xi_1)(q(s)+\xi_2)}\right]$ .... (8)

Now, $\frac{q(s)+k_t}{(q(s)+\xi_1)(q(s)+\xi_2)} = \frac{A}{(q(s)+\xi_1)} + \frac{B}{(q(s)+\xi_2)}$

$$\therefore q(s) + k_t = A(q(s) + \xi_2) + B(q(s) + \xi_1)$$

Put $q(s) = -\xi_1$

$$-\xi_1 + k_t = A(-\xi_1 + \xi_2) + B(0)$$

$$A = \frac{-\xi_1 + k_t}{\xi_2 - \xi_1}$$

Now put $q(s) = -\xi_2$

$$-\xi_2 + k_t = A(0) + B(-\xi_2 + \xi_1)$$

$$B = \frac{-\xi_2 + k_t}{-\xi_2 + \xi_1} = \frac{-(-\xi_2 + k_t)}{\xi_2 - \xi_1}$$

$$\therefore \frac{q(s)+k_t}{(q(s)+\xi_1)(q(s)+\xi_2)} = \frac{(-\xi_1+k_t)}{(\xi_2-\xi_1)(q(s)+\xi_1)} - \frac{(-\xi_2+k_t)}{(\xi_2-\xi_1)(q(s)+\xi_2)}$$

Equation (8) implies,

$$T(c_b) = c_0 p(s)\left[\frac{(-\xi_1+k_t)}{(\xi_2-\xi_1)(q(s)+\xi_1)} - \frac{(-\xi_2+k_t)}{(\xi_2-\xi_1)(q(s)+\xi_2)}\right]$$

$$T(c_b) = \frac{c_0}{(\xi_2-\xi_1)}\left[\frac{p(s)(-\xi_1+k_t)}{q(s)+\xi_1} - \frac{p(s)(-\xi_2+k_t)}{(q(s)+\xi_2)}\right]$$

$$c_b = \frac{c_0}{(\xi_2-\xi_1)}\left[(-\xi_1+k_t)T^{-1}\left(\frac{p(s)}{q(s)+\xi_1}\right) - (-\xi_2+k_t)T^{-1}\left(\frac{p(s)}{(q(s)+\xi_2)}\right)\right]$$

$$c_b = \frac{c_0}{(\xi_2-\xi_1)}\left[(-\xi_1+k_t)e^{-\xi_1 t} - (-\xi_2+k_t)e^{-\xi_2 t}\right]$$

Now put equation (8) in equation (6), we get

$$T(c_t) = \frac{k_b c_0 p(s)\left[\frac{q(s)+k_t}{(q(s)+\xi_1)(q(s)+\xi_2)}\right]}{q(s)+k_t}$$

$$T(c_t) = \frac{k_b c_0 p(s)}{(q(s)+\xi_1)(q(s)+\xi_2)}$$

Now,

$$\frac{p(s)}{(q(s)+\xi_1)(q(s)+\xi_2)} = \frac{A}{q(s)+\xi_1} + \frac{B}{q(s)+\xi_2}$$

$$\therefore p(s) = A(q(s)+\xi_2) + B(q(s)+\xi_1)$$

Put $q(s) = -\xi_1$

$$p(s) = A(-\xi_1+\xi_2) + B(0)$$

$$A = \frac{p(s)}{\xi_2-\xi_1}$$

Now put $q(s) = -\xi_2$

$$p(s) = A(0) + B(-\xi_2+\xi_1)$$

$$B = \frac{p(s)}{-\xi_2+\xi_1} = \frac{-p(s)}{\xi_2-\xi_1}$$

$$\therefore T(c_t) = \frac{c_0 k_b}{\xi_2-\xi_1}\left[\frac{p(s)}{(q(s)+\xi_1)} - \frac{p(s)}{(q(s)+\xi_2)}\right]$$

$$\therefore c_t = \frac{c_0 k_b}{\xi_2-\xi_1}\left[T^{-1}\left(\frac{p(s)}{(q(s)+\xi_1)}\right) - T^{-1}\left(\frac{p(s)}{(q(s)+\xi_2)}\right)\right]$$

$$\therefore c_t = \frac{c_0 k_b}{\xi_2-\xi_1}\left[e^{-\xi_1 t} - e^{-\xi_2 t}\right]$$

IV. CONCLUSION

we successfully used General integral transform for the models for drug diffusion through the compartments of blood and Tissue medium in the health Sciences. The results obtained by using general integral transform are same as the results obtained by Khanday et al [2] in 2017, where they used Eigen value method and Laplace transform.

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