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Implementation of the Sine - Cosine Algorithm to the Pressure Vessel Design Problem

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Abstract:- Engineering design problems usually include large-scale, nonlinear, or constrained optimization problems. Under a given circumstance, optimization methods aim to find the optimum solutions that give the extremum of a function. There are numerous methods for solving optimization problems. Some of these problems are solved by heuristic or evolutionary approaches. Genetic Algorithms (GA) and Particle Swarm Optimization (PSO) are two of the state-ofart heuristic optimization techniques. Additionally, one of the recently developed heuristic population-based optimization technique is the Sine-Cosine Algorithm.

The pressure vessel design problem has been solved using various methods in the literature. Fruit Fly Algorithm gives the best minimum design cost, which is 5896.9489. In this work, the Sine-Cosine Algorithm was used to optimize the design cost of the pressure vessel and obtained 5888.5213, which is better than the literature's best reported design cost.

Keywords:- *Genetic Algorithm, Particle Swarm Optimization, Sine* **-** *Cosine Algorithm, Pressure Vessel and American Society of Mechanical Engineers.*

I. INTRODUCTION

Mechanical design encompasses the designing of a variety of machine components, such as gears, bearings, clutches, and fasteners (Kannan & Kramer, 1994). The key design requirements for mechanical systems comprise wear, maintenance, liability, weight, size, functionality, safety, dependability, and manufacturability. Some basic design concerns are relevant to all mechanical systems, despite the fact that the number and importance of standards and criteria differ from product design to product design. These factors include the capacity for loading, deformation, stability, and durability. To optimize products, the modeling and analysis of the dependency of these assessment criteria on design variables are required. The mechanical design should satisfy the specifications provided by the American Society of Mechanical Engineers (ASME) (Sandgren, 1990).

Numerous researches have been conducted on the pressure vessel design problem utilizing a variety of techniques, including the genetic algorithm, particle swarm optimization algorithm, augmented lagrange multiplier method, branch and bound, differential evolution approach and fruit fly algorithm (Ke, et al., 2016).

Holland (1975) initially introduced the core ideas of the genetic algorithm (GA). GA employs techniques drawn from natural evolution to tackle optimization issues. The survival of the fittest and the mechanism of natural selection are the foundations of the GA. The three most significant stages in GA are mutation, crossover, and selection.

Eberhart and Kennedy (1995) proposed the Particle Swarm Optimization (PSO). The PSO algorithm imitates the group and individual foraging behaviors of a herd of animals and a flock of birds. The optimization process starts with a set of solutions generated at random, just like the GA algorithm. There is a second set called the velocity set in addition to the set of solutions generated that is used to define and store the speed of particle motion.

Kannan and Kramer (1994) described the augmented Lagrange multiplier method in conjunction with Powell's zeroth order method and, alternatively, the Fletcher and Reeves Conjugate Gradient method as a general approach for resolving mixed discrete, integer, zero-one, continuous optimization problems. Augmented Lagrangian method is one of the techniques for solving constrained optimization problems. They resemble penalty methods in the sense that they add a penalty term to the objective and swap out a constrained optimization problem for a sequence of unconstrained ones. The augmented Lagrangian approach, however, adds still another term that is meant to resemble a Lagrange multiplier. Although they are not identical, the augmented Lagrangian and the Lagrange multiplier approach are linked.

Sandgren (1990) proposed an algorithm that combined the Branch and Bound method with a quadratic programming method, and an exterior penalty function method. A branch and bound approach involve enumerating potential candidate solutions step by step while scouring the whole search domain. A rooted decision tree with all of the potential solutions is first created. The entire search space is represented by the root node. Each child node is a component of the solution set and a partial solution. Based on the ideal solution, we build an upper and lower bound for a specific problem prior to building the rooted decision tree.

Montes et al. (2007) developed the differential evolution (DE) approach. The algorithm explores the design space by keeping track of a population of potential solutions (individuals), and by combining potential solutions in accordance with a predetermined method, it generates new solutions. The candidates with the best objective values are retained in the algorithm's subsequent iteration in order to improve each candidate's new objective value and include it into the population.

Pan (2011) developed a new algorithm called Fruit Fly Optimization Algorithm. The algorithm is built on two primary foraging techniques: using the osphresis organ to

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smell the food supply and sensitive vision to move toward the appropriate area to find food. In terms of sensory perception, the fruit fly performs better than other species particularly in vision and osphresis. Fruit flies have osphresis organs that can identify a variety of smells in the atmosphere. When it is near a source of food, it uses its keen eyesight to locate both the food source and where its companions are congregating before flying in that direction.

The design of pressure vessels is a crucial component of structural engineering optimization, and it frequently looks for ways to reduce costs across board, including those associated with forming, welding, and material costs (Cagnina, 2008). There has been a lot of solutions to the Pressure Vessel Design Problem. Branch and Bound algorithm yielded \$8129.8000 (Sandgren, 1990), Genetic Algorithm yielded \$6288.7445 (Coello, 2000), Particle Swarm Optimization algorithm yielded \$6059.1313 (Hu et al., 2003) and Fruit Fly algorithm obtained \$5896.9489 (Ke et al.,2016), which is the literature's best method for obtaining the minimum design cost.

This paper is looking for a method which will do better than the current best method. Therefore, the sine - cosine algorithm is employed to examine its performance against the known methods in the literature.

II. DESIGN ANALYSIS AND FORMULATION

A. Design Analysis

The cylindrical pressure vessel with hemispherical heads on each of its ends is considered (Sandgren, 1990). The shell is constructed from two rolled plates that will be joined by two longitudinal welds to form a cylinder. Each head is forged, after which it is welded to the shell. Single-welded butt joints with a backing strip are used for all of the welds. The pressure vessel is set up so that the cylindrical shell's axis will be vertical.

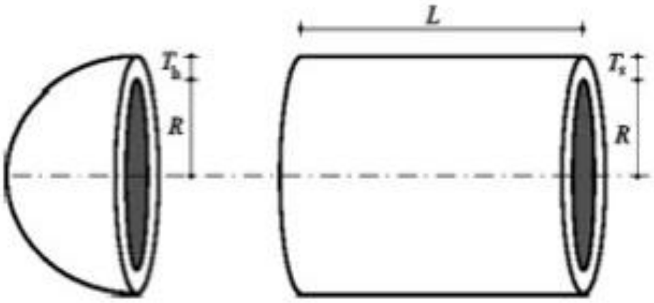

Fig. 1: Schematic of Pressure Vessel

B. Data

The data used in this work was taken from (Sandgren, 1990).

The material used in the vessel is carbon steel ASME SA 203 grade B.

Density of the carbon steel (D) = 0.2830 lb/in^3.

Approximate cost of welding (C_w) = \$8.00/*lb*.

Approximate material cost for the shell plate (C_s) = \$0.35/*lb*.

Approximate material cost for the hemispherical head plate (C_h) = \$1.00/*lb*.

C. Data Processing

The total cost comprises of the welding cost, material cost and forming cost.

a) Welding Cost:

1. Longitudinal welding cost:

$$\text{Longitudinal welding cost} = V_l \times D \times C_w \tag{1}$$

where,

$$V_l = \pi \left(\frac{T_s}{\cos 30^\circ}\right)^2 \times \left(\frac{60}{360}\right) \times L \times 2 \; (Sandgren, 1990) \tag{2}$$

Eqn(2) into (1):

$$Longitudinal\ welding\ cost = 3.1611 T_s^2 L \tag{3}$$

2. Circumferential welding cost:

$$\text{Circumferential welding cost} = V_s \times D \times C_w \tag{4}$$

where,

$$V_s = \pi\left(\frac{T_s}{\cos 30°}\right)^2 \times \left(\frac{60}{360}\right) \times 4 \times R \times \pi \ (Sandgren, 1990) \quad (5)$$

Eqn(5) into (4):

$$Circumferential\ welding\ cost = 19.86211T_s^2R \quad (6)$$

The total welding cost is the sum of the longitudinal and circumferential welding cost.

$$Total\ welding\ cost = 3.1611T_s^2L + 19.8621T_s^2R \quad (7)$$

b) Material and Forming Costs:
Material and forming costs = $2\pi DC_sRT_sL + 2\pi DC_hR^2T_h$ (*Sandgren*,1990) (8) = $0.6224RT_sL + 1.7781R^2T_h$ (9)

D. Formulation of the Problem

The objective is to minimize the total cost of material, forming, and welding. T_s(thickness of the shell, y_1), T_h(thickness of the head, y_2), R (inner radius, y_3) and L (length of cylindrical section of the vessel, y_4) are the four design variables. $T_s$and $T_h$are integer multiples of 0.0625 inch, the available thickness of rolled steel plates, and R and L are continuous. The total cost which we intend to minimize is then given as follows:

$$f(y) = 0.6224y_1y_3y_4 + 1.7781y_2y_3^2 + 3.1611y_1^2y_4 + 19.84y_1^2y_3 \quad (10)$$

The minimum wall thicknesses must be constrained by the constraint set. The minimum value of the tank and the length of the cylindrical shell are both constrained by the $T_s$and $T_h$from the ASME codes (Sandgren, 1990). These constraints are listed as:

Constraint of circumferential stress: $g_1(y) = -y_1 + 0.0193y_3 \leq 0$ (11)

Constraint of longitudinal stress: $g_2(y) = -y_2 + 0.00954y_3 \leq 0$ (12)

Constraint of volume: $$g_3(y) = -\pi y_3^2y_4 - \frac{4}{3}\pi y_3^3 + 129600 \leq 0 \quad (13)$$

Constraint of length:

$$g_4(y) = y_4 - 240 \leq 0 \quad (14)$$

$$1 \times 0.0625 \leq y_1, y_2 \leq 99 \times 0.0625,\ 10 \leq y_3, y_4 \leq 200$$

III. METHODOLOGY

A. Sine - Cosine Algorithm (SCA)

The Sine - Cosine Algorithm was developed in 2016 (Mirjalili, 2016). The sine cosine algorithm begins by generating random solutions known as search agents. The sine - cosine algorithm is tuned using four variables (r_1,r_2,r_3,r_4) and is given by:

X_i^{t+1}=X_i^t+ r_i^t× sin(r2) × |r3P_i^t– X_i^t| r4 <0.5

X_i^{t+1} =X_i^t + r_i^t× cos(r2) × |r3P_i^t– X_i^t | r4 $\geq$0.5

where $X_i^t$is the position of the current search agent in i–thdimension at t–thiteration. The r_1,r_2,r_3 and r_4 are random values. The parameter r_1 uses the expression below to control the exploration and exploitation during the search process.

$$r_1 = 2 - 2\left(\frac{t}{T_{max}}\right)$$

where t is the current iteration and $T_{max}$is the maximum number of iterations.

The parameter r_2 lies in the interval (0,2π) and specifies how far the movement should be toward or away from the destination point. The parameter r_3 has the interval [0,2] and gives random weight to the P_i^t which focuses on the exploration (r_3 >1) and exploitation (r_3 <1). The parameter r_4 lies in the interval (0,1). and toggles between sine and cosine components. Finally, P_i^t is the best destination point obtained so far and || indicates the absolute value.

B. The algorithm for sine - cosine technique:

1. Initialization of the set of search agents $\{X_1, X_2, \cdots, X_n\}$ using
   $x_{ij} = lb + rand(0,1) \times (ub - lb)$
   where x_{ij} is the components of each search agent, lb and ub are the lower and upper boundary respectively.
2. Fitness of each search agent is evaluated, $f(X_i)$
3. Memorize the best destination point, P_i^t
4. The Algorithm parameter r_1 is initialized and updated at every iteration using
   $r_1 = 2 - 2\left(\frac{t}{T_{max}}\right)$
5. Update each search agent using the SCA search equation.

$X_i^{t+1} = X_i^t + r_i^t \times \sin(r2) \times |r3P_i^t - X_i^t|$ $r4 < 0.5$

$X_i^{t+1} = X_i^t + r_i^t \times \cos(r2) \times |r3P_i^t - X_i^t|$ $r4 < 0.5$

6. Evaluate the fitness of each updated search agent $f(X_i^t)$.
7. Repeat steps 3, 4, 5 and 6 until the termination criteria is fulfilled.

The sine-cosine algorithm begins the optimization process with a set of random solutions. The algorithm then stores the best solutions thus far, labels it as the destination point, and modifies all other solutions in respect to it. The ranges of the sine and cosine functions are updated as the iteration counter increases to emphasize exploitation of the search space.

By default, when the iteration counter exceeds the preset number of iterations, the SCA algorithm terminates the optimization process. Any additional termination criteria may be taken into account, including the maximum number of function evaluations or the precision of the discovered global optimum.

IV. COMPUTATIONAL PROCEDURE

Sine - Cosine Algorithm was used in this work. All the codes were run 10 times with 5000 iterations in each run in Octave environment on a computer with the following specifications:
Operating System: Windows 10 Pro Education 64 - bit.
Processor: Intel(R) Celeron(R)
N4020 CPU @ 1.10GHz (2CPUs).
Memory: 4096MB RAM.

Input for the Sine - Cosine Algorithm:
Search Agents (X) = 150
Number of iterations (T_{max}) = 5000
Lower boundary $(y_{1,l}, y_{2,l}, y_{3,l}, y_{4,l})$ = (0.0625,0.0625,10,10),
Upper boundary $(y_{1,u}, y_{2,u}, y_{3,u}, y_{4,u})$ = (99 × 0.0625,99 × 0.0625,200,200), (Sandgren, 1990) Dimension = 4

$$r_1 = 2 - 2\left(\frac{t}{T_{max}}\right)$$

V. RESULTS

The following are the values of the best design variables obtained after running the program for the Sine - Cosine Algorithm:
$y_1 = 0.8259$
$y_2 = 0.3814$
$y_3 = 42.7444$
$y_4 = 168.7212$
$f(X) = 5888.5213$

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The Figure 1 below describes how the fitness value converges towards the 5000 iterations.

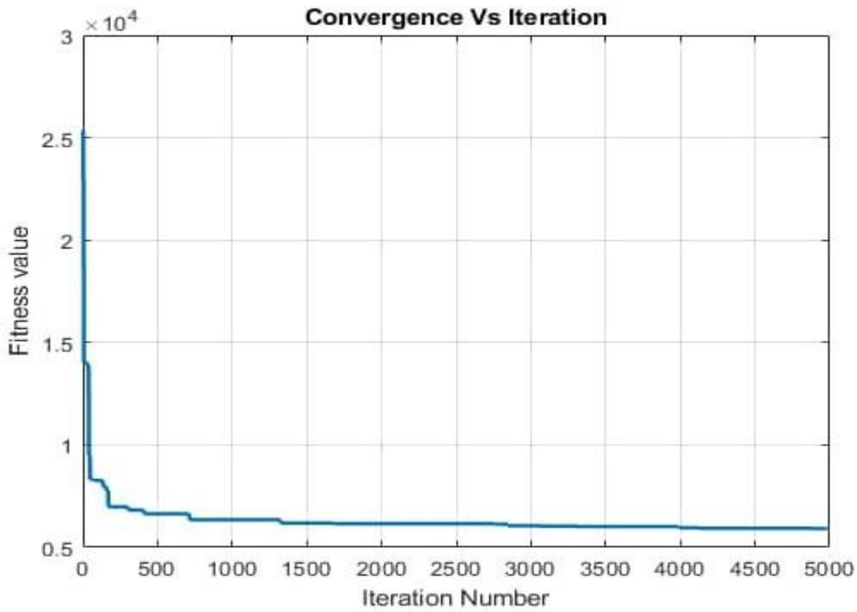

Fig. 1: The Convergence Vs Iteration Graph

From the graph or simulation, it was observed that the optimal cost of 5888.5213 was attained at the 4500th iteration after which it was maintained for the rest of the iterations.

VI. DISCUSSION

From the results section, we have:
Thickness of the shell (y_1) = 0.8259 inch.
Thickness of the head (y_2) = 0.3814 inch.
Inner radius (y_3) = 42.7444 inch.
Length of the shell (y_4) = 168.7212 inch.
Minimum cost $f(X)$ = $5888.521

The table 1 below shows the optimal cost calculated by the various methods found in theliterature as compared to the value o btained in this paper

Authors	Methods	y_1	y_2	y_3	y_4	$f(y)$
Sandgren(1990)	Branch and Bound	1.1250	0.6250	47.7000	117.7010	8129.8000
Zhang and Wang(1993)	Simulated Annealing	1.1250	0.6250	58.2900	43.6930	7197.7000
Kannan and Kramer (1994)	Augmented Lagrangian method	1.1250	0.6250	58.2910	43.6900	7198.2000
Deb and Gene (1997)	Genetic Adaptive	0.9375	0.5000	48.3290	112.6790	6410.3811
Coello (2000)	Genetic Algorithm	0.8125	0.4375	40.3239	200.0000	6288.7445
Coello and Montes (2002)	Improved Genetic Algorithm	0.8125	0.4375	42.0974	176.6541	6059.9460
Hu et al. (2003)	Particle Swarm	0.8125	0.4375	42.0985	176.6366	6059.1313
Gandomi et al. (2003)	Cuckoo Algorithm	0.8125	0.4375	42.0984	176.6366	6059.7143
He et al. (2004)	improved Particle Swarm	0.8125	0.4375	42.0984	176.6366	6059.7143
Lee and Geem (2005)	Harmony Search	1.1250	0.6250	58.2789	43.7549	7198.4330
Montes et al. (2007)	Differential Evolution	0.8125	0.4375	42.0984	176.6360	6059.7017
He and Wang (2007)	Effective Particle Swarm	0.8125	0.4375	42.0913	176.7465	6061.0777
Montes and Coello (2008)	Mixed Genetic Search	0.8125	0.4375	42.0981	176.6405	6059.7456
Cagnina et al. (2008)	New Particle Swarm	0.8125	0.4375	42.0984	176.6366	6059.7143
Kaveh and Talatahari (2009)	Hybrid Particle Swarm and Ant Colony Search	0.8125	0.4375	42.1036	176.5732	6059.0925
Kaveh and Talatahari (2010)	Improved Ant Colony Search	0.8125	0.4375	42.0984	176.6378	6059.7258
Coelho (2010)	Gaussian Quantum Behaved Particle Swarm	0.8125	0.4375	42.0984	176.6372	6059.7208
Akay and Karaboga (2012)	Artificial Bee Colony	0.8125	0.4375	42.0984	176.6366	6059.7143
Ke et al. (2016)	Fruit Fly Algorithm	0.7810	0.3863	40.4340	198.5049	5896.9489
Present Study	Sine - Cosine Search	0.8259	0.3814	42.7444	168.7212	5888.5213

Table 1: Comparison of the best solution for the pressure vessel design problem

From table 1, the difference between the results obtained from the Artificial Bee Colony technique and the Gausian Quantum Behaved Particle Swarm technique is 0.5063. Also, the difference between the results obtained from the Mixed Integer technique and the New Particle Swarm technique is 0.0313. These differences in the results were considered significant.

The current work optimizes the parameters such as the thickness of the shell, length, and radius of the pressure vessel using the Sine - Cosine Algorithm. The results are compared to various works which used other optimization methods and are shown in Table (6.1). It has been found that the optimal design cost, $5888.5213, obtained by the Sine - Cosine Algorithm is better as compared to the literature's best cost of $5896.9489.

The choice of the 5000 iterations, 150 search agents and the currency in dollars is that the comparing methods used the same information and therefore, the study did not want to introduce any variation to conflict the results. Also, the results hold for Carbon Steel ASME SA 203 grade B.

VII. CONCLUSION

In this paper, the optimal design cost of a pressure vessel was carried out through the sinecosine algorithm (SCA) by optimizing parameters which include thickness of the shell and head, length of the shell and inner radius of the pressure vessel. The results obtained are compared with the results of other optimization methods in the literature applied to the pressure vessel design problem.

Computational simulations indicate that the proposed SCA approach achieves the best result in terms of objective function (total design cost) minimization, being 5888.5213 which is 8.4276 (0.142%) better than fruit fly algorithm method, with 5896.9489, which is the best literature's best reported objective function value.

It can be concluded that by using the sine-cosine algorithm, the pressure vessel's optimal design parameters are found, and the objective of cost minimization by reducing pressure vessel weight is accomplished. The application of the sine-cosine algorithm to a pressure vessel problem with four design constraints and four variables has been demonstrated in this paper.

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