

Measurement of Yarkovsky Effect using Stefan Boltzmann Law

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Abstract:- Yarkovsky effect is a recoil force arising from the anisotropic thermal re-emission of absorbed solar radiation. This recoil force causes wide orbital change making it hard for the scientists to predict the orbit of potentially hazardous asteroids and their long term risk to the Earth.

Developing mathematics in order to calculate Yarkovsky effect in an efficient and simple way without getting into higher level astronomy, mathematics and physics is our main purpose. We have tried to give the insights about Yarkovsky effect and mathematics behind it using the simple concepts of Stefan Boltzmann law and Yarkovsky effect in a way so that even a high school student, amateur astronomers know about Yarkovsky effect and how it can be measured.

I. INTRODUCTION

When sunlight strikes a rotating asteroid the side where the sunlight falls gets heated as the asteroid turns the other side gets cooled and releases thermal radiation causing recoil force to generate which bring about orbital change. There are many potentially hazardous asteroids whose orbits need to be measured with great accuracy. Some measuring Yarkovsky effect plays a great role in orbital prediction of asteroid. In case the technology and advancement that we

have now fail to measure the orbit of asteroid- we may have to face the consequences such as asteroid strike over the Russian city Chelyabinsk in 2013 as at that time the recent technology failed to detect asteroid due to opposition effect.

Yarkovsky effect makes it harder for the scientists to predict the orbit of PHA due to wide orbital change. Although OSIRIS Rex satellite has been launched by NASA in order to measure Yarkovsky effect on asteroid 1999 RQ36, Yarkovsky effect needs to be explained in simple way without getting into complex mathematics and this is what we have presented in our paper which can lead to further development in the field of planetary defense system.

II. THEORY

This paper revolves around the implications of Stefan Boltzmann Law. This law states the total radiant heat power emitted per unit surface area of a black body is proportional to the fourth power of its absolute temperature T, i.e. $P \propto T^4$

Mathematically, Stefan – Boltzmann Law can be represented as: $P = e\sigma AT^4$

Where “e” is the emissivity of the object and its value is 1 for black holes and stars and σ is the Stefan Boltzmann constant whose value is $5.670374419 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$

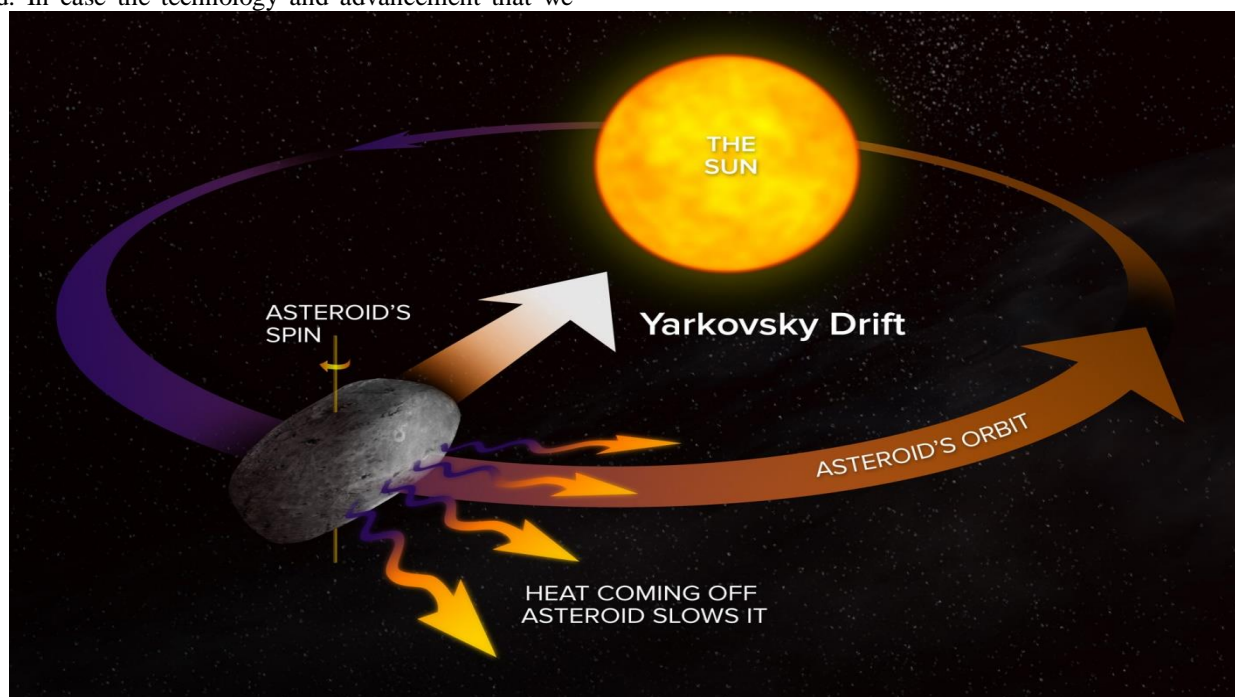


Fig. 1: Yarkovsky effect.

(Source:<https://public.nrao.edu/gallery/yarkovsky-effect/>)

According to this law, power emitted by the sun is
 $P_{\text{sun}} = \sigma AT^4$

Power received by an asteroid at a distance ‘r’ from the sun
 $P_{\text{asteroid}} = \frac{\sigma T^4 A}{4\pi r^2} \times \text{Surface area of the asteroid}$
 $P_{\text{asteroid}} = \frac{\sigma T^4 A}{4\pi r^2} \times A_1$

Where,

A= Surface area of the Sun

A₁ = Surface area of the asteroid

T = Temperature of the sun

r = distance between the sun and asteroid

As Yarkovsky effect is all about absorption and re-emission of energy. Now, let’s calculate energy received by an asteroid

Energy received during ‘t’ time interval (E) = $\frac{\sigma T^4 A}{4\pi r^2} \times A_1 (1-\phi) t$

Where φ is the albedo of the asteroid, Albedo is the measure of the diffuse reflection of solar radiation out of the total solar radiation and measured on a scale from 0, corresponding to a black body that absorbs all incident radiation, to 1, corresponding to a body that reflects all incident radiation.

According to Newton’s second law of motion, force experienced by a body is change in its momentum in certain interval of time.

$$\text{Force} = \frac{\text{Change in momentum}}{\text{time taken}}$$

$$= \frac{\Delta p}{\Delta t}$$

Let us suppose the momentum of asteroid at the starting of time period “t” be zero and after the “t”, let the momentum be p
 $= \frac{p}{t}$

$p = \sqrt{2mE}$, where E is the energy received by an asteroid during “t” interval of time

Now,

$F = \sqrt{\frac{2m\sigma T^4 AA_1(1-\phi)*t}{4\pi r^2}}$ which is the expression of the Yarkovsky force.

Mathematically, force is mass times acceleration. So,

$$ma = \sqrt{\frac{2m\sigma T^4 AA_1(1-\phi)*t}{4\pi r^2}}$$

$a = \sqrt{\frac{2\sigma T^4 AA_1(1-\phi)*t}{m4\pi r^2}}$, this is the expression of the Yarkovsky acceleration.

Yarkovsky acceleration is a recoil acceleration which leads to a small variation of the semi-major axis of an asteroid.

III. CONCLUSION

Although, Yarkovsky effect cannot be measured accurately using the method described in this paper, it can provide insights about the measurement of Yarkovsky effect by using the estimated values or previously discovered ones which is the prime objective of this paper. In this method many factors such as density, illumination, direction of spin are neglected as advanced equipment are needed to measure these parameters.

As our research may not provide the answers of all the queries but it will definitely provide the future lead in the field of planetary defense.

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