Approaches to Enhance Mathematical Thinking to Realize NEP 2020 Vision for Young Learners

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Abstract:- The present paper throws light on some approaches for teaching and learning of the young learners such that it leads to the enhancement of mathematical thinking as per the vision of NEP 2020.The National Education Policy emphasizes on strengthening the foundation of quality education. It pays focus on cognitive depth of the learner as an important aspect and on curbing the rote learning system. Thus, it presents a dire need to take into account each approach critically and understand its suitable applicability for the effective education in Mathematics. The approaches which are discussed in the present paper may prove to be of great use for the young learners and moreover the future of 'Incredible India'.

Keywords:- Approaches, Quality, Education, Learners, Teaching, Learning, Mathematical, Thinking, NEP2020.

I. INTRODUCTION

After independence, many significant steps were taken by the Indian government in the field of Education. The first action in this regard was the appointment of University Grants Commission or Radhakrishnan Commission on November 4, 1948 under the chairmanship of Dr. Sarvapali Radhakrishnan. The Commission was inaugurated on December 6, 1948 and greatly influenced the University Education in India. The next major effort came up on September 23, 1952,when the government of India appointed Secondary Education Commission on September 23, 1952 under the chairmanship of Dr. A.L. Swami Mudaliar. The Commission is also known as Mudaliar Commission and it played an important role in secondary education of India. In July 14,1964, Indian Education Commission was appointed by the government of India under the chairmanship of Dr. D.S. Kothari. This commission was also known as Kothari Commission and the whole system of Education was transferred to serve as a National system of Education and served as a complete survey of the entire educational situation. In 1986, National Policy of Education came up, which has been regarded as Magna Carta of the Education for years to come. Every aspect of the policy was framed keeping in view the human resource development and the needs of the 21st century. Then, in 1990, National Policy on Education Review Committee came up and in 1992, Revised National Policy on Education was presented. (Walia,2005)

All the commissions and policies which have been discussed had main focus on access and equity whereas the emphasis of National Education Policy i.e. NEP 2020 which is the first Education Policy of the 21st century is on quality education.

NEP 2020basically aims at revision of the structure of education for holistic development, quality teaching-learning and building up of best education system globally. The effort is made for building up a bright future of young learners and hence of 'Incredible India'. The persisting challenges are enforcing the change in the prevailing Education System. As per the vision of NEP 2020, competency-based learning is truly needed. (NEP, 2020)

The way a young learner thinks is seemed as the main focus in the present scenario in the field of teaching and learning. The main base of thinking is perception i.e. how a young learner perceives the world around. Observation, experience, interaction are the main pillars on which perception is based. Piaget, the famous psychologist suggested two processes as responsible for the thinking of the young learner: Perception (Consequence of direct contact) and Representation (Consequence of mental imagery when there is no direct contact). The development of thinking is a process which involves perception as the base, then sensory-motor experiences and finally thinking which may further transfer as critical thinking. Infact, the characteristics of cognitive development form the understanding of the development of insights for facilitation of effective teaching and learning. (NIOS)

II. MATHEMATICAL THINKING

As a subject, Mathematics plays a significant role in progress of a society, country and infact the world at large. Mathematical Thinking is the soul of learning of Mathematics. The five components (communicating, problem solving, reasoning, understanding & fluency) of Working Mathematically describe how content is explored or developed − that is, the thinking and doing of mathematics. – New South Wales Educational Standards Authority (NESA). (Coutts, 2019)

H. Weyl defined mathematical thinking as "By the mathematical thinking, I mean first that form of reasoning through which mathematics penetrates into the sciences of the external world and even into our everyday thoughts about human affairs." The development of mathematical thinking takes place through intuitive thinking i.e. experiencing using concrete ways and later through reflective thinking which refers to building of theory and generalization. There are large gaps in attainment of an in-depth understanding of the subject Mathematics. (NIOS)

Rote learning leads to reduction in the demands on working memory and the cognitive resources move at liberty which are direly needed for more advanced problem solving. Superficial rote learning strategies can be a major obstacle

to learning and using mathematics (Lithner 2000, 2003, 2008; Boesen et al., 2010).According to a review by Hiebert (2003), there are "massive amounts of converging data" showing that such teaching models fail to promote students' development of central mathematical competencies effectively and instead lead mathematics students to try to follow rote learning (i.e., by mechanical or habitual repetition) task-solution methods "like robots with poor memories" (p. 12).There are two ways for the enhancement of mathematical thinking: One is self-evolvement i.e. as the child evolves mathematical thinking keeps on evolving and the other is external drive which helps the enhancement pick up the pace.Improvement in mathematical thinking occurs with reading, experience and problem solving.The productive struggle is rooted in the fact that developing central mathematical competencies (e.g., reasoning ability and conceptual understanding) requires active engagement in corresponding challenging learning processes (e.g., non-routine problem solving). There is little or no transfer to such competencies from easier learning processes, such as imitation of given solution templates (Schoenfeld, 1985; Brousseau, 1997&Niss, 2007). (Lithner, 2017).

III. APPROACHES TO ENHANCE MATHEMATICAL THINKING AS PER NEP2020 VISION

There are ample approaches for teaching and learning of Mathematics and every approach is unique in itself in inculcating mathematical thinking. Before unfolding the approaches, one needs to understand the human information processing system. In context to the young learners, the main features of the system involve:

- Experience leads to learning i.e. Learning by Induction is the key process for learning of a human being.
- The human beings have a limited capacity of working memory i.e. more focus on understanding and obtaining skills and less burden on memory.
- Focus on not only learn but on learn how to learn i.e. meta-cognitive abilities of the human needs to be explored. (NIOS)

The approaches may either have steps or features or the approach presents an idea which the teacher can devise for inculcating the teaching learning process. In the present paper, in some approaches an idea is presented and the responsibility of devising a plan using the innovative efforts lies with the teacher whereas in some approaches the steps and features are discussed in detail. The approaches which are suitable as per the vision of NEP 2020 for the young learners are discussed as follows:

A. Manipulation of Objects

The approach serves as the base of acquisition of mathematical thinking by the young learners using the manipulation of concrete objects. Both novel as well as familiar variety of objects should be made available to facilitate a solid base for attaining skills of thinking in the subject. (NIOS)

B. Tasks in meaningful contexts

The approach helps the young learners learn quickly in their own creative way and explore in the real-life situations where the subject of mathematics serves the real purposes. Real life situations outside the school or can be created within the classroom or at large school and manipulated by the teacher using different methods of teaching, for e.g.: Role modelling etc. Apart from this an imaginative construct related to a real-life situation can also be built using different methods of teaching, for e.g.: Story Telling etc. Meaningful contexts lead the young learner to easily adjust and in fact understand the abstract representations. (NIOS)

C. Creative Mathematically founded Reasoning (CMR, Lithner 2008)

The approach using CMR helps in the development of creative mathematical reasoning.The studies done empirically has displayed CMR based on three criteria:

- Creativity: The learner creates a reasoning sequence not experienced previously, or re-creates a forgotten one (Silver 1997).
- Plausibility: There are predictive arguments supporting the strategy choice and arguments for verification, explaining why the strategy implementation and conclusions are true or plausible (Pólya 1954; Lithner 2008).
- Anchoring: The arguments are anchored in the intrinsic mathematical properties of the components of the reasoning (Lithner 2008).(Lithner,2017)

D. Representation in Multiple ways

The approach of representation in multiple ways encourages alternative strategies which serves as a platform for thinking. The ability of representing in multiple ways works as a path towards abstract thinking. The more representations the learner is tending to make, the more nearer is the mathematical thinking that is required. Different steps before representing can be followed using different methods and hence the representations can be explored. (NIOS).

E. Problem solving

The approach of problem solving leads to an in-depth understanding of the concepts of the learner.The problem solving approach involves five major steps i.e. Introducing the problem, understanding the problem, analyzing the problem, solving the problem, interpreting the problem which can be implemented using various strategies. With indirect support the learner can gain the problem solving abilities and hence gain mathematical thinking along with critical thinking. (NIOS)

F. Theory of didactical situations

The approach using theory of didactical situations curbs rote learning and suggests the ways for mathematical thinking. Brousseau's (1997) theory of didactical situations in mathematics abbreviated as TDSis used as the starting point for the design of a more constructive alternativeand it serves as a clarification of the characteristics and consequences of rote learning theoretically. The main features of the theory are:

- Firstly, it encourages intellectual original work and a struggle needed for enhancing mathematical ability.
- Secondly, it favours that learning by imitation is ineffective. An algorithm to solve a task without understanding its meaning is considered as an enhancer of rote learning.
- Thirdly, the aim of the theory is devolution of problems by the teacher i.e. a teacher arranges a didactic situation in the form of a suitable problem and students take up the responsibility of solving the problem and obtain the desired target of learning. (Lithner, 2017).

G. Concept Mapping

A concept can be developed in the form of a map containing all the related sub-concepts and the interlinks. The learners sometimes perceive the concepts as disjoint and are not able to make the relationships between them. But since more or less no concept is isolated in mathematics, interlinkage must be found by the learner for better and complete understanding of the subject. Thus, the approach using concept maps organises and interlinks the knowledge which is a very important aspect in teaching and learning process. (NIOS)

H. Experiential Learning

It is an approach where learners get engaged in an activity, reflect critically on it and gain an insight and hence learning. The approach enhances the mathematical thinking. The experience is their own and it becomes an integral part of the young one's behaviour. Apart from skill development, inculcation of values can also be done using different methods. The approach of Experiential Learning includes five steps which are as follows:

➤ The first step involves interpersonal interaction and gaining of experiences by the learner.
➤ The second step involves sharing of experiences and hence sharing their observations.
➤ The third step involves the examination of previously gained experiences which leads to the assessment of the learner by the group.
➤ The fourth step involves generalization of the concept i.e. the learner now begins to focus on the similar experiences observed in the third step.
➤ The fifth step involves application of the generalized concept in actual situations. (NIOS).

I. Structuralist Approach

It is an approach which involves visual, Hands-on Models to present basic but abstract ideas. The presentation of abstract ideas of course leads to the enhancement of Mathematical thinking among the young learners. In this approach, the teacher is to plan and prepare the structure suitable to the respective topic for explaining a particular abstract idea or theory. (Manizade, 2018)

J. Integrated- Environmentalist Approach

The present approach seeks to apply a global and systemic approach to solve socio-environmental problems and to check whether education for sustainable development helps to develop and encourage actions that promote sustainable development. The problem solving using mathematics definitely enhances mathematical thinking and the integrated approach at large may fulfil the goals of NEP 2020.In this approach, the practical problems on the socio-environment integrated issues need to be framed or chosen by the teacher and further a proper scale or test to check the development of the learner needs to be administered and interpreted.
(Camacho et al., 2019)

K. Formative Approach

In this approach, the teacher gains insight into every aspect of the developmental trend of the learner. Formative approach can give a comprehensive idea about the learner's mathematical cognition and conceptual development to the teacher. To make the respective approach successful the teacher needs to devise different ways to understand the strengths, weaknesses and capabilities of the learner.

L. Action Learning

In mathematics education, the genesis of approach of action learning is in the early childhood experience which has natural levels of maturity. In mathematics education, the motivation for action learning gradually changes from winning games to success in real-world ventures. The key to attaining success in building mathematical ability is to solve problems. According to research, curiosity can be characterized in terms of excitement about peculiar observations and unexpected phenomena. Reflection plays an equally important role which works as an internal control. In fact, action learning provides an effective and clear approach towards mathematical thinking and therefore mathematics education. (Abramovich et al., 2019).

M. Learner Centred Approaches

The focus of learner-centred approach is on the students to explore and construct their own knowledge using their own experiences and teacher is just a facilitator.

N. 5 E's Learning Model

The 5 E's Learning Model involves five phases:

➤ Engagement Phase-
The learners are engaged in any task in any form in the classroom where it works as an opportunity for the students to build a relationship between their previous knowledge and the existing ideas.
➤ Exploration Phase-
The learners discuss and explore in groups, getting involved with phenomenon and the materials. They build a platform for the common experiences and hence at large a ground of experiences.
➤ Explanation Phase-
The learner carrying the common experiences, begins to lay the foundation of abstract experiences and begin to clarify their misconceptions under the explanation of the teacher.
➤ Elaboration Phase-The learners elaborate their knowledge and hence expand their knowledge. In fact, the learners apply the concepts gained to make connections with other related fields and hence gain understanding of the real world.
➤ Evaluation Phase-

The diagnostic phase which determines whether the learner has acquired the required knowledge and understanding. (NIOS)

O. Interpretation Construction Design Model
It is a model which contains seven steps:

- Step 1: The learners observe the situation or the problem before proceeding for the solution.
- Step 2: The learners relate the situation or the problem to their previous experiences.
- Step 3: The learners are made to analyse and interpret the situation or the problem through brainstorming.
- Step 4: The learners collaborate to make discussions in groups and explore to gain the understanding of their analysis and interpretations.
- Step 5: The learners analyse the knowledge which is constructed and generate an interpretation of their own.
- Step 6: The learners use the interpreted knowledge and multiple interpretations are made by them in lieu of solving the problem.
- Step 7: The learners apply multiple interpretations to attain multiple solutions of the problem. (NIOS)

P. Flipped Classroom Model for Teaching Mathematics
In a Flipped Classroom all form of teaching content and activities are provided online to the students in advance so that they get access and go through the materials before attending the classroom lesson. Therefore, when the learners attend the classroom, they are entirely familiar with the teaching learning material. As a result of the collaborative instruction mode, the young learners can avail the chance of getting engaged in the process of teaching and learning thoroughly. The respective teaching model promotes independent learning along with improved learning awareness and hence encouraging students to work together with peers.(Umam et al., 2019; Yousufi, 2020)

Q. The approaches identified by Webb (1992)
The three approaches for the young learners are: the topics approach, the process approach or operational approach, and the conceptual fields approach.

R. Topics approach
The first advantage which is proposed by the topics approach is that if the teachers follow the curriculum and teach the topics week by week as the order is prescribed then obviously the young learners will attain some proficiency in the subject. The second related perceived advantage may be that the order and progression of the topics are carefully planned such that the conceptually preceding concepts are presumably taught prior to the more advanced topics. Both the perceived advantages of the topics approach are premised on the assumption that teachers already have knowledge of the underlying mathematical principles and properties and have a clear understanding of the challenges that the young learners will face. The disadvantage of the topics approach may be that it can bypass the necessity for advanced mathematics knowledge. It is in fact the teacher who interacts with the learners keeping in mind their current proficiency and the various paths towards abstract concepts, further promoting learning.

S. Process approach
The theoretical foundation of the approach is derived from Piaget and collaborators, who proposed that conceptions and competences are attained through activity (Piaget 1952; Piaget &Inhelder 1969). Dewey, who proposed that the thoughtful methods employed in problem-solving may be likened to the work of advanced mathematicians (1910, cited in Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Olivier & Wearne 1996); and then to Polya (1957). In this process approach, the focus is more directly on the learner and on the development of skills obtained through engagement with problems.The advantage of this process approach, when the lessons have been well conceptualised and planned, is that having learners engage with contextually relevant and therefore meaningful problems will propel their curiosity, exploration, and the discovery of new learning (Hiebert et. al., 1996). A disadvantage of the approach arises when the teacher unthinkingly presents the whole class with a problem which is either out of thegeneral zone of proximal development of the learner or it is not conceptualised for learning purposes. The second failing of the approach is that the teacher underestimates the extent and degree of planning and of both indirect and direct teaching that underpins such an approach.

T. Conceptual fields approach
The conceptual fields approach draws primarily on the work of Vergnaud (1983; 1988), in which he responds to both the complexity of mathematics knowledge and the gradual acquisition of this knowledge by learners by positing a complex conceptual framework. One of the challenges of mathematics education noted by Vergnaud (1988) is that arguably every mathematics concept is rooted in situations and problems, and in consequence a single concept may be applied to multiple problem situations; at the same time one specific situation or problem may require many distinct mathematics concepts. The reality from a cognitive perspective is that related concepts do not develop in isolation (as in separate steps in a sequence), but simultaneously and in conjunction with other concepts. Building on the notion of a conceptual field we note that addition and subtraction are not inherently separate rather they are related concepts. The disadvantage of a conceptual fields approach may be that it requires more advanced mathematical knowledge. A second challenge may be that more attention is required in regard to the existing understanding of the learner. (Long & Dunne, 2014)

IV. CONCLUSION

The various views of NEP 2020 regarding the subject of Mathematics teaching and learning impact directly on teachers. In lieu with the vision of NEP 2020, the listed approaches in the paper may help in the enhancement of mathematical thinking. A teacher may have an implicit view of mathematics and her approach to teaching will reflect the suggested approaches in one way or another. The professional development programmes may extend the understanding and reflection of teacher in the teaching and learning of Mathematics and pave the path to attain the level of mathematical thinking required for the young learners.

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