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Flow of Blood in Asymmetric Stenosed Artery with Velocity Slip at the Interface: A Two–Layered Casson Model

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Abstract:- The current study looks at a two-layered Casson model of blood flow inside an asymmetrically stenosed artery with velocity slip at the interface. The model consists of a central layer of red blood cells suspended in a Casson fluid, a middle layer of Newtonian plasma, and a peripheral layer of Newtonian plasma. Analytical equations foraxial velocity, flow rate, wall shear stress, apparent viscosity, and pressure gradient are obtained and their variations with different parameters are illustrated in figures and their variations of flow variables has been discussed in this article.In addition, the physiological consequences of this theoretical modeling for blood flow circumstances are briefly examined.

Keywords:- *Stenosed artery, shear stress, Casson fluid, resistance to flow, wall shear stress.*

I. INTRODUCTION

Many scholars are becoming increasingly interested in theoretical modeling of various blood flow scenarios.Many attempts have been made to conduct viscometric investigations and theoretical modeling on blood flow, as well as to investigate flow factors in a vascular channel such as velocity profile, flow rate, pressure drop, apparent viscosity, pressure-flow relationship and resistance to flow (Pedly1980, Fung1984, Puniyani and Nimi 1998,Biswas 2000).Many authors have looked at blood flow from various viewpoints using analytical models (Skalak 1982, Biswas and Chakraborty 2009) and experimental investigations (Bugliarello and Hayden 1962, Bugliarello and Sevilla 1970, Young and Tsai 1973).As a result, various theoretical, analytical, and numerical models in the subject of Biomechanics have been suggested.Furthermore, due to the fact that many cardiovascular (cvs) disorders are closely related with the flow phenomena in blood vessels, i.e. in CVS, a great deal of curiosity, attention, and enthusiasm among scientists in this particular domain of Biomechanics has gained traction (Dintenfass 1981, Caro 1981, Chien 1982). Blood behaves like a Newtonian fluid at high shear rates and big diameter arteries, where the shear stress against rate of strain relationship is linear (Fung, 1981; Taylor, 1959). Young (1968) investigated the impact of stenosis on blood flow behavior. It has been observed that as the size of the stenosis grows, so does the resistance to flow and the wall shear stress. Young's work was expanded by Forrester and Young (1970), who looked at the consequences of flow separation in an artery with modest stenosis. Young and Tsai presented the results of their experimental work on models of vascular stenosis (1973). The pulsatile blood flow through a stenosed artery was studied by Sanyal and Maiti (1998). They used graphs to illustrate numerical solutions for axial velocity profiles and pressure gradients. Blood flow in a catheterized curved artery was examined by Jayaraman and Tewari (1995). They treated blood as an incompressible Newtonian fluid and analyzed the flow using the standard zero-slip boundary condition. By treating blood as a Newtonian fluid, Sarkar and Jayaraman (1997) proposed a model of blood flow through a catheterized stenosed artery. Many writers have reported on the importance of slip velocity at flow borders, both theoretically and experimentally (Vand, 1948; Bloch, 1962; Brunn, 1975; Nubar, 1967; Bugliarello and Hayden, 1962; Bennet, 1967). The influence of slip velocity on blood flow via a stenosed artery was studied by Chaturani and Biswas (1984). Biswas and Chakraborty studied pulsatile blood flow in a stenosed artery using an axial velocity slip at the stenosed wall and treating blood as a Newtonian fluid (2010).Biswas and Chakraborty explore the annular flow of blood in a catheterized tapering artery (2009). Biswas and Laskar (2012) investigated a Newtonian fluid model of blood flow in a catheterized multi-stenosed artery with an axial velocity slip condition at the stenotic wall in a catheterized multi-stenosed artery with an axial velocity slip condition at the stenotic wall. Biswas and Paul recently proposed mathematical modeling of blood (Newtonian fluid) flow through an angled non-uniform stenosed artery (2013).Casson's equation, according to Charm and Kurland (1965), can be used to analyze blood flow over a wide range of hematocrit and shear rates. Oka controls the flow of Casson fluid in tapered tubes (1973). Chaturani and Samy (1982) used a stenosed two-layered mode with a Newtonian fluid peripheral layer and a Casson fluid core area. Using the Casson fluid model of blood, Dash et al. (1995) investigated the changing flow pattern and assessed the increase in flow resistance in a small catheterized artery.Many writers have looked at Casson fluid models of blood flow for various flow circumstances and geometries (Chaturani and Palanisamy, 1990; Biswas and Bhattacherjee, 2003; Nagarani and Sarojamma, 2008). By considering blood as a Casson fluid, Sankar (2009) presented a two-layered pulsatile flow of blood inside a catheterized artery with core area as Casson fluid and plasma in a constricted rigid artery. Verma et al (2011). The aberrant growth of stenosis at the artery wall is generally non-symmetrical, as can be shown. With the aforementioned rationale, a 2-layered model of asymmetric stenosis was created to investigate the impacts of slip (at the asymmetric stenosis) and the influence of flow variables (wall shear stress, velocity, flow rate, pressure gradient, apparent viscosity).The flow is supposed to be constant and laminar in motion.

Theoretical modelings on different blood flow situations have drawn an increasing interest and keen attention of many researchers. There could be noticed a great many attempts which are motivatedto the viscometric studies and theoretical modeling onblood flow as well as in the investigation of flow variables, such as velocity profile, flow rate, pressure drop, apparent viscosity, pressure-flow relationship and resistanceto flow, in a vascular channel(Pedly1980,Fung1984, Puniyani and Nimi 1998,Biswas 2000). Many authors have examined blood flow through analytical models(Skalak 1982, Biswas and Chakraborty 2009) and experimental (Bugliarello andHayden 1962, Bugliarello and Sevilla 1970, Young and Tsai 1973) works withdifferent perspectives. Consequently, a good many theoretical, analytical and numerical models have been proposed in the field of Biomechanics. Further, a great interest, attention and enthusiasm among the investigators in this particular domain of Biomechanics, have gained a momentum due to the fact that many cardiovascular (cvs) diseases are closed associated with the flow phenomenon in blood vessels i.e. in CVS (Dintenfass 1981, Caro 1981, Chien 1982). It is already said that at high shear rate and large diameter arteries, blood behaves likes a Newtonian fluid where in shear stress versus rate of strain relation is linear(Fung, 1981; Taylor, 1959). Young (1968) has examined the effects of stenosis onflow behavior of blood. It has been noticed that resistance to flow and wall shearstress increase with the increase of stenosis size. Forrester and Young (1970)extended the work of Young (1968), including the effects of flow separation in anartery with mild stenosis. Results of experimental work on models of arterial stenosis, have been presented by Young and Tsai (1973). Sanyal and Maiti (1998) have addressed the pulsatile blood flow through a stenosed artery. They have discussednumerical solutions of axial velocity profile and pressure gradient graphically. Jayaraman and Tewari (1995) have studied blood flow in a catheterized curved artery. They assumed blood as an incompressible Newtonian fluid and usual zero-slip boundary condition is used to analyze the flow. Sarkar and Jayaraman (1997) have put forward a model of blood flow through a Catheterized stenosed artery by considering blood as a Newtonian fluid. The importance of slip velocity at flow boundaries has reported by many authors both theoretically and experimentally (Vand, 1948; Bloch, 1962; Brunn, 1975; Nubar, 1967; Bugliarello and Hayden, 1962; Bennet, 1967). Chaturani and Biswas (1984) have examined the effect of slip velocity on blood flowthrough a stenosed artery. Pulsatile flow of blood in a stenosed artery by using an axial velocity slip at the stenosed wall and by considering blood as Newtonian fluid has been dealt by Biswas and Chakraborty (2010). The annular flow of blood in a catheterized tapered artery, is investigated by Biswas and Chakraborty (2009). Biswasand Laskar (2012) have studied a Newtonian fluid model of blood flow, in a catheterized Multistenosed artery with an axial velocity slip condition at the stenotic wall. Recently, Mathematical modeling of blood (Newtonian fluid) flow through inclined non-uniform stenosed artery has been proposed by Biswas and Paul (2013). Charm and Kurland (1965) have reported the utility of Casson's equation which can be used to analyze blood flow over a wide range of hematocrit and shear rates. The flow of Casson fluid in tapered tubes is dealt by Oka (1973). Chaturani and Samy (1982) have taken a stenosed two-layered mode with peripheral layer of Newtonian fluid and the core region of Casson fluid. Dash *et al.* (1995) have studied the changedflow pattern and estimated the increase of flow resistance in a narrow catheterizedartery, by using the Casson fluid model of blood. Casson fluid models of blood flow for different flow situations and geometries have investigated by many authors (Chaturani and Palanisamy, 1990; Biswas and Bhattacherjee, 2003; Nagarani andSarojamma, 2008). Sankar (2009) has proposed a two-layered pulsatile flow ofblood inside a catheterized artery with core region as Casson fluid and plasma in aconstricted rigid artery, by considering blood as a Casson fluid, has been studied byVerma *et al*.(2011).It is seen that the abnormal growth of stenosis at the arterial wall is mostly non-symmetrical.With the above motivation, an attempt has been made to study theeffects of slip (at the asymmetric stenosis) and the influence of flow variables (wallshear stress, velocity, flow rate, pressure gradient, apparent viscosity) for 2-layeredmodel of Casson fluid flow through asymmetric constricted vessel with velocity slipat interface. The motion of flow is assumed to be steady and laminar.

II. MATHEMATICAL FORMULATION

We consider the steady, laminar, and fully developed flow of blood through a restricted artery with an axially non-symmetrical but radially symmetrical stenosis with constriction (assumed to be incompressible in the axial (Z) direction).The arterial constriction occurs in the lumen of the artery and is characterized by modest and progressive changes.The form of the stenosis is considered asymmetric in this investigation. The artery length is believed to be long enough in comparison to its radius to allow for the absence of entrance, exit, and special wall effects. The model is made up of a red blood cell suspension core in the middle.

In the outer layer, there are two plasma layers: a central plasma layer and a peripheral plasma layer (as shown in Fig.6.1). The rheology of blood in the core region has been classified as a non-Newtonian fluid obeying the Casson fluid model and a Newtonian fluid with differing viscosities K_c and μ_1 respectively. In Fig. (1), R(z) is the radius of the tube with stenosis, Ro is the constant radius of the tube, $R_1(z)$ is the radius of the artery in the core region such that $\alpha = \frac{R_1(z)}{R_0}$, L0 is the length of the stenosis, L is the length of the tube, d is the stenosis location, δ_s and δ_i are the maximum height of the stenosis in the PPL and the core region respectively at $z = d + \frac{L_0}{2}$ such that the ratio of the stenotic height to the radius of the artery is very much less than unity i.e. $\frac{\delta}{R_0} \ll 1$

The equation of motion for a one-dimensional flow can be expressed as

$$c+\frac{\mu}{r}\frac{d}{dr}\left[r\frac{d}{dr}u(r)\right]=0 \quad (2.1)$$

$$\sqrt{\tau}=\sqrt{\tau_0}+\sqrt{\eta\dot{\gamma}}$$

which implies that $\dot{\gamma}=\frac{1}{k_c}\left(\sqrt{\tau}-\sqrt{\tau_0}\right)^2, \tau \geq \tau_0$ (2.2)

$$= 0,\ \tau \leq \tau_0 \quad (2.3)$$

Where τ is the shear stress, τ_0 is the yield stress, k_c is the casson's viscosity and $\dot{\gamma}$ is the rate of shear strain. To find strain rate ($\dot{\gamma}$), we integrate equation (2.1) twice and use the boundary condition of normal no-slip at wall (Schlichting, 1968) at r=R(z), tube radius, u=0 and symmetry condition: at r=0 (tube axix), u is finite, there yields the solution

$$u(r)=\frac{cR^2(z)}{4\mu}\left\{1-\left(\frac{r}{R(z)}\right)^2\right\}, \quad 0 \leq r \leq R(z)$$
(2.4)

Shear stressing can be applied at any distance r from the tube axis (Schlichting, 1968)

$$\tau_{rz}=\mu\frac{du}{dr}$$

$$=\mu\frac{cR^2(z)}{4\mu}\left\{0-\frac{2r}{R^2(z)}\right\}$$

$$=-\frac{cr}{2}$$

$$=\frac{r}{2}\frac{dp}{dz} \quad (2.5)$$

Wall shear stress $\tau_{w=\tau_{rz}}|r = R(z)$

$$=\frac{R(z)}{2}\frac{dp}{dz} \quad (2.6)$$

At the stenotic wall, shear stress is τ_w and at a certain distance from the tube axis it is τ_0, the yield stress (Fung, 1981) whose correspondence on the horizontal axis is r_c, a critical radius (fig. 1).

Taking τ_{rz}=τ_0, the expression for yield stress leads to the form:

$$\tau_0=\frac{r_c}{2}\frac{dp}{dz}=-c\frac{r_c}{2} \quad (2.7)$$

There are two cases for the two stress τ_0 and τ_w:

- Blood will not flow if the shear stress τ_{rz} is smaller than the yield stress i.e. r<r_c and hence u(r)=0, when $\frac{dp}{dz}<\frac{2}{R(z)}\tau_0$
- If $\tau > \tau_0$ i.e then blood will flow and for that u=u(r) when $\frac{dp}{dz}>\frac{2}{R(z)}\tau_0$. Thus the casson's equation (2.2-2.3) can be represented as

$$\dot{\gamma}=\frac{1}{k_c}\left(\sqrt{\tau_{rz}}-\sqrt{\tau_0}\right)^2, \tau_{rz}\leq\tau_0 \quad (2.8)$$

$$=0, \quad \tau_{rz}\leq\tau_0 \quad (2.9)$$

Where the vanishing of the strain rate i.e. $\dot{\gamma}=0$

$$\Rightarrow \frac{d}{dr}u(r)=0$$

$\Rightarrow u(r)=constant=u_c$ Where $\tau_{rz}=\tau_0$ where u_c is the core velocity at r=r_c (core radius). Thus for blood flow when $r_c < R_1(z)$, there will be two zones for the blood flow viz. $0\leq r\leq r_c$ and $r_c\leq r\leq R_1(z)$

For the zone $0\leq r\leq r_c$,

We have $\frac{du(r)}{dr}=0$

$\Rightarrow u(r)=u_c$, for $0\leq r\leq r_c$ this indicate that velocity profile will be flat and for that region $r_c\leq r\leq R_1(z)$, u(r) will show deviations from flat profile, necessitating the use of Casson's equation (2.8) , (Young,1984). The equation (2.8) is translated from the previous considerations and equations (2.5), (2.7) and (2.8), the governing equations of motion for one dimensional steady, laminar flow in an asymmetric stenosed artery of a Casson fluid (Biswas and Nath, 2003)

$$\frac{d}{dr}u_1(r)=\frac{1}{k_c}\left(\sqrt{\frac{-cr}{2}}-\sqrt{\frac{-cr_c}{2}}\right)^2, r_c\leq r\leq R_1(z),$$
(2.10)

$$=0,\ 0\leq r\leq r_c$$

And for the Peripheral region

$$c+\frac{1}{r}\frac{d}{dr}\left(r\mu_1\frac{du_2}{dr}\right)=0,\ R_1(z)\leq r\leq R(z)$$
(2.11)

Where u_1 and u_2 are the velocities in the core and PPL, $c=-\frac{dp}{dz}$ is the pressure gradient, r is the radial co-ordinate, r_c is a critical radius corresponding to τ_y, the yield stress.

III. BOUNDARY CONDITIONS

Forsolving the above equations (2.10—2.11) we use the following boundary conditions:

- u_2 =0 at r=R(z), (no slip at the stenotic wall) (3.1)
- $u_1 - u_2 = u_s$ at r=R(z), (slip at interface) (3.2)
- $\frac{\partial u_2}{\partial r}=0$, at r=0 (3.3)
- $u_1(r)$=u_c at r=r_c (3.4)

IV. SOLUTION OF THE PROBLEM

The solution of equations (2.10-2.11) with the above boundary conditions are obtained as

$u_2 = \frac{c}{4\mu_1}(R^2(z) - r^2)$, $R_1(z) \leq r \leq R(z)$ (4.1)

$u_1 = u_s + \frac{c}{4k_c}\left[[R_1^2(z) - r^2] + 2r_c(R_1(z) - r) - \frac{8}{3}\sqrt{r_c}\left(\sqrt{R_1^3(z)} - \sqrt{r^3}\right)\right] + \frac{c}{4\mu_1}(R^2(z) - R_1^2(z))$, $r_c \leq r \leq R_1(z)$ (4.2)

Also $u_1 = u_s$ for $0 \leq r \leq r_c$ where u_c is the core velocity, which can be defined as

$u_c = u_s + \frac{c}{4k_c}\left[\left[\sqrt{R(z)} - \sqrt{r_c}\right]^3\left(\sqrt{R_1}(z) + \frac{\sqrt{r_c}}{3}\right)\right] + \frac{c}{4\mu_1}(R^2(z) - R_1^2(z))$ (4.3)

The volumetric flow rate Q is calculated as follows

$Q = 2\pi\int_0^{R(z)} ru(r)dr$ is obtained as using equations (4.1-4.2)

$= \mu R_1^2(z)u_s + \frac{\mu c R^4(z)}{8\mu_1}\left\{1 - \left(\frac{R_1(z)}{R(z)}\right)^4\right\} - \frac{\pi R_1^4(z)}{8kc}\left(\frac{dp}{dz}\right)\varphi(\beta)$ (4.4)

$$Where\,\psi(\beta) = \left(1 - \frac{16\sqrt{\beta}}{7} + \frac{4\beta}{3} - \frac{1}{21}\beta^4\right), l = \frac{r_c}{R_1(z)}, \beta = l^3$$

From equation (4.4), the pressure gradient term can be express as

$\frac{dp}{dz} = \frac{8\mu_1}{[1-\mu_1'\psi(\beta)R_1^4(z) - R^4(z)]}\left(\frac{Q}{\pi} - u_s R_1^2(z)\right)$ (4.5) where $\mu_1' = \frac{\mu_1}{k_c}$

Integrating between the limits $P=P_i$ at z=0 and $P=P_0$ at z=L, where L is the length of the tube

We get **the pressure drop** as:

$p_i - p_0 = 8\mu_1\int_{z=0}^{L}[R^4(z) - (1 - \mu_1'\psi(\beta))R_1^4(z)]^{-1}\left(\frac{Q}{\pi} - u_s R_1^2(z)\right)dz$ (4.6)

The resistance to flow (λ) is determined by

$\lambda = \frac{p_i - p_0}{Q} = 8\mu_2\left[R_0^4\left\{1 - \left(-\mu_1'\Psi\left(\sqrt{\frac{r_c}{\alpha R_0}}\right)\right)\alpha^4\right\}\right]^{-1}\left[\frac{1}{\pi} - Q_1^{-1}(\alpha R_0)^2 u_s\right]\left(\frac{L-L_0}{L}\right) + \int_{z=d}^{d+L_0}\left[R^4(z)\left(1 - \mu_1'\Psi(\beta)\right)R_1^4(z)\right]^{-1}\left(\frac{1}{\pi} - Q^{-1}u_s R_1^2(z)\right)dz$ (4.7)

Where $Q_1 = Q$, at $R(z) = R_0$

The average pressure gradient in the axial direction can be defined as

$\left(\frac{dp}{dz}\right)_{av} = \frac{\int_{r=0}^{R(z)} r\left(\frac{dp}{dz}\right)dr}{\int_{r=0}^{R(z)} rdr}$

$= \left(\frac{dp}{dz}\right)$ (4.8)

The apparent viscosity can be expressed from the formula $\mu_a = \frac{\pi c R^4(z)}{8Q}$ as

$\mu_a = \left[\frac{8u_s}{cR^2(z)}\left(\frac{R_1(z)}{R(z)}\right)^2 + \mu_2^{-1}\left\{1 - (1 - \mu_1'\Psi(\beta))\left(\frac{R_1(z)}{R(z)}\right)^4\right\}\right]^{-1}$ (4.9)

Stress component at stenotic wall (τ_w), interface $(\tau_{R_1(z)})$ and yield stress (τ_y) can be obtained from the formula $\mu_r = \left(\frac{-cr}{2}\right) = \frac{r}{2}\frac{dp}{dz}$ in the following forms

$\tau_w = -\mu_1\frac{\partial u_2}{\partial r}$ at r=R(z)

$= -\frac{c}{2}R(z)$

$= \frac{R\{z\}}{2}\left(\frac{dp}{dz}\right)$ (4.10)

$\tau_{R(z)} = \left(\frac{-c}{2}\right)R_1(Z) = \frac{R_1(z)}{2}\left(\frac{dp}{dz}\right)$ (4.11)

$\tau_y = \left(-\frac{c}{2}\right)r_c = \frac{r_c}{2}\left(\frac{dp}{dz}\right)$ (4.12)

The following non dimensional variables can be used to express the flow variables in their non-dimensional form

$\bar{z} = \frac{z}{R_0}$, $\bar{d} = \frac{d}{R_0}$, $\bar{R} = \frac{R}{R_0}$, $\overline{R_1} = \frac{R_1}{R_0}$, $\bar{\delta} = \frac{\delta}{R_0}$, $\overline{\left(\frac{dp}{dz}\right)} = \frac{\frac{dp}{dz}}{\left(\frac{dp}{dz}\right)_0}$, $\bar{A} = \frac{A}{R_0^{n-1}}$, $(\bar{L}, \overline{L_0}) = \frac{(L, L_0)}{R_0}$

$\bar{u} = \frac{u}{u_0}$, where $\bar{\lambda} = \frac{\lambda}{\lambda_0}$, $\bar{Q} = \frac{Q}{Q_0}$, $u_0 = \frac{cR_0^2}{4\mu_1}$, $\tau_{R_0} = -\left(\frac{dp}{dz}\right)\frac{R_0}{2}$, $Q_0 = \frac{\pi c R_0^4}{8\mu_2}$,

$\left(\frac{dp}{dz}\right)_0 = -\frac{8\mu_1 Q_0}{\pi R_0^4}$, $\lambda_0 = \frac{8\mu_1}{\pi R_0^4}L$

Velocity functions:

$\overline{u_2} = \bar{R}^2\left[1 - \left(\frac{r}{R(z)}\right)^2\right]$, $\frac{R_1(z)}{R(z)} \leq \frac{r}{R(z)} \leq 1$, (4.13)

$$\overline{u_1} = \overline{u_s} + \mu_1'[\overline{R_1}^2 - \overline{R}^2\left(\frac{r}{R(z)}\right)^2 + 2\overline{r_c}\left(\overline{R_1} - \overline{R}\left(\frac{r}{R(z)}\right)\right) -$$

$, \frac{r_c}{R} \leq \frac{r}{R} \leq \frac{R_1}{R}$ (4.14)

$$\overline{u_c} = \overline{u_s} + \mu_1'\left(\sqrt{\overline{R_1}} - \sqrt{\overline{r_c}}\right)^3\left(\sqrt{\overline{R_1}} + \frac{1}{3}\sqrt{\overline{r_c}}\right) + \left(\overline{R^2} - \overline{R_1^2}\right)$$

$$, 0 \le \frac{r}{R} \le \frac{r_c}{R} \qquad (4.15)$$

Flow Rate: $\overline{Q} = 2\overline{u_s}\,\overline{R_1^2} + \left[\overline{R^4} - \overline{R_1^4}\left(1 - \mu_1'\psi(\overline{t^2})\right)\right]$

(4.16)

Where $\psi(\overline{t^2}) = \psi\frac{\overline{r_c}}{R_1}, \mu_1' = \frac{\mu_1}{k_c}$

Pressure gradient :

$$\overline{\left(\frac{dp}{dz}\right)} = \left[\overline{R^4} - \left\{1 - \mu'\psi(\beta)\right\}\overline{R_1^4}\right]^{-1}\left(\overline{Q} - 2\overline{u_s}\,\overline{R_1^2}\right) = \overline{\left(\frac{dp}{dz}\right)}$$

(4.17)

Apparent viscosity:

$$\overline{\mu_a} = \left[1 + 2\frac{\overline{u_s}}{\overline{R^2}}\left(\frac{\overline{R_1}}{\overline{R}}\right)^2 - \left\{1 - \mu_1'\psi\left(\overline{\beta}\right)\right\}\left(\frac{\overline{R_1}}{\overline{R}}\right)\right]^{-1}$$

(4.18)

Flow geometry: For PPL

$$\overline{R}\left(\overline{z}\right) = 1 - \overline{A}\left[\overline{L_0^{n-1}}\left(z - d\right) - \left(\overline{z} - \overline{d}\right)^n\right], \overline{d} \le z \le \overline{d} + \overline{L_0}$$

=1, otherwise

For the core region:

$$\overline{R_1}\left(\overline{z}\right) = \alpha - \overline{A}\left[\overline{L_0^{n-1}}\left(z - d\right) - \left(\overline{z} - \overline{d}\right)^n\right], \overline{d} \le z \le \overline{d} + \overline{L_0}$$

$= \alpha$, otherwise

V. RESULTS AND DISCUSSIONS

A. Variation of velocity:

In the present analysis three successive growths at luman of an artery, slip and no-slipcases at interface for investigating two-layered in uniform stenosed tube are employedin Fig1.Blood is supposed to act as a non-Newtonian fluid in the model, with a yield feature known as Casson fluid. The shear stress versus vs rate of strain connection in such a visco-inelastic fluid is non-linear, and because the Casson fluid has a finite yield stress, there may be two flow scenarios. Specifically, (a) if the shear stress τ_{rz} at a radial distance r is not greater than the yield stress τ_y ($\tau_{rz} \le \tau_y$), blood will not flow, and (b) if the shear stress is less than the yield stress τ_y ($\tau_{rz} \ge \tau_y$), blood will flow. Variations in axial velocity of both regions, rate of flow, pressure gradient, shear stress, and apparent viscosity at the interface of fluids are calculated using analytical expressions.

There are three zones in this two-layered blood flow: $0 \le r \le r_c$(criticalradius), $r_c \le r \le R_1$ and $R_1 \le r \le R$, wherer $_c$ is a critical radius, whenr=r_c, thenu(r)= u_c (a constant velocity) andτ_{rz} = τ_y. (b) when R(z) = R_0 = R_1(z), τ_y = 0 (or r_c = 0) itleads to Poiseuille flow of blood (behaving as a Newtonian fluid) with slip or zero-slipat thevessel wall.

The variations of velocity are shown in Fig. (2-5). As expected, velocity attains aconstant magnitude in the yield stress zone and thereafter shows parabolic trends incoreregion and PPL. As slip velocity increases, velocity increases in coreand PPL regions. Velocity attains the highest magnitude in all three layers for mild formation of stenos is and the lowest one in severe case of constriction. Although velocity shows an on-parabolic trend in coreregion but in PPL region, itexhibitsaparabolic profile. It increases with shape parameter n = 2 (symmetric case) to n> 2(asymmetricstenosis).

B. Variationofflowrate:

For all values of the shape parameter n, the highest magnitude is obtained at the two ends of the stenosis, however the lowest magnitude is at the throat for n=2 (symmetric form) and towards the termination position for n > 2. (Asymmetric case).Although the stenotic throat has the smallest magnitude, it is the maximum for moderate stenosis and the lowest for severe stenosis.However, flow rate is larger with slip velocity than with zero-slip velocity in all situations of stenosis.

C. Variation of pressure gradient:

The variation of pressure gradient, shown in Fig. 12), reveals that pressure gradient decreases asshape parameter increases(fromn=2ton>2). It is lowest with slip and increasing slip velocity for symmetric and asymmetric forms of stenosis.

D. Apparent viscosity:

The variation of apparent viscosity exhibited in fig.(8-9), shows that as yield stress increases, it increases in all three stenosis formations. It decreases with velocity slip. The apparent viscosity attains the minimum value for mild stenosis case and the maximum magnitude for severe growth. It is found that $\overline{\mu_a}(mildcase) \le \overline{\mu_a}$(moderatecase)$\le \overline{\mu_a}$(severe growth)

E. Wall shear stress:

As illustrated in Figures (10) and (11), shear stress on the wall decreases as slip velocity increases. In severe stenosis, it reaches its highest magnitude, while in moderate stenosis, it reaches its lowest magnitude. When Q rises, so do all three types of growth.

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VI. CONCLUSIONS

In Fig. 1, the constant, laminar, and one-dimensional (1-D) flow in the two-layered region of a homogeneous asymmetric stenosed artery is discussed.

An analysis has been developed here to explore the influence of velocity slip at the fluid interface in the situation of mild, moderate, and severe arterial stenosis using a slip condition at the fluid interface. The fluctuations of axial velocity in both sections, rate of flow, pressure gradient, shear stress, and apparent viscosity at the interface of fluids in the constricted region, among other factors, have been graphically displayed (Figs. 2-12)

The flow variables are given a second form, and their fluctuations are graphically depicted.

It is discovered that velocity is a function of R, z, k_c, L_o, and other variables.

The following observation can be made based on the current study:

- In a two-layered Casson fluid flow, there are two regions for flow (for flow situation), namely $\tau_{rz} \leq \tau_y$ and $\tau_{rz} \geq \tau_y$. In the former instance, no flow occurs, however in the latter case, blood flow is feasible.
- There are three locations for two-layered blood flow within a uniformly stenosed artery region: $0 \leq r \leq r_c$, $r_c \leq r \leq R_1(z)$ and $R_1(z) \leq r \leq R(z)$.
- It comprises (i)Poiseuille flow of blood (acting as a non-Newtonian fluid) with wall-slip or at the vessel border (ii) one-dimensional stenosed flow of bothNewtonianfluidandCassonfluidwithsliporzero-slipatuniform arterialwall.
- The behavior of Velocity is presented in the Fig (2-5). it attains a fixedmagnitude in yield stress zone and alters parabolically in core and PPL regions.It increases with shape parameter n = 2 (symmetric case) to n > 2 (asymmetricstenosis). It attains the highest magnitude for mild growth and lowest value forseverestenos is. However, in all forms of stenos is development, velocity increases with a velocity slip as well as with an increasing magnitude of slip.
- (e)The flow rate variations are presented in Figs.(6-7), demonstrating that the magnitude achieved with n = 2 is lower than with n > 2.

As a result of the foregoing study, it can be concluded for flow variables that using axial velocity slip at the fluids interface in a confined uniform channel can lessen damages to a diseased or blocked artery. Slip has been used in this analysis to speed up the flow in the congested two-layered region on the one hand, and to minimize the resistance to this uni-directional flow on the other. However, some artificial measurements of slip were utilized in the analysis, such as us=0,.5,.1, n=2,6, 9; K_c=2cp, and stenotic wall at stiff, non-porous, and visco-elastic as well as flow is constant and uni-directional.This theoretical model might be enhanced if the degree of a velocity slip, blood yield stress, PPL thickness, two-dimensional flow, and other factors were taken into account. The current model could be utilized as a starting point for developing a better model of blood flow through a two-layered uniformly stenosed artery. Apart from having a finite yield stress (τ_y), blood has been shown to have Newtonian behavior in blood flow simulations.

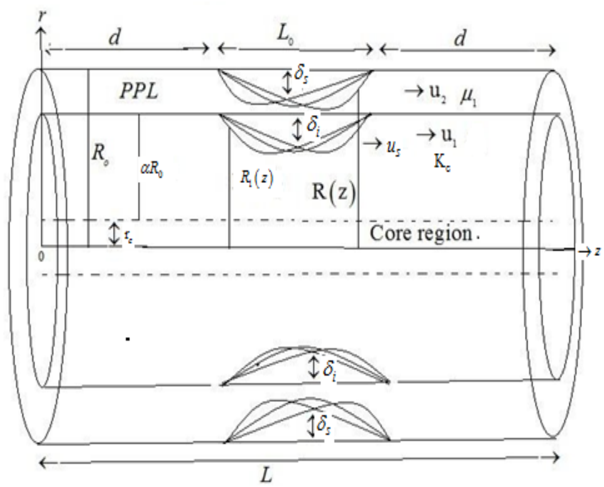

Fig.1 Schematic diagram of two-Layered Casson Model of blood Flow inside Asymmetric stenosed Artery

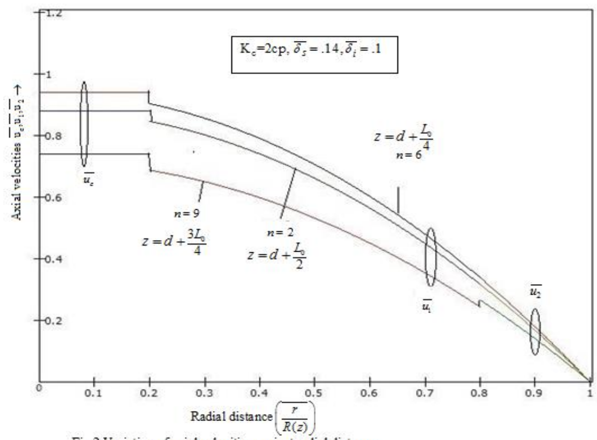

Fig.2 Variation of axial velocities against radial distance for different values of n and $\overline{u}_s = 0.00$, $K_c = 2cp$

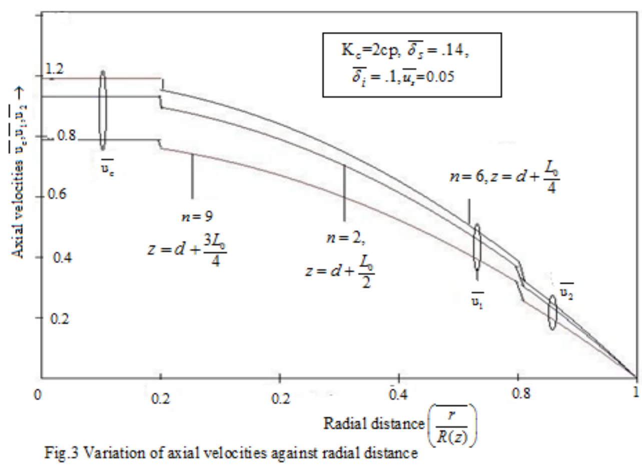

Fig.3 Variation of axial velocities against radial distance for different values of n and $\overline{u}_s$=0.05, K_c=2cp

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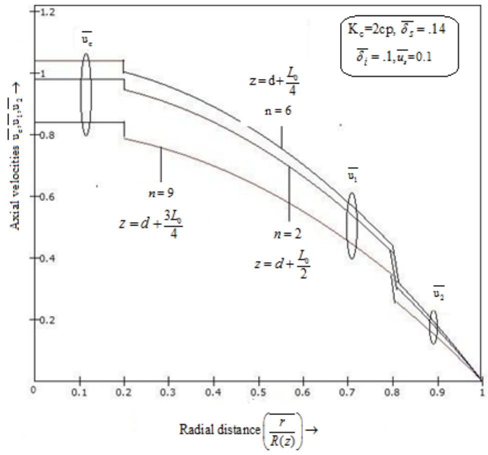

Fig.4 Variation of axial velocities against radial distance for different values of n and $\overline{u_s}$=0.1, K_c=2cp

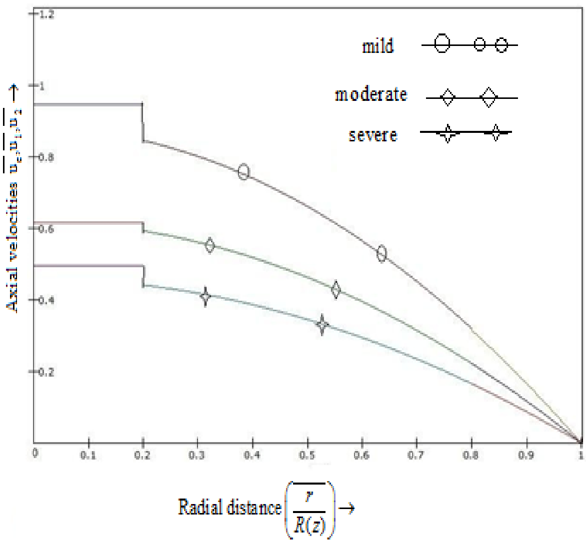

Fig.5 Variation of axial velocities against radial distance for n=6, $\overline{u_s}$=0.0, K_c=2cp

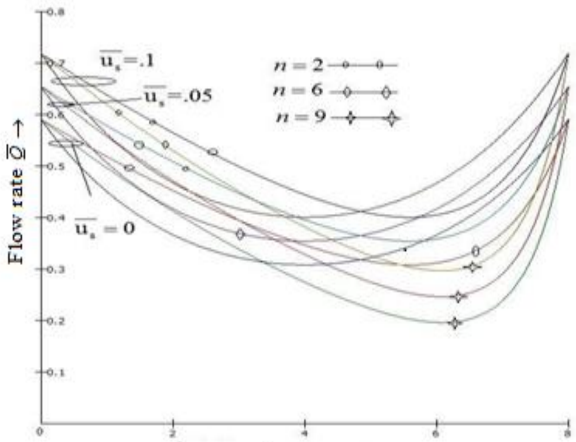

Fig.6 Variation of flow rate against axial distance for n=2,6,9, $\overline{u_s}$=0,.05,0.1,K_c=2cp

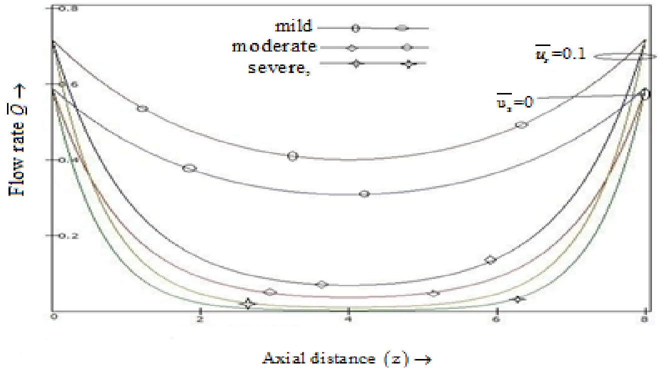

Fig.7 Variation of flow rate against axial distance for n=2, $\overline{u_s}$=0.0, 0.1, kR_0 = 0.1

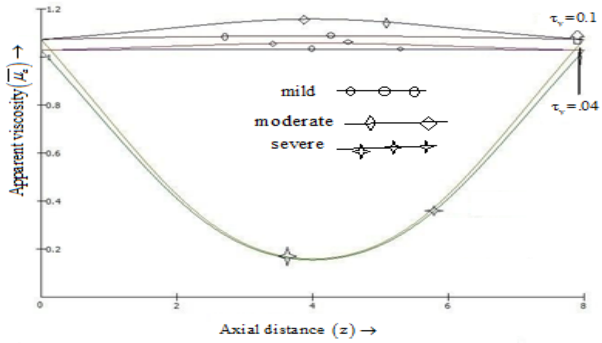

Fig.8 Variation of apparent viscosity against axial distance for n=2, τ_y=0.1,.04, K_C=2cp

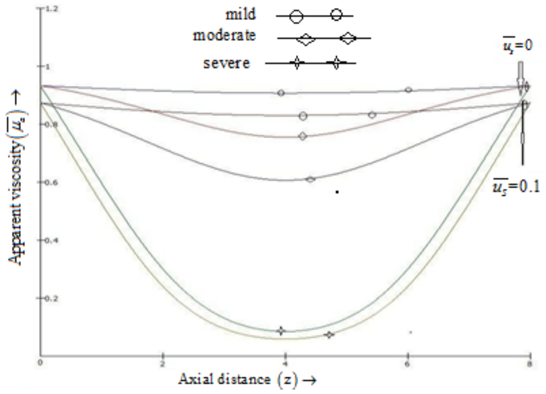

Fig.9 Variation of apparent viscosity against axial distance for n=2, τ_y=0, K_C=2cp and u_s = 0.1

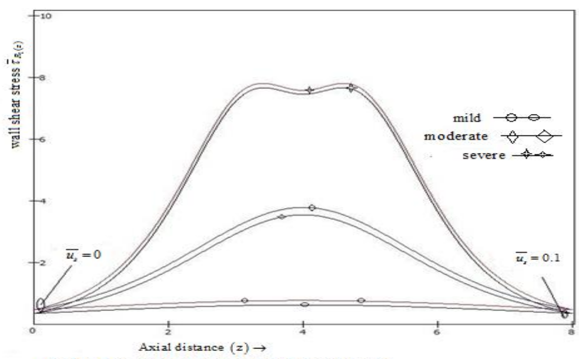

Fig.10 Variation of wall shear stress against axial distance for n=2, u_s = 0,0.1

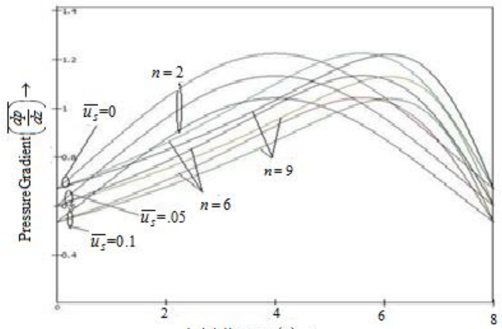

Fig.11 Variation of pressure gradient against axial distance for n=2,6,9, $\bar{u}_s$=0,.05,0.1,K_c=2cp

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