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Statistical Physical Significance for the Constant Pi other than Circular Geometry

Dr. Ismail Abbas
Senior Lecturer at MTC, Cairo University

Abstract:- Since the time of Archimedes 200 BC and up to the present day, it is still generally accepted that the Greek letter π (Pi) is a mathematical constant approximately equal to 3.142 and is defined as the ratio of the circumference of a circle to its diameter.

But it is quite mysterious how it appears in many formulas in most fields of mathematics and physics, even when the geometry of the considered space does not have circular symmetry.

In fact, the statistical significance of 2 Pi is that, the energy dissipation time rate (dU / dt) of a system multiplied by its own parameters (absorbent area / volume in unit values) is equal to 2 Pi times its total energy U for any geometry.

This article, where we attach statistical physical significance to the constant Pi other than circular geometry, is just a small step on a long road and it is recommended to extend it to different 3D spatial geometries.

I. INTRODUCTION

The Greek letter π (Pi) is a mathematical constant approximately equal to 3.142 and is defined in Euclidean geometry as the ratio of the circumference of a circle to its diameter.

From the time of Archimedes 200 BC and up to the present day, it is still generally accepted. But it is quite mysterious how it appears in many formulas in most fields of mathematics and physics, even when the geometry of the considered space does not have circular symmetry.

Different hypotheses lead to different theories. There are many approaches to classical statistical mechanics depending on hypotheses and presupposed spaces such as that of the continuous phase space N ^ 6 of the BBGKY hierarchy theory [1], the famous statistical mechanics of Maxwell and Boltzmann and finally the statistic of the matrix chain B proposed for the discrete numerical solution of the boundary value problems.

We assume that current statistical quantum mechanics CSM and classical Maxwell-Boltzmann statistics as well as phase space statistics BBGKY are unable to define the constant Pi because they separate the spatial and temporal independent variables x, t while the statistical significance of Pi itself is linked to the entanglement of x, t in an inseparable 4D block.

In the classic treatment for discrete numerical solution, the independent variables space and time are separated as for example the use of the classical numerical methods with finite differences such as FDM and FEM [2,3] for the Laplacian PDE.

The separation tells nothing but the entanglement of space and time in the 4D block reveals an inherent natural behavior.

Note that 4D in the sense of CSM is different from that of a light cone of special and general relativity, but it is a characteristic inherent in the new statistic of the chains of matrix B proposed for the solution of the problem of the limit values.

In short, If we assume that the rise / fall time of the dynamical system follows a time jump trajectory of exponential stability, i.e. U (t) = U (0). Exp (-Alpha .N d t) and that the average absorption coefficient of the internal walls of the bounded system is Sav.

Sav=(A1.S1+A2.S2+. . . . . +An. Sn)/ (A1+A2+ . . . . +An)

Where A is the surface area.

Sav is element of the closed interval [0,1].

A striking result emerges,

Alpha. S . (n + 1) ^ 2 = 6.28 = 2 Pi .... (1)

[Where n represents the number of equidistant free nodes in the 3D geometrical space].

Equation 1 is valid for all cubes of different n, S.

Note that Equation 1 is equivalent to Sabines' semi-empirical formula for reverberation time in audio rooms with good precision.

The statistical significance of the equation 1 is that, the energy dissipation time rate (dU / dt) of a diffusion system multiplied by its own parameters(area over volume in unit values)is equal to 2 Pi times its total energy U for any geometry.

Finally, we recall the quote from the QM scientist N. Bohr with a slight modification to the CSM:

Anyone who thinks they can easily explain quantum mechanics and / or classical statistical mechanics has not yet sufficiently understood the subject.

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II. THEORY

The theory of the new branch of statistical mechanics proposed for the boundary value diffusion problem has been explained repeatedly in references 5,6,7 and is based on the recurrence formula in x-t space.

$U^{(N+1)}_{i,j,k} = B\,(U^{N}_{i,j,k} + b + S)$. . . (2)

B is the stochastic transition matrix of the proposed statistical model.

b is the vector of boundary condition value arranged in the proper order.

S is the source / sink term vector placed at the free nodes concerned and expressed in adequate units, ie Joule m ^ -3 sec ^ -1.

Cartographic coordinates, x = i dx, y = j dy, z = kdz.

Obviously the time t is given by N dt where dt is the time step or the time jump, in other words Ndt = t is the time necessary for the evolution.

Eq. 2 is a rigorous physical assumption meant to describe the statistical nature of the diffusion process and means that the boundaries of the physical system act as a source / sink term from the first instant t = 0.

It follows that for the Dirichlet boundary conditions independent of time, zero source term.

$U^{(N+1)} = B\,(U^{N} + b + S). + B^{N}.U(0)$. . . (3)

U (0) are the initial conditions at t = 0.

In the case of absorbent or dissipative boundary conditions such as walls in audio rooms [6],

We apply the same equation (2,3) but with a major difference:

The boundary conditions are not frozen in time as in the case of Dirichlet conditions but they are oscillating absorbent walls varying in time.

We can show that the equation. 3 changes to [4],

U (i, IT) = B (I, k, IT) * U0 (k) + B (i, j, IT) * BC (j, 1). . . . . . . (4)

Note that Eq.4 corresponds to the superposition of the two terms proposed by Chiara [4]

The integer IT in equation 4 corresponds to the number of iterations or time steps N.

Here, the transition matrix B itself is multiplied by the absorption coefficient S at the frontier walls. Likewise, the vector BC (b) in Eq, 5 is multiplied by (S) ^ N, where N is the number iterations in order to take into account the time dependence of boundary conditions.

In order not to worry too much about the details of the theory, let's go right into 3D illustrative applications and numerical results.

III. APPLICATIONS AND NUMERICAL RESULTS

Our proposal to find statistical physical significance for the constant Pi other than circular geometry is to apply Equation 4 for different absorption coefficients A and different cubic space geometries for the time dependent BCs and to inspect how the mathematical constant Pi appears spontaneously as explained in EQ 1.

The simplest 3D configuration is a Cube of 8 equidistant free nodes shown in Fig. 1

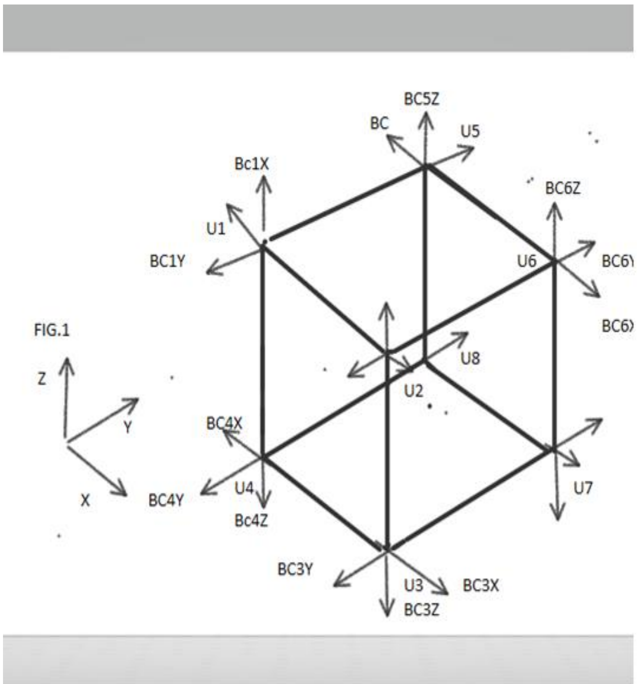

Fig 1: 3D case of a cube of 8 equidistant free nodes and 8 time dependent boundary conditions

The sound source is suddenly cut off, i.e. S = 0 at t = 0

For simplicity, we have assumed a uniform spatial distribution of the initial conditions U (r, 0) = constant and use equation 4 to calculate the time evolution of U (t) = U (0) .Exp (-ALPHA.N dt).

The results obtained are presented in Table I

Table I. Numerical values for Alpha vs absorption coefficient A For Fig. 1.

Absorption coefficient S 1.0 0.9 0.8 0.7 0.6 0.5 0.4
Alpha. . . . . , 0.69 0.8 0.92 1.05 1.2 1.39 -----
Alpha * A * $(n + 1)^2$ For figure I.
= Alpha * $(2 + 1)^2$ 6.21 6.48 6.624 6.39 6.48 6.255

Go further for the slightly more complex case of the 3D cube of 27 equidistant free nodes and 27 time dependent boundary conditions illustrated in figure 2.

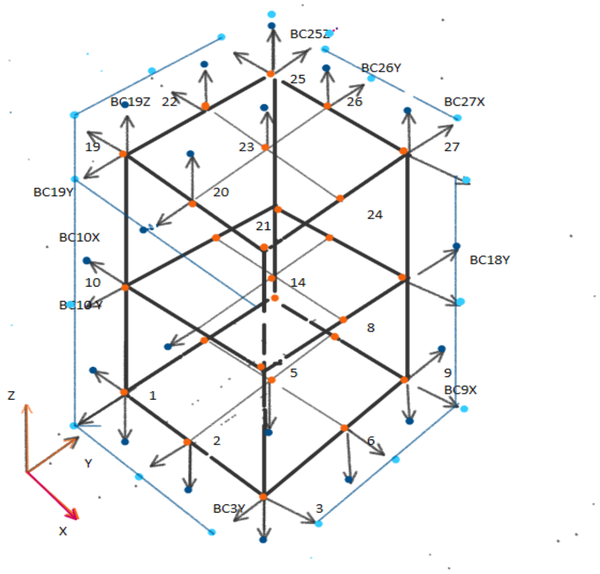

Fig 2: 3D case of a cube of 27 equidistant free nodes and 27 time dependent boundary conditions.

Similar to case of Fig.1 , the sound source is suddenly cut off, i.e. S = 0 at t = 0

For simplicity, we have assumed a uniform spatial distribution of the initial conditions U (r, 0) = constant and use equation 4 to calculate the time evolution of

U (t) = U (0) .Exp (-ALPHA.N dt).

The results obtained are presented in Table II

Table II. numerical values for Alpha vs absorption coefficient A

Absorption coefficient S 1.0 0.9 0.8 0.7 0.6 0.5 0.4
Alpha.. . . . . . 0.38 0.46 0.58 0.68 ---- ------
Alpha * A * (n + 1) ^ ^{2}. . 6.08 6.62 7.42 7.6--- For figure II = Alpha * A * (3 + 1) ^ ^{2}

An in-depth study of Tables I and II shows that,

Alpha. S . (n + 1) ^ 2 = 6.28 = 2 Pi . . . . . (5)

Which is the statistical significance given by equation 1.

IV. SABINE FORMULA FOR REVERBERATION TIME

Sabine's semi-empirical formula 100 years ago is the main formula for calculating reverberation time in audio rooms and known to be fairly accurate,

...
RT60 = 53.46 V / C A S ,. . second. . . . . (6)

V is the volume of the acoustic room, S the area of its interior walls, C is the speed of sound and A is the average sound absorption coefficient.

For C = 340 m / s ,it is reduced to

RT60 = 0.162 V / A S ,. . dry. . . . . . . (7)

In fact, Sabine's formula is at the heart of classical statistical mechanics and can be reduced to,

RT60 = 2 Pi / (n + 1) ^ 2 * A in accordance with Eq. 1

For example a cubic piece of side length = 11 m and Absorption A = 0.8 leads to n = 10 and the reverberation time RT60 = 6.3 / ((10) ^ 2 *). 8)

Sabine Eq. 7 and Eq, 1 give an RT60 almost equal to 1.48 s.

V. CONCLUSION

The mathematical constant 2 Pi has its place in classical statistical mechanics.

In this paper, we attach and test a statistical significance of 2 Pi is that, the energy dissipation time rate dU / dt of a system multiplied by its own parameters (area over volume in unit values) is equal to 2 Pi times its total energy U for any geometry.

This study is only a small step in a long way and should be followed by further extents at different 3D spatial geometries.

N.B. All calculations in this article were produced with the author's own double precision algorithm to ensure maximum possible precision, followed by ref. 8 for example

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