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Unconstrained Global Optimization Method Based on a Novel Filled Function Approach

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Abstract:- In this paper, we propose a new single-factor filled function only where this new filled function is not only easily decomposable but also uniformly approximated by a continuously differentiable function. Therefore, a thumbnail of the proposed filled function can be obtained simply by using the local optimization algorithm. Then, the obtained minimizer is taken as a prime number to reduce the objective function, and by repeating this process, a better miniature is found. And we will finally have a global minimizer. Through the numerical results of the proposed new filled function, it becomes clear to us the effectiveness of this method.

Keywords:- *Global Optimization; Filled Function Method; Global Minimize; Local Minimizer; Strict Local Maximizer; Smoothing Technique.*

I. INTRODUCTION

Most practical problems in applied sciences of various fields are presented as global improvement problems. Many researchers competed in the field of global improvement, where many recent contributions were made to solving global improvement problems, either theoretically or by using computers. The field of global optimization is fundamentally concerned with the properties and calculations of multimodal capacities. That is, existing techniques can be ordered into two strategies: deterministic strategies (see, e.g.,[1]) and probabilistic techniques [2], where the first method is a model for filled function methods (FFM) [3], the path method [4], the tunnel method [5] and the covering method [5], while the second method is a model for the assembly method [5], the methods mentioned in [6], the simulated annealing method [7], and genetic algorithms [7]. Moreover, some hybrid deterministic and probabilistic algorithms (see, e.g., [8]) have been proposed to solve practical problems. The great challenge of the global optimization problem is that there are multiple local minimums for the general non-normative objective function. Thus, there are two main problems for global improvement. Finding a way to find out a minimum thumbnail of the target function from a known local thumbnail. The other is during out how to converge and, accordingly, designing stopping criteria. FFM was first suggested by Ge in [9], which was used to solve the global miniaturization of the unconstrained multiextremum function. Subsequently, many researchers have made valuable efforts to improve this method (see, e.g., [10, 3, 11]). In any case, conventional filled functions are frequently undefined (see, e.g., [12]), need more than one modifiable parameter (see, e.g., [3]), or contain unseemly terms Conditional (see, e.g., [3]). To beat these weaknesses, some parameter functions (see, e.g., [13]) and some without parameter functions (see, e.g., [14]) are proposed; nevertheless, they are typically unclear, regularly cousing extra local miniaturization. Some ceaselessly differentiable filled functions with a single parameter (see, e.g., [14]) have been proposed; however, the parameter is not difficult to tune. To manage this issue, another class of functions, filled with a solitary operand, which can be reliably recognized and effectively tuned by parameters, is proposed in this paper. In light of this, another single operand filled function strategy has been proposed; the algorithm is mathematical soundness. Also, the proposed strategy can be utilized to tackle the multi-layered issueIn this article, we investigate the following objective functions:

$$min\{f(x) : x \in R^n\} \tag{1}$$

where $f(x)$ is a twice continuously differentiable function on $R^n$and $\boldsymbol{\Omega} = \prod_{i=1}^{n}[l_i, u_i] \subset R^n$. Generally, we assume that $f(x)$ has only a finite number of minimizers and the set of minimizers is denoted as $L_m = \{x^*{}_i | i = 1,2, \ldots, I\}\ in\ \boldsymbol{\Omega}$ (I is the number of minimizers of $f(x)$).

The following are some essential principles and notations:

x_1^*: A local minimizer of $f(x)$ on $\boldsymbol{\Omega}$ found so far;
$S1$: Set $S_1 = \{x | f(x) \geq f(x_1^*), x \in \boldsymbol{\Omega} \backslash \{x_1^*\}\}$;
$S2$: Set $S_2 = \{x \in int\boldsymbol{\Omega} | f(x) < f(x_1^*)\}$;
m: A constant satisfying $m = min_{i,j\in\{1,2,\ldots,I\}, f(x_i^*)\neq f(x_j^*)} |f(x_i^*) - f(x_j^*)|$;
M: A constant satisfying $M = \max_{x,y\in\boldsymbol{\Omega}} \|x - y\|$.

Assumption. The local minimizers of the function f(x) lie within the interior of $\boldsymbol{\Omega}$.

Definition 1: The basin $B(x_1^*)$ [15] of a function at x_1^* is a connected domain that envelopes the point x_1^*, where the descent sequences of the function start from any point x and converge to x_1^*, while the minimization sequences of the function $f(x)$ start from any point outside of $B(x_1^*)$ do not.
It is obvious that if $x \in B(x_1^*)$, then $f(x) > f(x_1^*)$. If there is another minimizer x_2^* of $f(x)$ and $f(x_2^*) <$ or $\geq f(x_2^*)$, then the basin $B(x_2^*)$ of $f(x)$ at x_2^* is said to be lower (or higher) than $B(x_1^*)$ of $f(x)$ at x_1^*.

Definition 2: A function $G(x)$ is supposed to be a filled function of $f(x)$at the point x_1^*, assuming that it satisfies the accompanying properties:

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- x_1^* is a strict local maximizer of $G(x)$ over $\boldsymbol{\Omega}$;
- $G(x)$ has no stationary point in the set $S_1 - \{x_1^*\}$;
- If the set S_2 is not empty, then there exists a point $x' \in S_2$ such that x' is a local minimizer of $G(x)$.

II. A NEW FILLED FUNCTION AND ITS PROPERTIES

Suppose that we find a local minimizer x_1^* of $f(x)$. Now let us study the following function of problem (1):

$$G(x, x_1^*, a) = (\tan(\min\{f(x) - f(x_1^*), 0\}) - \mathrm{a}) \times \|x - x_1^*\|^2 \quad (2)$$

where a is a parameter. The proof of the next theorems will demonstrate that expression (2) represents a filled function that fulfills definition 2. We will apply to our newly filled function all the properties and theorems contained in [16].

Theorem 1: Suppose x_1^* is a local minimizer of $f(x)$, and $G(x, x_1^*, a)$ is defined by (2), then x_1^* is a strict local maximizer of $G(x, x_1^*, a)$ for all $a > 0$.

Proof. Since x_1^* is a local minimizer of $f(x)$, there exists a neighborhood $N(x_1^*, \epsilon) \subset int\boldsymbol{\Omega}$ of $x_1^*, \epsilon > 0$ such that $f(x) \geq f(x_1^*)$ for all $x \in N(x_1^*, \epsilon)$. For all $x \in N(x_1^*, \epsilon)$, $x \neq x_1^*$ one has

$$G(x, x_1^*, a) = -a \times \|x - x_1^*\|^2 < 0$$
$$= G(x_1^*, x_1^*, a) \quad (3)$$

Thus, x_1^* is a strict local maximizer of $G(x, x_1^*, a)$. □

Theorem 2: Suppose x_1^* is a local minimizer of $f(x)$, x is a point in set S_1, then x is not a stationary point of $G(x, x_1^*, a)$ for all $a > 0$.

Proof. Due to $x \in S_1$, one has $f(x) \geq f(x_1^*)$ and $x \neq x_1^*$, so

$$G(x, x_1^*, a) = -a \times \|x - x_1^*\|^2$$
$$\nabla G(x, x_1^*, a) = -2a \times (x - x_1^*) \neq 0$$

That is x is not a stationary point of $FF(x, x_1^*, a)$. □

Theorem 3: Suppose x_1^* is a local minimizer of $f(x)$ but not a global minimizer of $f(x)$, which means that S_2 is not empty, then there exists a point $x' \in S_2$ such that x' is a local minimizer of $G(x, x_1^*, a)$ when $0 < a < m$.

Proof. Since x_1^* is a local minimizer of $f(x)$, and x_1^* is not a global minimizer of $f(x)$, there exists another local minimizer x_2^* of $f(x)$ such that $f(x_2^*) < f(x_1^*)$.

By the definition of m and continuity of $f(x)$, there exists a point $\bar{x}$ in rectangular area $[x_1^*, x_2^*]$, such that

$$f(x_1^*) - f(\bar{x}) = a \quad (4)$$

So

$$G(x_1^*, x_1^*, a) = G(\bar{x}, x_1^*, a) \quad (5)$$

By x_1^* is a local minimizer of $f(x)$, there exists a point $\bar{x} \in B(x_1^*) \cap [x_1^*, x_2^*]$ such that $G(\bar{x}, x_1^*, a) < 0$. Then, there exists a point $x' \in [x_1^*, x_2^*] = \{x \in \mathbb{R}^* \,|\, \min\{|x_1^*|_i, |x_2^*|_j\} \leq x_i \leq \max\{|x_1^*|_i, |x_2^*|_j\}\} \subset int\boldsymbol{\Omega}$ which is a minimizer of $G(x, x_1^*, a)$.

By **Theorem 2**, we have $f(x') < f(x_1^*)$. We know that of theories 1, 2, and 3 when there is a better local minimizer x_2^* of $f(x)$ than x_1^*, there is a point x' and a miniature for $G(x, x_1^*, a)$. Located in the basement. This means that if we reduce $f(x)$ using the initial point x', it will find a better minimizer for $f(x)$.

We can do it in the event that x_1^* is definitely not a global minimizer for the meaningful function, then, at that point, $G(x, x_1^*, a)$ is non-differentiable sooner or later in Ω. Gradient algorithms cannot be utilized for neighbourhood improvement for the minimizer of $G(x, x_1^*, a)$. Here a smoothing strategy to approximate $G(x, x_1^*, a)$ is utilized here as follows.

Let

$$G_b(x) = [-a + \frac{1}{b}\log(1 + e^{\tan(f(x) - f(x_1^*))})] \times \|x - x_1^*\|^2 \quad (6)$$

where b is a positive parameter. Clearly $G_b(x)$ It is differentiable. And , because

$$G_b(x) - G(x, x_1^*, a) = [\frac{1}{b}\log(1e^{\tan(f(x) - f(x_1^*))})\text{-} \tan(\min\{f(x) - f(x_1^*), 0\})] \times \|x - x_1^*\|^2$$
$$\geq [\frac{1}{b}\log(2e^{\tan(\min\{f(x) - f(x_1^*), 0\})})\text{-}0] \times \|x - x_1^*\|^2 = \frac{\log 2}{b}\|x - x_1^*\|^2,$$

we have the inequality

$$0 \leq G_b(x) - G(x, x_1^*, a) \leq \frac{\log 2}{b}\|x - x_1^*\|^2$$
$$\leq \frac{\log 2}{b}A^2, \quad (7)$$

holds. We can see from the previous discussion that when b becomes larger, $G_b(x)$ gradually converges to $G(x, x_1^*, a)$. As a result, the minimization of $G(x, x_1^*, a)$ could be substituted by the minimization of $G(x, x_1^*, a)$ by choosing a suitably large b.

$$\min_{x \in \Omega} G_b(x) \quad (8)$$

To produce a rather more accurate minimizer of $G(x, x_1^*, a)$, a should be large enough in $G_b(x)$ by solving $G_b(x)$. Furthermore, if this value of b is too large, the function values $G_b(x)$ will be overwritten. To avoid this, a contraction factor r is inserted into $G_b(x)$ to provide a fixed and sufficiently big b (eg, b=10^{α} A,$3 \leq \alpha \leq 8$) that ensures that the $G_b(x)$ correctly approximates $G(x, x_1^*, a)$.

Thus, to avoid numerical computing difficulties, a large b might be used. Therefore, the expression $G_b(x)$ may be expressed as

$$G_b(x) = [-a + \frac{1}{b} \times \|x - x_1^*\| \times \log(1 + e^{\tan(f(x) - f(x_1^*))})] \times \|x - x_1^*\|^2 \quad (9)$$

By doing so, current flaws can be addressed. □

III. THE ALGORITHM OF THE PROPOSED METHOD

From the study of the previous theorems, we can propose a new filled function algorithm to find an overall minimization of f(x). Next, we can give some explanations of this algorithm. We detail it as follows:

Step 1: Select an initial value $a = a_0$, a lower bound of a, (denoted by h), sufficiently large b. Directions $d = (cosi, sini), i = 2 * \frac{pi}{100} : 2 * \frac{pi}{100} : 2 * pi$. Set $k := 1$.

Step 2: Minimize $f(x)$ at $x_k \in \Omega$ and determine x_k^* .

$$x_k = fminsearch(@(x)obj(x), x_0)$$

Step 3: Define the function

$$G_b(x) = [-a + \frac{1}{b} \times \|x - x_1^*\| \times \log(1 + e^{\tan(f(x) - f(x_1^*))})] \times \|x - x_1^*\|^2,$$

$$x_k = fminsearch(@(x)ff(x, a.x_k, b), x_k).$$

Step 4: for $i = 2 * \frac{pi}{100} : 2 * \frac{pi}{100} : 2 * pi$ set $x_k = x_k^* + ep * d$ then proceed to **Step5**; Othewise, proceed to **Step 6**.

Step 5: The point x is used as an initial point to minimization of $G_b(x)$, assuming that the minimization arrangements of $G_b(x)$ go out of Ω, set $x_0 = x_0^* + ep$ and return to **Step 4**; Othewise, a minimizer x of $G_b(x)$ can be determined in Ω and set $x_k = x'$ $a = a_0$, $k = k + 1$ and return to **Step 2**.

Step 6: In the case $h \le ep$, the algorithm is terminated and x_k^* is chosen as the global minimizer of $f(x)$; Else, proceed to **Step 3**;

We must first supply basic additional details on the above-mentioned filled function algorithm before beginning the experiments.

1. To minimize $f(x)$ and $G_b(x)$, we must first choose a local optimization approach. The trust region approach is used in the suggested algorithm.
2. In **Step 4**, Step 4 requires a smaller *ep* to pick properly; in our technique, the *ep* is chosen to ensure that $\|G_b(x)\|$ is larger than a threshold.

(e.g., take the threshold as 10^{-3}).

3. **Step 5** shows that when a local minimizer x of $G_b(x)$ is discovered in Ω and with $f(x') < f(x_k^*)$, a superior minimizer of f(x) would be obtained using x as the starting point to minimize $f(x)$.

IV. NUMERICAL EXPERIMENTS

Problem 1: (Three- hump back camel function)

$$min\ f(x) = 2x_1^2 - 1.05x_1^4 + \frac{1}{6}x_1^6 - x_1x_2 + x_2^2$$

$s.t\ \ -3 \le x_1 \le 3,\ -3 \le x_2 \le 3$

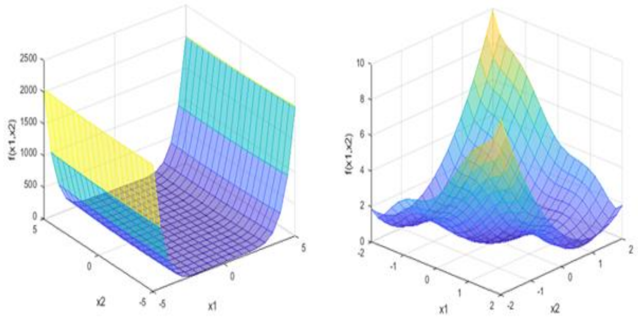

Fig 1: Three-hump back camel function

The global minimum solution is $x^* = (0,0)^T$.

Problem 2: (Six-hump back camel function)

$$min\ f(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 - x_1 x_2 - 4x_2^2 + 4x_2^4$$

$s.t\ \ -3 \le x_1 \le 3,\ -3 \le x_2 \le 3$

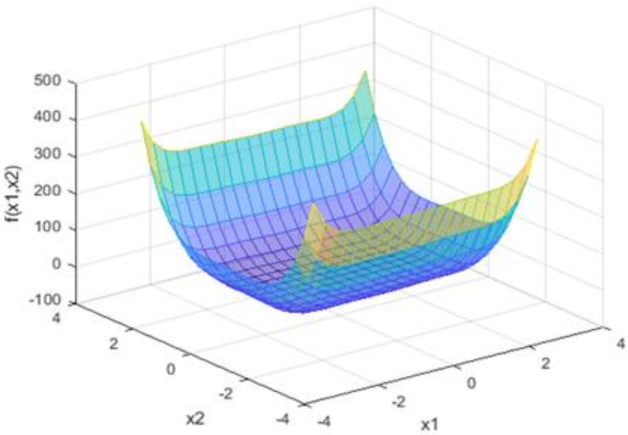

Fig 2: Six-hump back camel function

Where $x^* = (-0.0989, -0.7127)^T$ *or* $x^* = (0.0989, 0.7127)^T$.

Problem 3: (Treccani function)

$$min\ f(x) = x_1^4 + 4x_1^3 + 4x_1^2 + x_2^2$$

$s.t\ \ -3 \le x_1 \le 3,\ -3 \le x_2 \le 3$

The global minimum solution are $x^* = (0,0)^T$ and $x^* = (-2,0)^T$.

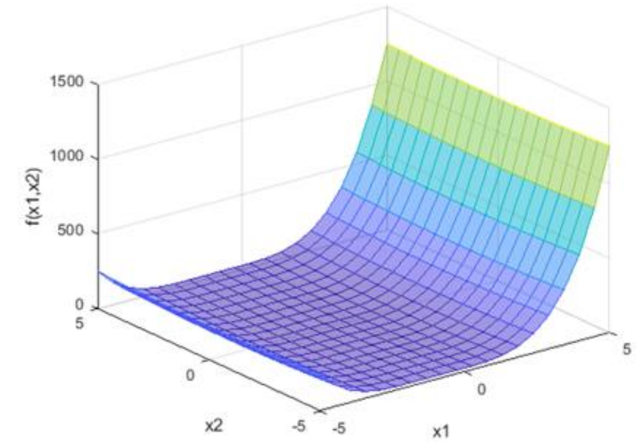

Fig 3: Treccani Function

Problem 4: (The Goldstein price function)

$minf(x) = (1+(x1+x2+1)^2(19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2)) \times (30+(2x_1-3x_2)^2(18-32x_1+12x_1^2+48x_2-36x_1x_2+27x_2^2))$

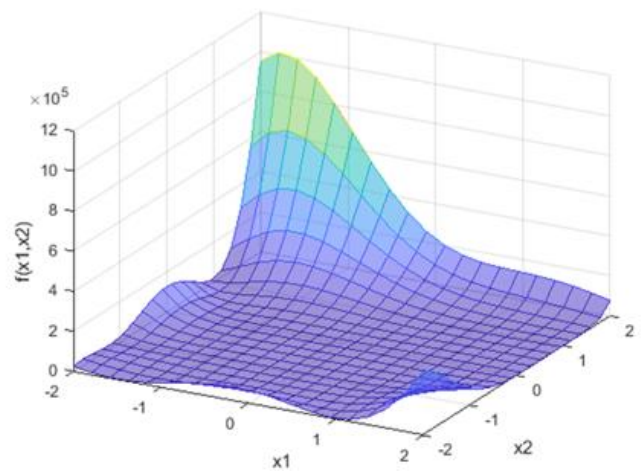

Fig 4: the Goldstein price function

This function has 4 local minimizers in the domain $-2 \le x_i \le 2, i = 1,2,3,4, \ldots$ but only one global minimizer $x^* = (0,-1)^T$.

Problem 5: (Banana function)

$$min\ f(x) = 100 \times (x_2 - x_1^2)^2 + (1-x_1)^2$$
s.t $-3 \le x_1 \le 3,\ -3 \le x_2 \le 3$

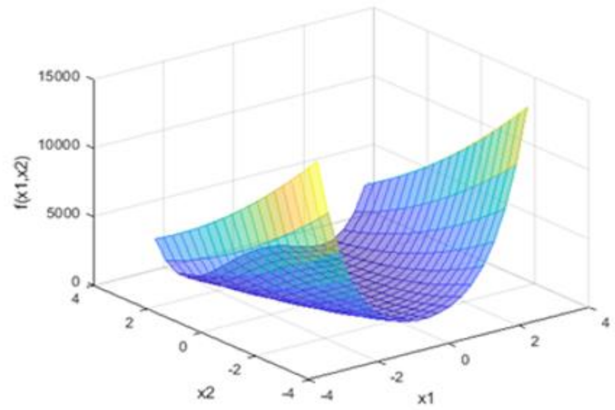

Fig 5: Banana function

Problem 6: (Two-Dimensional Shubert function)

$\min f(x) = \{\sum_{i=1}^{5} icos[(i+1)x_1]+i\}\{\sum_{i=1}^{5} icos[(i+1)x_2]+i\}$
s.t $0 \le x_1 \le 10, 0 \le x_2 \le 10$

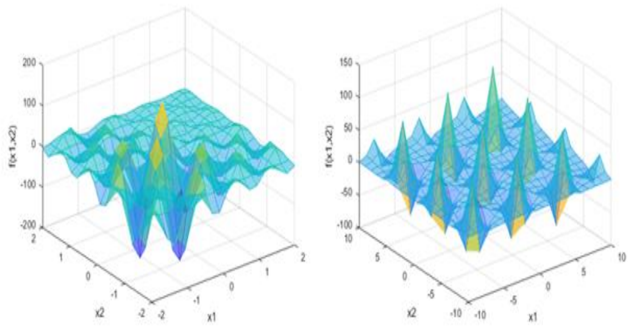

Fig 6: Two-Dimensional Shubert function

There are 760 minimizers in all for this function, and $f(x^*) = -186.7309$.

Problem 7: (Two-Dimensional function)

$$minf(x) = [1 - 2x_2 + csin(4\pi x_2) - x_1]^2 + [x_2 - 0.5\,sin(2\pi x_1)]^2$$
s.t $0 \le x_1 \le 10, -10 \le x_2 \le 0$

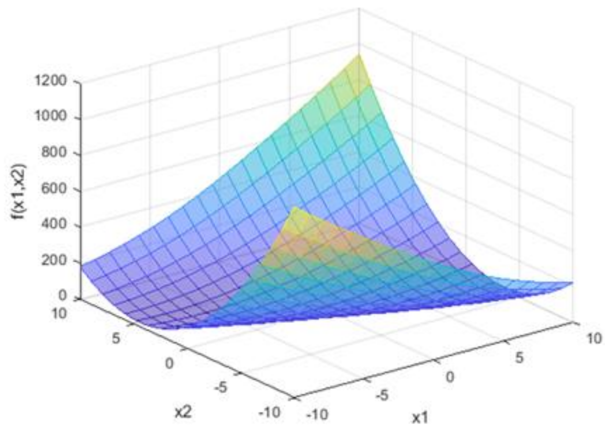

Fig 7: Two-Dimensional function

where $c = 0.2, 0.5, 0.05$. and $f(x^*) = 0$ for all c.

Tables 1 and 2 provide the results obtained of all of the experiments.

Table 1: Computational results for problems with initial point x_0

Problem No.	n	$\boldsymbol{x_0}$	$\boldsymbol{x^*}$	$\boldsymbol{f(x^*)}$
1	2	(-3.000, 3.000)	(-0.9031, -0.5256)e-07	1.4328e-14
2	2	(-3.000, 3.000)	(-0.0989, -0.7127)	-1.0316
3	2	(-3.000, 3.000)	(-2.0000, 0.0000)	2.2584e-17
4	2	(-3.000, 3.000)	(0.0000, -1.0000)	3.0000
5	2	(-10.000, 10.000)	(1.000, 1.000)	6.1225e-12
6	2	(-10.000, 10.000)	(-1.4251, -0.8003)	-186.7309
7($\boldsymbol{c = 0.2}$)	2	(-10.000, 10.000)	(0.1216, 0.3458)	2.5657e-12
7($\boldsymbol{c = 0.5}$)	2	(-10.000, 10.000)	(0.4128, 0.2606)	7.4092e-12
7($\boldsymbol{c = 0.05}$)	2	(-10.000, 10.000)	(0.4025, 0.2874)	4.4232e-16

Table 2: The results obtained by our algorithm

Problem No.	n	$\boldsymbol{f_m}$	$\boldsymbol{f_b}$	$\boldsymbol{f_c}$
1	2	2.9086e-14	1.4328e-14	60
2	2	-1.0316	-1.0316	48
3	2	2.6819e-14	2.2584e-17	68
4	2	3.0000	3.0000	682
5	2	3.5146e-11	6.1225e-12	94
6	2	-186.7309	-186.7309	72
7($\boldsymbol{c = 0.2}$)	2	2.9666e-09	2.5657e-12	73
7($\boldsymbol{c = 0.5}$)	2	8.3229e-10	7.4092e-12	156
7($\boldsymbol{c = 0.05}$)	2	7.7740e-10	4.4232e-16	72

In Tables 1 and 2, we will observe that our algorithm is powerful and impacted by the underlying value of a and the determination of a lower bound of a. The bigger introductory worth of a, the less nearby minimizer will be found, and furthermore the lower calculation cost will be; in the interim, assuming the function worth of the current neighbourhood minimizer is shut to that of the worldwide minimizer, then, at that point, the adequately little lower bound of an is essential, while a moderately enormous starting worth of a will cause expanding of the number of cycles. Hence, the underlying worth of an and lower bound of an are should have been chosen precisely. The determination of a lower bound of a guarantees the exactness of the worldwide minimizer, so that the adequately little lower bound of an and suitable little introductory an should be chosen.The following symbols are used in this paper:

x_0 The starting point.

x^* The local minimizer.

$f(x^*)$ The function value of the local minimizer.

f_e Total number of functions evaluations.

f_m The ten-run average of such best value.

f_b Best value out of ten runs.

V. CONCLUSION

The filled function technique is a thoughtful and successful strategy for generally speaking improvement. There are a few issues and imperfections in the present filled functions; for instance, some portions of these filled functions cannot be recognized, some of them contain more than one setting parameter, some contain fill terms conditions, etc. These deformities might prompt disappointment or trouble in the calculation in tracking down the ideal arrangement worldwide. To address this deficiency, another filled function is proposed in this paper. Albeit the new filled function cannot be demonstrated at certain places, it tends to be approximated consistently by a differentiable constant function. The basic lack in proximal function can be overcome with basic treatment. The adequacy of the proposed filled function strategy has been demonstrated by mathematical investigations on some optimization test issues.

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