ISSN No:-2456-2165

Modelling Turbulence Using the Staggered Grid and Simplec Method

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Abstract:- In a natural convection, local density differences and a resulting pressure gradient accelerate the fluid. In this paper a numerical study of a turbulent, natural convection problem is performed with an incompressible fluid in a rectangular enclosure. At the heated wall, the temperature distribution is a function of temperature gradients. The objective of this study is to conduct a numerical investigation of turbulent natural convection in a 3-D cavity using the staggered grid and the SIMPLEC method. The statistical-averaging process of the mass, momentum and energy governing equations introduces unknown turbulent correlations into the mean flow equations which represent the turbulent transport of momentum, heat and mass, namely Reynolds stress () and heat flux (), which are modeled using k- SST model. The Reynolds-Averaged Navier-stokes (RANS), energy and k- SST turbulent equations are first non-dimensionalized and the resulting equations are discretized using a staggered and solved using SIMPLEC. From the results, both the experimental data and simulation using the staggered grid and SIMPLEC return a non-dimensional temperature of 0.5 at the core of the cavity and almost zero towards the cold and the natural turbulence flow is responsible for temperature distribution. Further, convective mass exchange is dominant in the centre of the enclosure. The investigated Rayleigh number of this study lies at Ra = 1.58.

***Keywords**:- Turbulence, Natural Convention, Staggared Grid, SIMPLEC Method.*

I. INTRODUCTION

In fluid dynamics, turbulence is a flow regime characterized by chaotic and stochastic changes. This includes low momentum diffusion, high momentum convection and rapid variation of pressure and velocity in space and time. Turbulent flows exist everywhere in nature from the jet stream to the oceanic currents. Convection flows are one of the fundamental problems in fluid dynamics due to their role in meteo- rology where they appear as wind as an outcome of solar radiation in the atmosphere eand in industrial applications, where they are used in cooling systems to reduce possible noise exposures and technical failures. Many analytical, experimental and numerical investigations have been performed in the past, but the findings are not exhaustive and so these flows are still of substantial interest. He focus of this paper is computational study of the lift due to the iccured convection and the resultant thermal mixing of the flow in an enclosure described at 2 below.

II. LITERATURE REVIEW

Natural turbulent convection in cavities attracts considerable interest from thermal scientists given that it can be found in many industrial and/or civil engineering applications such as energy transfer in rooms and buildings, nuclear reactor cooling, solar collectors and electrical component cooling. A significant number of experimental and theoretical works have been carried out in the past in an attempt to understand turbulent natural convective flows in enclosures.

Among the work is that of Kulacki (1975) who studied natural convection in a horizontal fluid layer boundary bounded by upper isothermal surface and bottom insulated plate. For Prandtl numbers varying from 2:75 to 6:85 and Rayleigh numbers up to 2 10 12, the experimental data of Kulacki (1975) were correlated by the following expression:

$$\boldsymbol{Nu_{top}} = \mathbf{0.403}\boldsymbol{Ra}^{0.226} \quad \ldots\ldots 1$$

Further research on natural convention in an enclosure with localized heating and cooling has been studied by Gatheri et al. (1994). Gatheri et al. (1994) further investigated how to use False Transient Factors for the Solution of Natural Convection Problems and has well done a parametric Study of an Enclosure with Localized Heating and Cooling, Gatheri (1997). Sigey (2004), not only did research on Numerical Free Convection Turbulent Heat Transfer in an Enclosure but also carried out parametric studies on a rectangular enclosure using the standard k-model. Further work on natural turbulent convection has been about the use of mesh generation for the solution of natural convection problems, Gatheri (2005), use of a variable False transient for the solution of Coupled elliptic Equations and use of Buoyancy Driven Natural Convection heat transfer in an enclosure and Magnetohydrodynamic (MHD) free convective flows past an infinite vertical porous plate with Joule heating, Gatheri (2005).

ISSN No:-2456-2165

The validation of and a coarse DNS models for turbulent natural convection in a differentially heated cavity containing a fluid with P r = 0:71 and Rayleigh numbers ranging from 1.5810 9 to 10 12 was performed by Aounallah et al. (2007). The conclusion of Aounallah et al. (2007) was that the k-!-SST provided superior outcomes than the other models analyzed, though it was not able to reproduce accurately the mean flow.

Awuor (2012) did a study to assess the performance of three numerical turbulence models; to find the model with a better approximation to the experimental data in predicting heat transfer profiles due to natural convection inside an air filled cavity. The results showed that model is a more accurate layer simulation under high temperature gradient as compared with the models. A numerical data was then obtained for a test problem using the best model, the resultant numerical data stratified into three regions: a cold upper region, a hot region in the area between the heater and a warm lower region. The research work by Awuor (2012) is the motivation behind this paper.

III. MATHEMATICAL FORMULATION

In this paper, modelling turbulent heat transfer in a natural convection flow using the staggared grid within a cavity is conducted. The geometry is illustrated in figure 1. It consists of a hot surface, located on the left side of the rectangular cavity wall, and a cold surface on the right side. The enclosure is heated on the hot wall (Red color) and cooled on the cold wall (blue color). The measurement of Ampofo and Karyiannis (2003) were used.

The walls measures 0.75m by 0.75m wide by 1.5m. The hot and cold walls of the cavity were isothermal at 323K and 2830.15K respectively, giving a Reyleigh number of 1.58. Each of the remaining walls are adiabatic. Fluid flow will depend only on the temperature difference given as ΔT. Aspect ratio is 0.5. Furthermore, the Boussinesq Approximation (1903) is assumed and is presented below. In this research, we will study the variables as used by Ampofo and Karyiannis (2003).

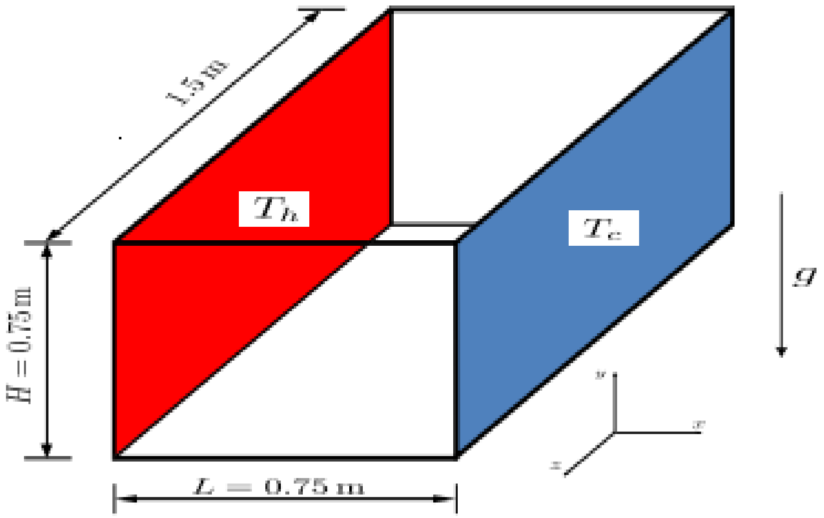

Fig 1:- Geometry of the 3-D numerical model

The convection cell can be described by the parameters of the Rayleigh number Ra, the Prandtl number Pr, the Nusselt number Nu and its aspect ratios Γ x ,Γ z . These parameters are defined as;

$$\left.\begin{array}{l} Ra = \dfrac{g\Delta T\,\beta H}{\alpha\,\upsilon} \\ Pr = \dfrac{\upsilon}{\alpha} \quad Nu = \dfrac{\partial T}{\partial y}\dfrac{H}{\Delta T} \\ \Gamma = \dfrac{L}{H} = 0.5 \end{array}\right\} \quad \ldots\ldots\ldots 2$$

Where g is the gravitational acceleration, the temperature difference between the horizontal walls, β the coefficient of therml expansion, H the height of the container, the thermal diffusivity, the kinematic viscosity, and ∂T∂y the temperature gradient directly at the heated walls (index w). The Nusselt number characterises the heat flux in the container. All parameters in Equation (1) correspond to the mean temperature field T mean = T cold + ΔT2 = 303.15 K between both heated walls of the setup. If the fluid exceeds a critical value of the Rayleigh-number, it starts moving, controlled by the temperature difference of the vertical, heated walls. The Prandtl number lies in this study at Pr = 0.71. The simulation assumes a Boussinesq fluid.

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IV. GOVERNING EQUATIONS

The resulting equations in general form after non dimensionalisation of the time averaged equations of motion, become;

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}\left(\rho U_i + \overline{\rho u_j}\right) \qquad \ldots\ldots.3$$

$$\frac{\partial}{\partial t}\left(\rho U_i + \overline{\rho u_i}\right) + \frac{\partial}{\partial x_j}\left(\rho U_i U_j + U_i \overline{\rho u_j}\right)$$

$$= -N_1 \frac{\partial P}{\partial x_i} + N_2 p g_i + \frac{\partial}{\partial x_j}\left(N_3 \tau_{ij} - U_i \overline{\rho u_i} - \rho \overline{u_i u_j} - \overline{\rho u_i u_j}\right)$$

$$= 0 \qquad \ldots\ldots.4$$

$$\frac{\partial}{\partial t}\left(c_P \rho \Theta + c_P \overline{\rho \Theta}\right) + \frac{\partial}{\partial x_j}\left(c_P \overline{\rho U_j \Theta}\right)$$

$$= L_1 \left[\frac{\partial p}{\partial t} + U_j \frac{\partial p}{\partial x_j} + \overline{u_j \frac{\partial p}{\partial x_j}}\right] + \frac{\partial}{\partial x_j}\left(L_2 \lambda \frac{\partial \Theta}{\partial x_j} - c_P \overline{\rho \Theta} + c_P \rho \Theta\right)$$

$$+ L_3 \emptyset \qquad \ldots\ldots..5$$

V. VARIABLE ARRANGEMENT ON THE STAGGARED GRID

Before describing the discretization scheme, choice of arrangement on the grid requires some consideration. Instead of a collocated grid, we used a staggered grid arrangement for this paper computation in order to evaluate the velocity components at the control volume faces while the rest of the variables governing the flow field, such as the pressure, temperature, and turbulent quantities, are stored at the central node of the control volumes. A typical arrangement is depicted in figure 3, which is in 2-D, for convenience.

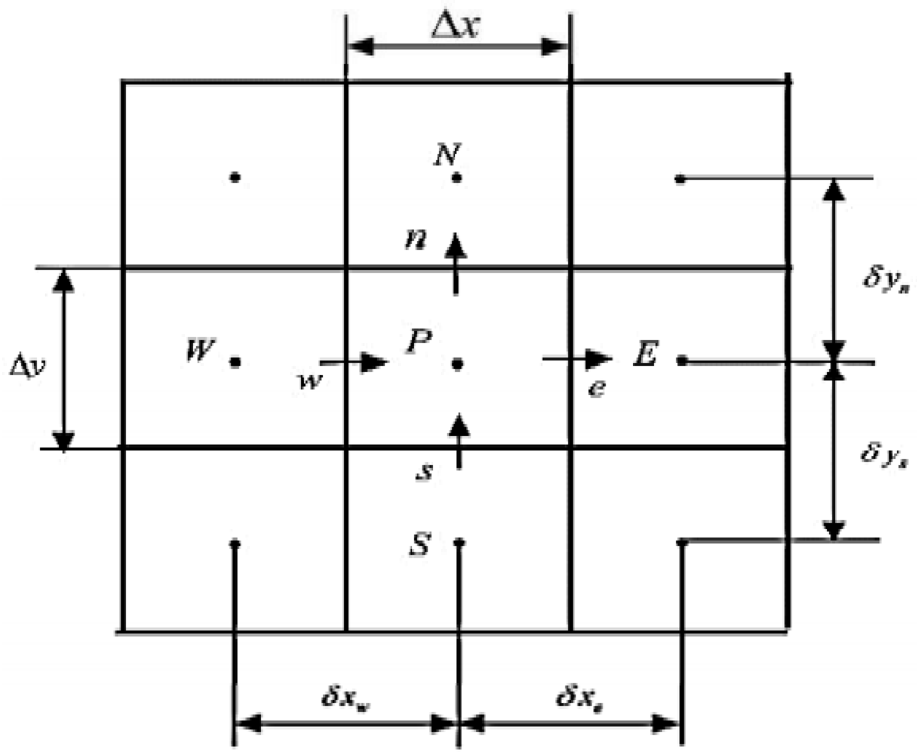

Fig 2:- control volumes in 2D.

It can be demonstrated that the discrete values of the velocity components, , from the momentum equation are evaluated and stored at the east, , and the west, , faces of the control volume. By evaluating the other velocity components using the momentum and momentum equations on the rest of the control volume faces, these velocities allow a straightforward evaluation of the mass fluxes that are used in the pressure correction equation. This arrangement therefore provides a strong coupling between the velocities and pressure, which helps to avoid some types of convergence problems and oscillations in the pressure and velocity fields.

VI. THE SIMPLEC SOLUTION ALGORITHM

The SIMPLEC Algorithm follows the same steps as SIMPLE algorithm, with the difference that the momentum equations are manipulated so that the SIMPLEC velocity correction equations omit terms that are less significant than those omitted in SIMPLE.

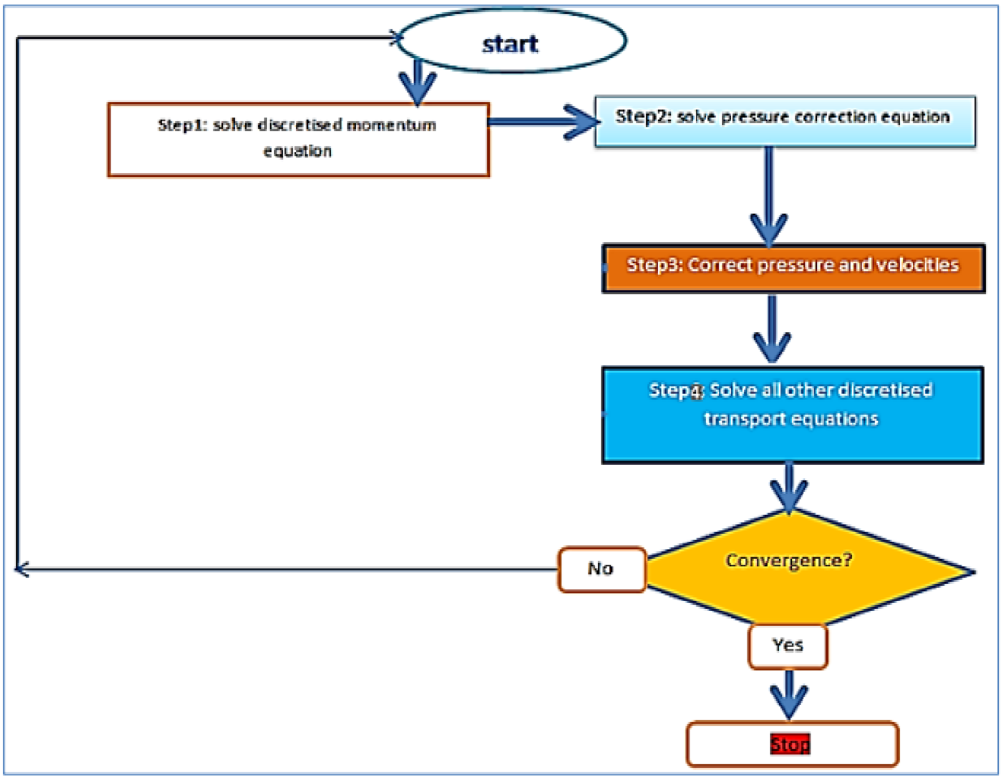

Fig 3:- SIMPLEC Algorithm flow chart

VII. RESULTS AND DISCUSSION

The results presented here were obtained by solving equations (3), (4) and (5)) by SIMPLEC algorithm after discretization using the staggered grid as shown in figures(2) and together with the boundary conditions gave the following numerical solutions. The numerical results we have found were validated against the experimental data provided by Ampofo and Karayiannis (2003). This benchmark is at a Rayleigh number of.

A. Solution Convergence by SIMPLEC Method

Convergence was monitored with residuals, whereby a decrease in residuals by three orders of magnitude was to indicate at least qualitative convergence whereby case residual plots would show when the residual values have reached the specified tolerance. For SIMPLEC, the residual convergence criterion for each variable was achieved and the residual imbalance became negligible after 350 iterations in a duration of 1hr, 15min.

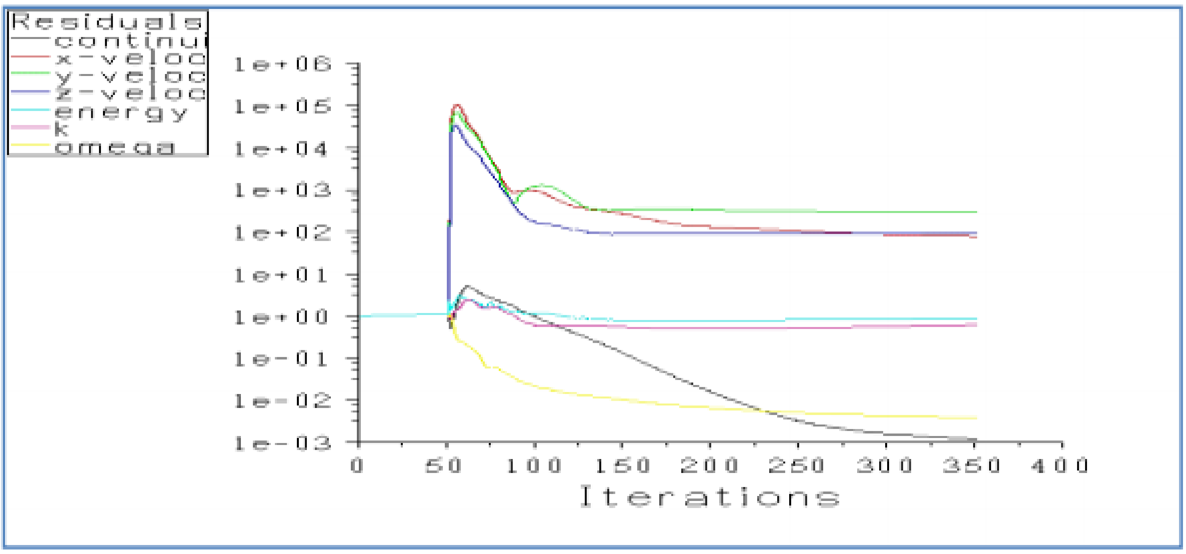

Fig 4:- Scaled Residuals by SIMPLEC

B. Validation of Results

Verification and Validation to assess the accuracy and reliability of results in this numerical code was done against the experimental solutions obtained from Ampofo and Karayiannis (2003).

➢ *Temperature Profiles*

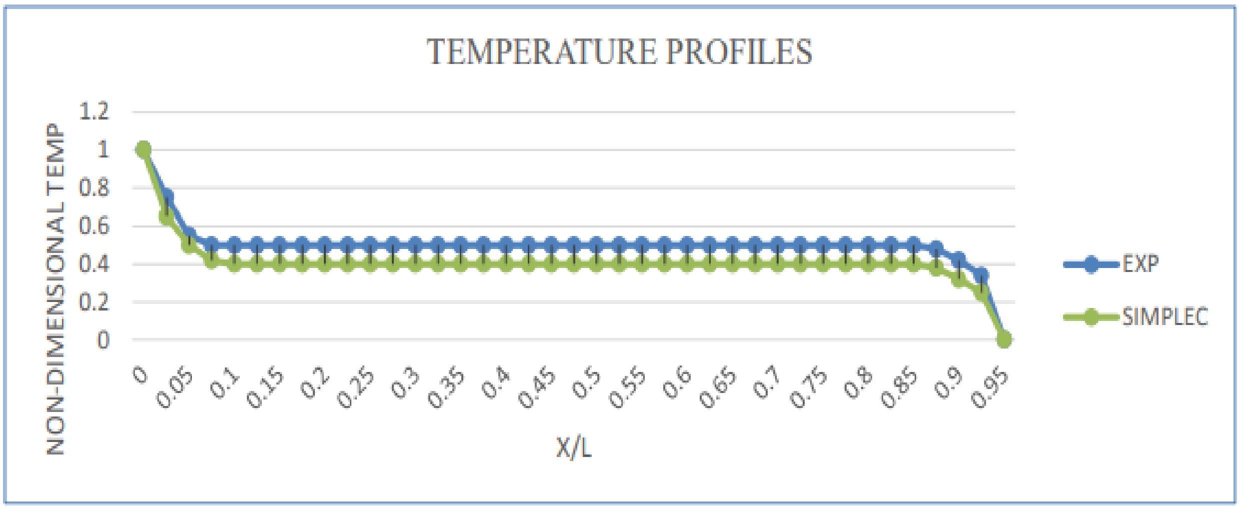

Fig 5:- Comparison of the Mean Temperature at y/H=0.5

From fig. 5, the mean temperature profiles show an almost uniform distribution in the enclosure core. This shows that in the enclosure core region, there is very little activity as the mean temperature is nearly uniform. The predicted temperatures by SIMPLEC show a minimum which is lower than the experimental values in the core of the cavity. Again this shows insufficient mixing with the laminar core. In the enclosure core region, there is very little activity; the mean temperature is nearly uniform.

➢ *Mean Vertical Velocity*

Figure 6 shows the profiles for rate of change of vertical displacement of the fluid particles with time. The profiles are asymmetrical and with a peak near the heated surface.

As seen in figure 6, there is good agreement between the experimental data and the predicted data in terms of the mean velocity. The peak value of velocity is particularly well captured by SIMPLEC method. In the enclosure core region, there is very little activity and hence the fluid velocity is very small.

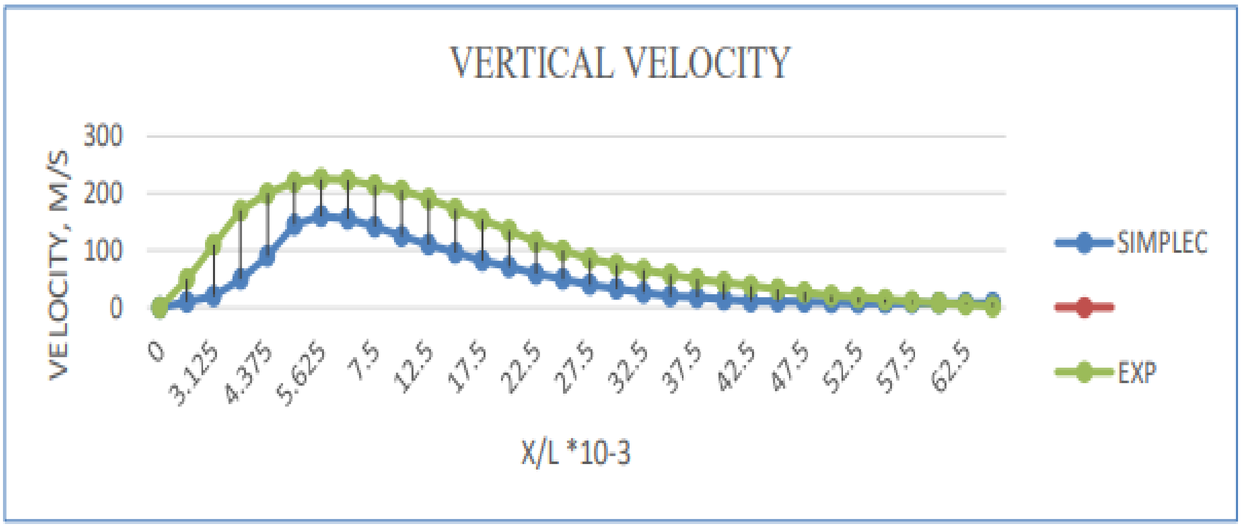

Fig 6:- Comparison of the vertical velocity

➢ *Mean Horizontal Velocity*

Figure 7 shows the profiles of the rate of change of horizontal displacement of the fluid with time.

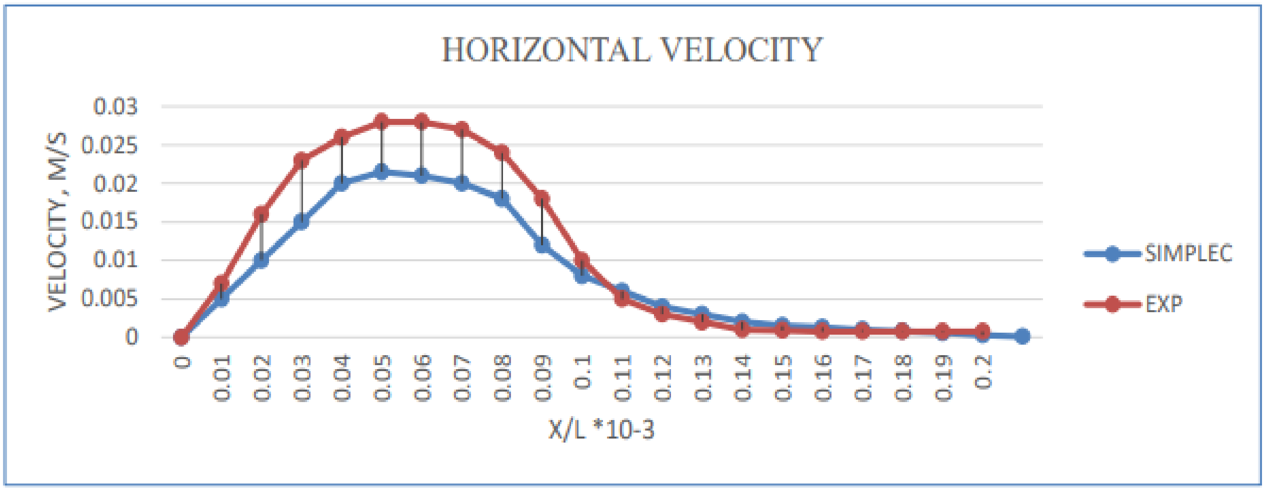

Fig 7:- Comparison of the Horizontal velocity

The rise of velocities near the heated surface of the cavity is as a result of fluid gaining kinetic energy from the heated wall causes an increased convective heat transfer coefficient, while there is a drop of velocities after 0.04.

All in all, there is good agreement between the experimental data and the predicted data in terms of the mean horizontal velocity, as in fig. 7,

VIII. CONCLUSION

➢ From the numerical data, the numerical method produced a solution which approached the exact solution by Ampofo and Karayiannis (2003) as the grid spacing reduced to zero. Further, the method is stable, the governing equations consistent, as evidenced by the damping errors as the numerical method proceeded and the initial data did not cause wild oscillations of divergence. Therefore using Lax's equivalence theorem, Lax and Richtmyer (1956), this code is valid, stable and consistent.
➢ In this numerical investigation, both the experimental data and simulation using SIMPLEC return a non-dimensional temperature of 0.5 at the core of the cavity and almost zero towards the cold Therefore
➢ The results show that in an enclosure environment, the natural turbulence flow is responsible for temperature distribution.

ACKNOWLEDGEMENT

My sincere gratitude to Prof. F. K. Gatheri and Dr. K. O. Awuor for their valuable contributions and inspiration.

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