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The Strategy of High School Students in Solving Probability Problems

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Abstract:- Students' strategies in solving probability problems are still rarely studied in depth although this ability is needed by students in solving problems that contain an uncertainty element. The purpose of this study was to describe the strategies that were used by high school students in solving probability problems. Subject in this study was a male student of grade 11 science who had studied probability material and had low math ability in which the determination of student's category was based on the score of math ability test. The research data were obtained through in-depth interviews and problem solving tasks. The data credibility test used time triangulation with data convergence to the same meaning. The results showed that in solving the probability problem, the strategy used by the student was calculating the number of the sample space members during the drawing of 2 coins to 6 coins. Subject calculated the number of sample space members in the child birth order event by relating them to the 6 coin draw event, then he used the classic definition of calculating the odds of each event to be compared so that he could decide the correct answer. A person's ability to the aspect of probability thinking strategies in solving probability problems varies depending on the ability in understanding the information and knowledge owned by each person in facing the probability situations.

Keywords:- *Strategy, probabilistic thinking, mathematical skills, solving probability problems.*

I. INTRODUCTION

Probability is closely related to everyday life and has become an integral component in various profession fields [1]. To solve problems related to probability, namely problems that have various possibilities or contain an uncertainty element, a person is required to be able to think probabilistically. Probabilistic thinking also plays a role in decision making, namely as a tool for decision making when a person faces a situation that contains an uncertainty element [2].

Probabilistic thinking is very important so that the ability to think probabilistically needs to be developed, among others, through education. In education world, probability material which is part of mathematics can develop students' probabilistic thinking. In many countries, probability and statistics are included in mathematics curricula. Curriculum 2013 (K-2013), which is the current national curriculum in Indonesia, also includes statistics and probability as parts of mathematics subject. At high school level, the materials of statistics and odds are learned when students are in class XI. The materials contain some daily problems faced by students in the form of problems containing probability situations. To solve the problems, students are required to be able to think logically or make reasoning in providing arguments or explanations so that later they can become smart users and can make important decisions from information.

One's understanding of the probability of an event can be seen through one's ability to identify and justify which of the two or three events that are most likely or least likely to occur [3]. Strategy is one aspect that can be observed related to students' probabilistic thinking. Strategy is seen as a goal-directed procedure that facilitates problem solving and the proficiency of a specialized knowledge domain. Strategy is also seen as potentially conscious and controllable [4]. In this study, strategy is the procedure or ways and reasons used by students in solving probability problems.

English (2007) in revising his research findings on children aged 4 years 6 months to 9 years 10 months found five strategies used by students when solving probability problems [4]. The five strategies are named by McGalliard III: (1) trial and error, (2) emerging strategy, (3) a cyclic pattern, (4) odometer with errors, and (5) odometer complete [5]. The research results on 55 students of grades 10 and 11 in Slovakia to identify the strategies used by students in solving combinatorial-related problems showed that there were 10 strategies used by students which were categorized according to the students' combinatorial thinking level suggested by Jones et al. (1997) were: unordered list of the elements of set of outcomes (Level 1 subjective), table of the elements of set of outcomes (Level

2 transitional), figure (vertex graph) (Level 2 transitional), tree diagram (Level 2 transitional), expression using informal algebra (Level 3 informal quantitative), expression using numbers (Level 3 informal quantitative), expression using variables (Level 3 informal quantitative), verbal reasoning and justification (Level 4 numerical), combinatorial formula (Level 4 numerical), mental calculation without written procedure (Level 5 extended abstract) [6]

The ability of a person to aspects of probabilistic thinking strategies in solving probability problems is different. It is influenced by a person's ability to understand the information provided by the problem and the differences in knowledge owned by each person in the situation. Other factors that cause students' differences in probabilistic thinking include: language, beliefs, and experiences [7]; cultural factors [8]; and students' math abilities [9].

Therefore a research is needed to reveal in depth how the strategies are used by high school students in solving probability problems. The results of this study can be used to improve the quality of the learning process and develop students' abilities in practicing their probabilistic thinking.

II. METHOD

A. Setting and Participants of the Study

This research was conducted at one of the State Senior High Schools in Jayapura. The subject of this study was a male student in grade 11. The selection of research subjects was based on the condition in which the research subjects had studied teaching materials about probability and had low mathematical abilities.

The first step of subject selection is selecting a group of male students to be given a math ability test. Through the results of the test analysis on test scores, a low-ability research subject was selected.

B. Design

The research approach was carried out as it was so that the research process took place naturally and could develop according to field conditions. This research was conducted by focusing on research subject to obtain a description of strategies in solving probability problems. Because the research result was specifically oriented to one research subject, the research results could not be generalized, so that it could only draw conclusions on actual events in the field in accordance with the research objectives.

This research used a qualitative approach because the data obtained were through observation, in-depth interviews, and documentation and the results were described descriptively. The type of this research is an exploratory descriptive study. Qualitative data is richer in terms of description and explanation [10].

C. Instrument and Validation

By referring to the type of research, namely qualitative, the main instrument in this study was the researcher herself who acted as a planner, collected data through direct in-depth interviews with research subject, analyst, interpreter, and reported the results of research. The researcher as the main instrument must be neutral and impartial.

The supporting instruments for obtaining a profile of probabilistic problem solving strategies were Mathematics Ability Test (TKM) questions which were used to determine the subject's mathematical ability category, and a Probability Problem Solving Task (TPMP) question sheet given to selected subject to be solved and an interview guide instrument.

Before the instruments were used, the instruments were consulted with the supervisor and validated by three experts (judgment experts). The criterion for the validity of the question instruments included content validity, construct validity and the language used. The following are problem-solving tasks given during the study.

1. Group A wants to form a committee of 2 people chosen from 10 people. Group B wants to form a committee of 8 people chosen from 10 people. Which of the following statements is correct?
a. The number of committee kinds that Group A can form are smaller than Group B.
b. The number of committee kinds that Group A can form are the same as Group B.
c. The number of committee kinds that Group A can form are larger than Group B
Give reasons of your answer.
2. In a survey of families with six children, the birth order was found as follows: LPLLLL, PLPLLP, and LLLPPP. (L: boy and P: girl)
i) If the birth order of LPLLLL is compared to PLPLLP, which one has the greater odds of occurring?
a. LPLLLL
b. PLPLLP
c. Both have the same odds
Explain the reasons of your answer.
ii) If PLPLLP birth order is compared to LLLPPP, which has the greater odds of occurring?
a. PLPLLP
b. LLLPPP
c. Both have the same odds
Explain the reasons for your answer.

Fig 1:- The problem solving task

Checking the validity of the data was done by testing the degree of credibility. Credibility testing was done through triangulation (time triangulation), which was comparing data from interviews I and II. The data credibility criterion used was the convergence of data to the same [11]; [12].

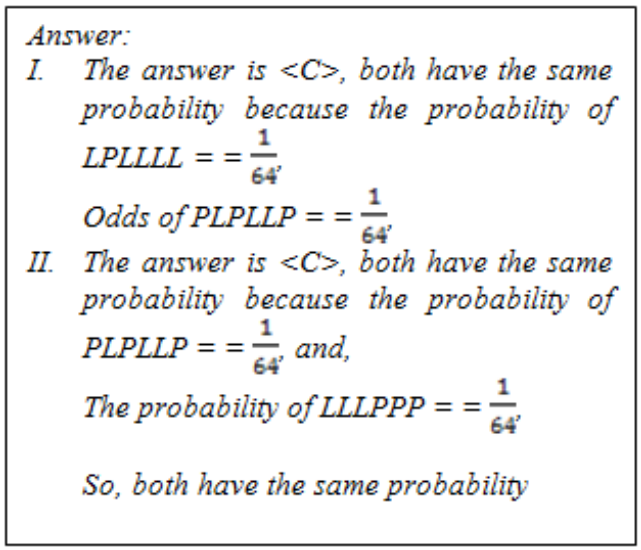
D. Data Collection and Analysis Procedures

After the research subject was selected, data collection started by providing validated supporting instruments. Subject was given probabilistic problem solving questions to be answered and then an interview was conducted. All data collection activities starting from working assignments to interviews were documented visually, audio, and audio visually. After conducting several interviews and collecting data, the consistency of data collected was reviewed by comparing each data obtained.

The research data were analyzed using Constant Comparative Method with modifications according to the data and focus of this study [13]; [14]

III. RESULT AND DISCUSSION

This study showed the strategies of a male student with low math ability in solving probability problems. Probability problem solving strategies included identifying problems, calculating odds, calculating the number of sequences that occured, and deciding by comparing. Based on the activity of giving probabilistic problem solving questions, student's answers were obtained in writing as follows:


Answer:

I. The answer is <C>, both have the same probability because the probability of $LPLLLL = = \frac{1}{64'}$

$Odds\ of\ PLPLLP = = \frac{1}{64'}$

II. The answer is <C>, both have the same probability because the probability of $PLPLLP = = \frac{1}{64'}$ and,

$The\ probability\ of\ LLLPPP = = \frac{1}{64'}$

So, both have the same probability

Fig 2:- The research subject's Answers

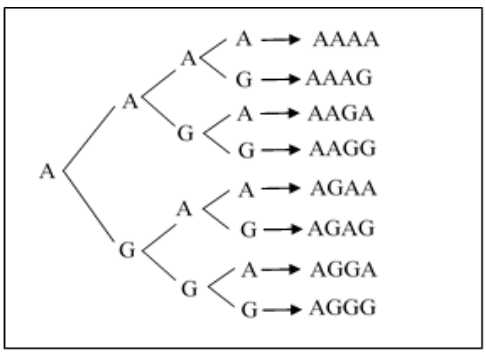
Based on the results of the interview, it was revealed that to solve this problem, the subject first used coins, calculating the number of sample space members during several draw events. The following is a snippet of the interview:

Finally, complete content and organizational editing before formatting. Please take note of the following items when proofreading spelling and grammar:

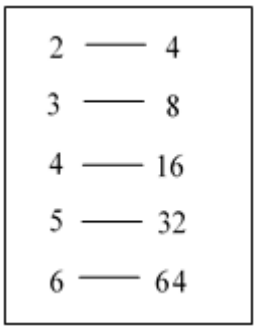
P : *How to solve this problem?*
S : *This is a question about the birth order of a family with 6 children. It asks about the odds. It means we have to find out how many birth orders there are.*
P : *How do you search for the number of possible birth orders?*
S : *I try to calculate for 2 coins, 3 coins, 4 coins first.*
P : *Why do you use coins?*
S : *Because the coins are similar with this problem. Coins have numbers and pictures while in this question there are males and females. So they are similar, both have two sides.*
P : *Why don't you calculate directly for 6 coins?*
S : *I don't know how to calculate directly for 6 coins.*

Subject used the tree diagram to calculate the number of sample room members at some coin draw events. In the 2-coin draw, there were 4 possible outcomes. In the 3-coin draw, 8 possible outcomes were obtained. Then he recalculated the number of results that might occur on the 4-coin draw. The following is a snippet of the interview:

P : *What is next?*
S : *I'm searching again for 4 coins using the tree diagram*
P : *How?*
S : *This is the way (making a tree diagram):*



P : *How many?*
S : *For the number side (A) on the first coin, the result is 8. Then for the image side (G) on the first coin there is also 8. Thus, if 4 coins are drawn, the possible results are 16.*
P : *Next?*
S : *Next I calculate it like this (writing):*



P : *How can you get thirty-two?*
S : *Thirty-two is got from 16 + 16, for 5 coins. For 6 coins,* $32 + 32 = 64$
P : *In general, what is the formula?*
S : *I do not know. I cannot make the general formula, Ma'am.*
P : *Next?*
S : *It is about coins, while the question is about the birth order of 6 children. Suppose that 6 coins are equal to 6 children, the number of possible birth orders in a family with 6 children is 64.*

After knowing the number of birth orders that can occur or the number of sample space members, subject calculated the probability of each event occurring, as in the following interview snippet:

P : *What are the odds of LPLLLL?*
S : *The LPLLLL odds are (writing):*

$$P < LPLLLL >= \frac{1}{64}$$

P : *How did you get one?*
S : *One is got from LPLLLL. There is only 1 possible birth order. 64 is the number of possible birth orders.*

The same thing was done to calculate the probability of PLPLLP and LLLPPP birth orders.

Based on the interview, it was also revealed that to calculate the number of birth orders that could occur in a family with 6 children, subject could not calculate it using a formula. Therefore subject used coins, calculated the number of sample room members at the drawing of 2 coins, 3 coins, and 4 coins. In the 2-coins draw, he paired side A of the first coin with each side (A and G) of the second coin, then he paired side G of the first coin with each of the sides (A and G) of the second coin. Thus, the sample space members obtained were AA, AG, GA, and GG.

In the 3-coin draw, he paired side A of the first coin with each side (A and G) of the second coin, also systematically paired each side (A and G) of the third coin. The same way was done for side G of the first coin, paired with each side (A and G) of the second coin and systematically paired with each side (A and G) of the third coin. Thus, finally it used up both constant items and the sample space members were AAA, AAG, AGA, AGG, GAA, GAG, GGA, and GGG.

In the 4-coin draw, he paired side A of the first coin with each side (A and G) of the second coin, then paired each side (A and G) of the third coin, also systematically paired it with each side (A and G) of the fourth coin. The same way was done for G side of the first coin, so that finally it used up both constant items and the sample room members were: AAAA, AAAG, AAGA, AAGG, AGAA, AGAG, AGGA, AGGG, GAAA, GAAG, GAGA, GAGG, GGAA, GGAG, GGGA, and GGGG.

It appeared that to calculate the number of sample space members during the drawing of 2 coins to 4 coins, the student used strategy 5 (odometer complete) because he used the odometer pattern consistently and completely without any shortcomings [4]; [5].

Meanwhile, to calculate the number of sample space members in the 5-coin draw event, subject calculated them by adding the results obtained in the previous draw (4 coins) with the same result, so that the number of sample space members obtained was $16 + 16 = 32$. In the same way, subject calculated the number of sample space members in the event of the 6-coin draw was $32 + 32 = 64$. Although subject could calculate the number of sample space members on the drawing of 2 coins to 6 coins, subject could not find the general formula. To calculate the number of birth orders that might occur in a family with 6 children, subject calculated them by relating them to the 6-coin draw, assuming 6 children were 6 coins.

After knowing the number of possible birth orders, subject calculated the odds value of each event. The odds of LPLLLL, PLPLLP and LLLPPP was $\frac{1}{64}$. The value of 1 in the numerator was obtained from each event that occured only once while 64 in the denominator was the number of all birth orders that could occur. According to subject, each event had the same odds to occur, namely $\frac{1}{64}$. [15]. Furthermore, to answer the questions, subject compared the odds of birth order of LPLLLL with PLPLLP and PLPLLP with LLLPPP. All of them had the same odds, so subject decided that the correct answer to the two questions was c: both had the same odds.

In solving this probability problem, subject used a tree diagram to list the sample room members. This is in line with the statement that working on probability, as in other mathematical topics, is done through various representations and models, such as tables, Venn diagrams, tree diagrams, formulas, etc. [1]. The same thing is expressed by Maher & Ahluwalia (2014) that the aspect of representation is one of the aspects needed to solve probability problems [9]. Representation will help students understand the concept of probability. On the other hand, the use of tree diagrams can be seen as one of the strategies used by subjects to solve probability problems related to the room sample [6].

IV. CONCLUSION

The strategies used by students with low math abilities in solving probabilistic problems are as follows: He calculates the number of sample space members on the draw of 2-coin to 6-coin. He calculates the number of sample space members or the number of birth orders that may occur in a family with 6 children by relating them to the 6-coin draw event, assuming 6 children are equal to 6 coins. It is done because he cannot find a formula to calculate the number of the sample space members. He uses the classic definition of calculating the odds of LPLLLL, PLPLLP, and LLLPPP; then he compares the odds of LPLLLL with PLPLLP and PLPLLLP odds with LLLPPP to decide the correct answer.

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