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Effect of Heat Source on Magnetohydrodynamic Free Convection Through a Channel With a Wall Having Periodic Temperature

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Abstract:- The analytical examination of the effects of velocity and temperature parameter variations and heat source effects on Couette free convective flow was studied on an electrically conducting optically thin, viscous and incompressible fluid through a porous medium with periodic wall temperature. The solutions to the set of coupled ordinary differential equations arising from the formulation of the problem were obtained by method of undetermined coefficient. Thereafter with realistic parameter values; the results were displayed and discussed with plots.

Keywords:- *Free Convection, Magnetohydrodynamic, Heat Source, Periodic Wall Temperature.*

I. INTRODUCTION

Magnetohydrodynamic fluid is a fluid that in electric and magnetic fields conducts electricity. It associates fluid dynamics and electromagnetic assertions to describe concurrent effects of magnetic field on the flow and vice versa. The eruption of scholarly published papers in this area has been enormous due to its value in controlling the velocity of molten metal in metallurgical industry, etc. Achogo W. H. et al.(2020) considered magnetohydrodynamic convective periodic study in inclined plane. Fazle Mabood and Halima Usman (2019) studied the multiple effects on MHD nanofluid considering thermo-solutal flow. Harisingh N. S. et al.(2014) investigated MHD free convective Couette with thermal radiation. Buggaramulu J. and Venkata M. K(2017) underwent a study of MHD convection flow of Kuvshinski fluid. Sharma P. R.et al.,(2014) examined the effects of free convection on MHD flow in a medium having periodic wall temperature.

Difference in temperature gives rise to the concept of free convection of heat in an enclosure or near a heated or cooled flat plate. Much scholarly papers have come up on this concept due to its application in engineering such as high – speed aircraft, rocket nozzles, cooling of nuclear reactors, etc.. Joydeep Borah and Hazarika G. C. et al(2018) studied free convective MHD flow over a stretching sheet in presence of radiation and viscous dissipation with variable viscosity and thermal conductivity. Pooja Sharma and Ruchi Sboo(2017) examined free convective flow of walter's liquid model –B . Free convective flow and heat transfer was investigated by Ahmed N. and Sarma D.(1997). Md S. K. et al.,(2012) investigated unsteady MHD free convection boundary-layer flow of a nanofluid. Mandal H. K. et al.,(2014) considered free convection in the study of transient free convection done in a vertical channel. Uddin Z. and Kumar M.(2010) studied free convection in inclined plate. Sandeep N. et al.,(2012) examined MHD free convective flow through vertical plate.

The concept of heat transfer becomes very interesting when internally generated heat is involved. Choudhury K and Ahmed N(2018) examined soret effect on transient MHD convection with radiation. Krishnendu Bhattacharyya(2011) investigated the effect of heat source/sink on MHD flow and heat transfer over a shrinking sheet with mass suction. Mamta Thakur(2017) studied magnetohydrodynamic free convection and radiation with heat source.

In this present paper, we examine the effect of heat source on magnetohydrodynamic free convective Couette flow through a porous medium with periodic wall temperature.

II. FORMULATION OF THE PROBLEM

We examine the effect of heat source on unsteady Couette MHD free convective flow through a prous medium with peirodic wall temperature. We made the following assumptions.

- The fluid is viscous, incompressible and electrically conducting optically thin fluid embedded by two vertical parallel plates with porous medium in it. x'-axis is taken in a vertical upward direction along one of the plate and y'-axis normal to the plate.
- The plate has a magnetic field of magnitude β_0 which is uniform placed normal to it. The transverse applied magnetic field and magnetic Reynold's number are assumed to be very small so that the induce magnetic field is negligible.The temperature of the both walls are assumed high enough to induce radiative heat transfer.
- The fluid has constant kinematic viscosity and constant thermal conductivity and the Boussineq's approximation has been used.

Therefore, under the confinement of framework; using the stipulated assumptions above and the usual Boussiqness approximation the governing equations are as follows:

$$\frac{\partial u'}{\partial t'} = \upsilon\frac{\partial^2 u'}{\partial y'^2} - \frac{\vartheta}{K'}u' - \frac{\sigma B_0^2}{\rho}u' + g\beta\,(T' - T_0') \quad (1)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} + S'\,(T' - T_0') \quad (2)$$

where $u'(y', t')$ axial velocity, t' is the time, ν' is the kinematic viscosity, σ is electrical conductivity, k is the thermal conductivity, C_p is the specific heat at constant pressure, ρ is the fluid density, ω is the frequency of oscillation, T' is the temperature of the fluid, B_0 is the magnetic field, g is the acceleration due to gravity, q' is the radiation flux, K' is the permeability of the porous medium and S' the heat source.

The boundary conditions expedient to the study are given as;

$$y' = 0 : u'(y', t') = \cup_0(1 + \varepsilon e^{i\omega' t'}), T'(y', t') = T_1' + (T_1' - T_0')$$
$$y' = h : u'(y', t') = 0, T'(y', t') = T_1' \quad (3)$$

By Cogley et al.(1968) the heat flux is expressed as;

$$\frac{\partial q'}{\partial y'} = 4\alpha^2 (T' - T_0') \quad (4)$$

where α is the mean absorption coefficient.

Put eqn(5) into equations(1-4)

$$y = \frac{y'}{h},\ u = \frac{u'}{\cup_0},\ t = \frac{t'\nu}{h^2},\ \omega = \frac{\omega' h^2}{\nu},\ \theta = \frac{T' - T_0'}{T_1' - T_0'},$$

$$\Pr = \frac{\nu\rho C_p}{K},\ K^2 = \frac{h^2}{K'},\ Gr = \frac{g\beta h^2 (T' - T_0')}{\nu \cup_0},\ M^2 = \frac{\sigma \beta_0^2 h^2}{\rho\nu},\ R = \frac{4\alpha h^2}{k},$$
$$S = \frac{S' h^2}{K} \quad (5)$$

we obtain

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta - (K^2 + M^2)u \quad (6)$$

$$\Pr\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} - (R^2 - S)\theta \quad (7)$$

where R , S, u, $\cup_0$, y, t, θ, Gr, Pr, K, M is the radiation parameter, heat source parameter, dimensionless velocity, mean flow velocity, the dimensionless coordinate axis normal to the plates, dimensionless time, dimensionless temperature, thermal Grashof number, Prandtl number, porosity parameter and magnetic parameter respectively.

The corresponding boundary conditions in dimensionless forms are stated as;

$$y = 0 : u(y, t) = 1 + \varepsilon e^{i\omega t}, \theta(y, t) = 1 + \varepsilon e^{i\omega t}$$
$$y = 1 : u(y, t) = 0, \theta(y, t) = 1 \quad (8)$$

III. METHOD OF SOLUTION

Equations 6 and 7 are second order coupled partial differential equation. Hence we assumed the solutions of the form below since $\varepsilon \ll 1$

$$u(y,t) = u_0(y) + \varepsilon u_1 e^{i\omega t} + O(\varepsilon^2)$$
$$\Theta(y,t) = \Theta_0(y) + \varepsilon \Theta_1 e^{i\omega t} + O(\varepsilon^2) \quad (9)$$

Now put (eqn 9) into (eqns 6-8) and neglecting the coefficient of $O(\varepsilon^2)$. We obtain

$$\frac{\partial^2 u_0}{\partial y^2} - (K^2 + M^2)u_0 = -Gr\theta_0 \quad (10)$$

$$\frac{\partial^2 \theta_0}{\partial y^2} - (R^2 - S)\theta_0 = 0 \quad (11)$$

$$\frac{\partial^2 u_1}{\partial y^2} - (K^2 + M^2 + i\omega)u_1 = -Gr\theta_1 \quad (12)$$

$$\frac{\partial^2 \theta_1}{\partial y^2} - (R^2 - S + i\omega\Pr)\theta_1 = 0 \quad (13)$$

and the boundary conditions are

y=0: $u_0 = 1, u_1 = 1,\ \theta_0 = 1,\ \theta_1 = 1$

y=1: $u_0 = 0, u_1 = 0,\ \theta_0 = 1,\ \theta_1 = 0$ (14)

Solving eqns(10-13) under the appropriate boundary conditions. Therefore the values of u_0 (y), u_1 (y), θ_0 (y) and θ_1 (y) are given below through method of undetermined coefficient as;

θ_0 (y)=$D_1 e^{\alpha_1 y} + D_2 e^{\alpha_2 y}$

θ_1 (y)=$D_3 e^{\alpha_3 y} + D_4 e^{\alpha_4 y}$

u_0 (y)= $D_5e^{\alpha_5 y} + D_6e^{\alpha_6 y}$ + $D_7e^{\alpha_2+\alpha_1 y} + D_8e^{\alpha_1 y}+D_9e^{\alpha_2 y}+D_{10}e^{\alpha_1+\alpha_2 y}$
u_1 (y)= $D_{13}e^{\alpha_7 y} + D_{14}e^{\alpha_8 y} + D_{15}e^{\alpha_4+\alpha_3 y} + D_{16}e^{\alpha_3+\alpha_4 y}$ (15)

By substituting (eqn15) into (eqn9), $\theta(y,t)$ and u(y,t) are therefore obtained as
u(y,t)= $D_5e^{\alpha_5 y} + D_6e^{\alpha_6 y}$ + $D_7e^{\alpha_2+\alpha_1 y} + D_8e^{\alpha_1 \alpha y}$ + $D_9e^{\alpha_2 y}$ + $D_{10}e^{\alpha_1+\alpha_2 y}$ + $\varepsilon\,[D_{13}e^{\alpha_7 y} + D_{14}e^{\alpha_8 y}+D_{15}e^{\alpha_4+\alpha_3 y} + D_{16}e^{\alpha_3+\alpha_4 y}]e^{i\omega t}$
$\theta(y,t)$=$D_1e^{\alpha_1 y} + D_2e^{\alpha_2 y}+\varepsilon\,[D_3e^{\alpha_3 y} + D_4e^{\alpha_4 y}]e^{i\omega t}$ (16)

where the constants are stated accordingly at the appendix

The shear stress and Nusselt number on the wall from the physical stand can be obtained by

$$\tau = \left(\frac{\partial u(y)}{\partial y}\right)_{y=0}$$

$$Nu = -\left(\frac{\partial \theta(y)}{\partial y}\right)_{y=0}$$

IV. RESULTS AND DISCUSSION

The effects of physical parameters such as radiation parameter, heat source parameter, Prandtl number on the temperature profile are such that; the radiation parameter decreased the temperature profile as observed in figure 1. This is physically true because the radiation of heat causes a decrease in the temperature of a substance. The heat source parameter causes a decreament in the temperature profile as noticed in figure 2 which is true since the addition of heat to a substance increases the temperature of the substance. Furthermore, the Prandtl numberreduces the temperature profile as noticed in figure 3.In figure 4, we observed that increase in magnetic parameter causes the velocity of the fluid to decrease this is as a result of Lorentz force in the magnetic field. In figure 5 that increase in the Prandtl number does not give rise to any significant change in the velocity profile. Increase in thermal Grashof number increases the velocity of the fluid as observed in figure 6. This is true since thermal bouyancy increases the boundary layer of the fluid hence a consequential rise in the velocity. Incrementally varying radiation parameter decreases the velocity of the fluid in figure 7 because of a decrease in the momentum boundary layer. Increase in heat source parameter causes the velocity profile to increase as shown in figure 8. Figure 9 shows a decrement in the velocity of the fluid owing to a decrement in the porosity.

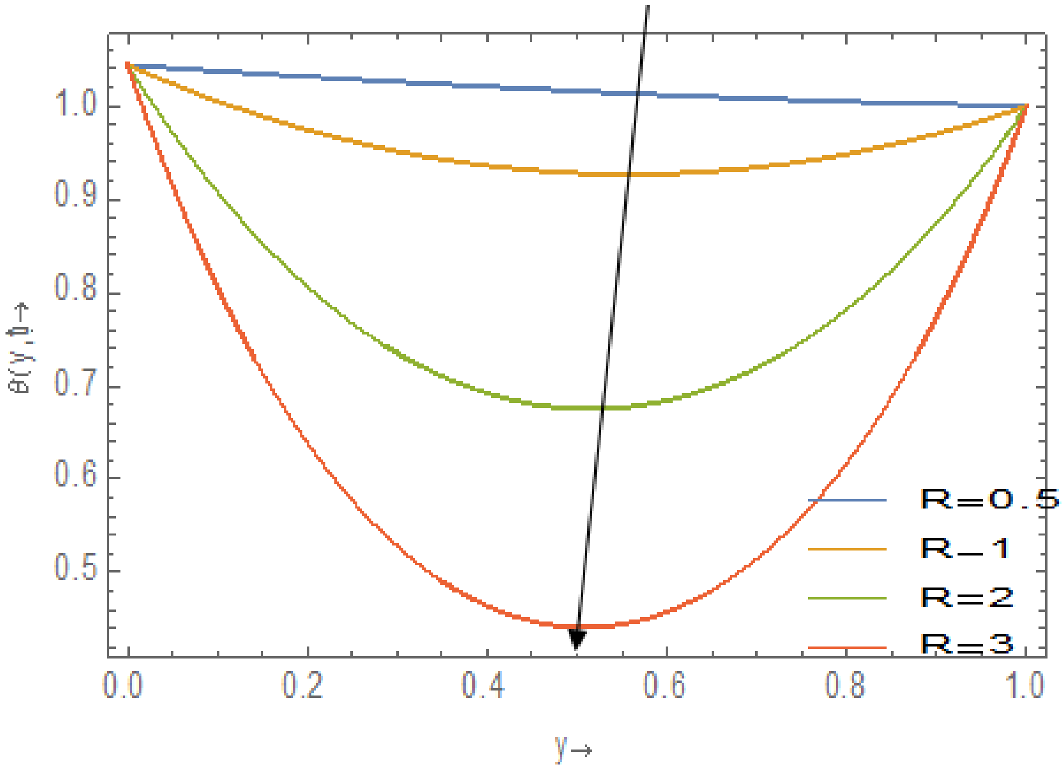

Fig 1:- Variation of R with S=0.2, Pr=0.71,ω=0.5, ϵ=0.05, t=1

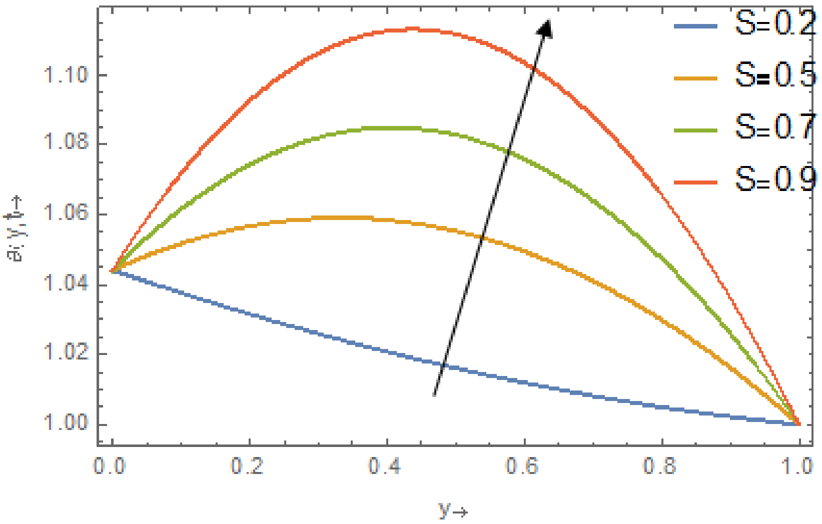

Fig 2:- Variation of S with R=0.5, Pr=0.71,ω=0.5, ϵ=0.05, t=1

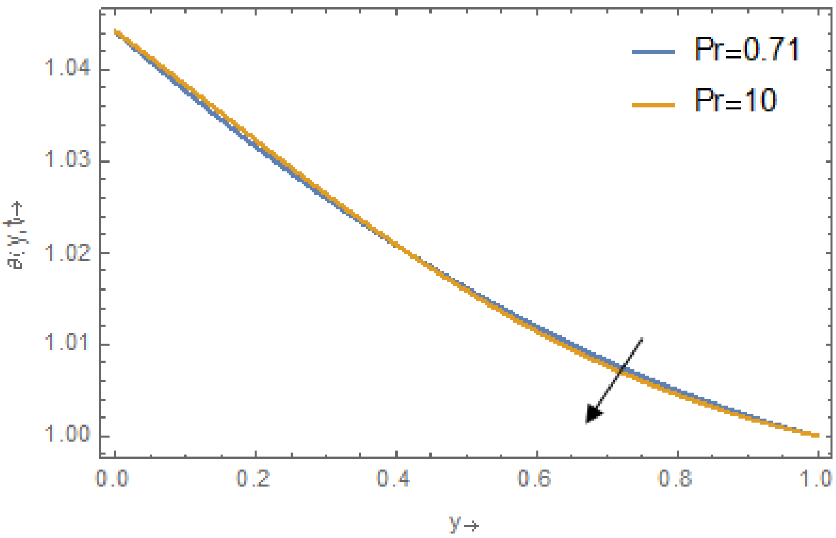

Fig 3:- Variation of Pr with Pr=0.2, R=0.5,ω=0.5, ϵ=0.05, t=1

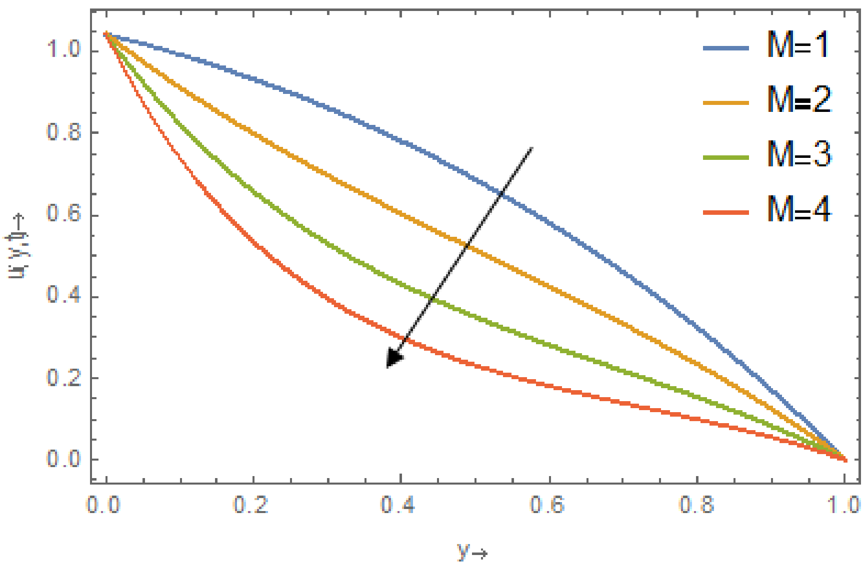

Fig 4:- Variation of M with Pr=0.71, R=0.5, S=0.2, Gr=2, K=0.3,ω=0.5, ϵ=0.05, t=1

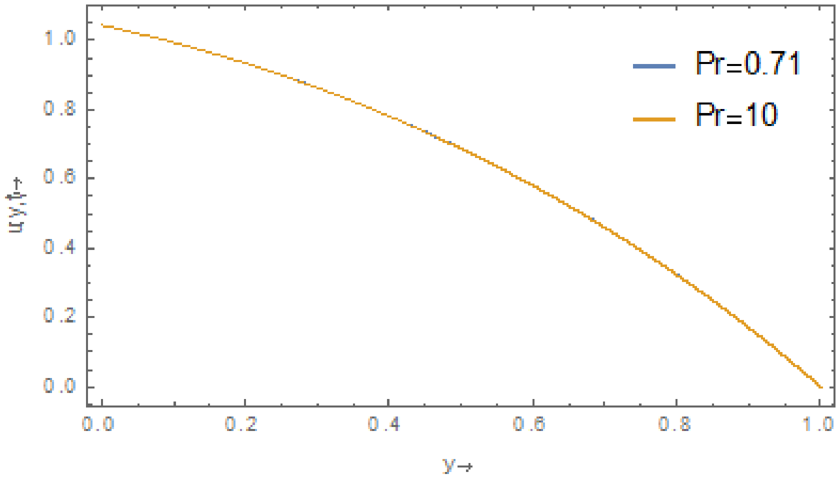

Fig 5:- Variation of Pr with M=1, R=0.5, S=0.2, Gr=2, K=0.3,ω=0.5, ϵ=0.05, t=1

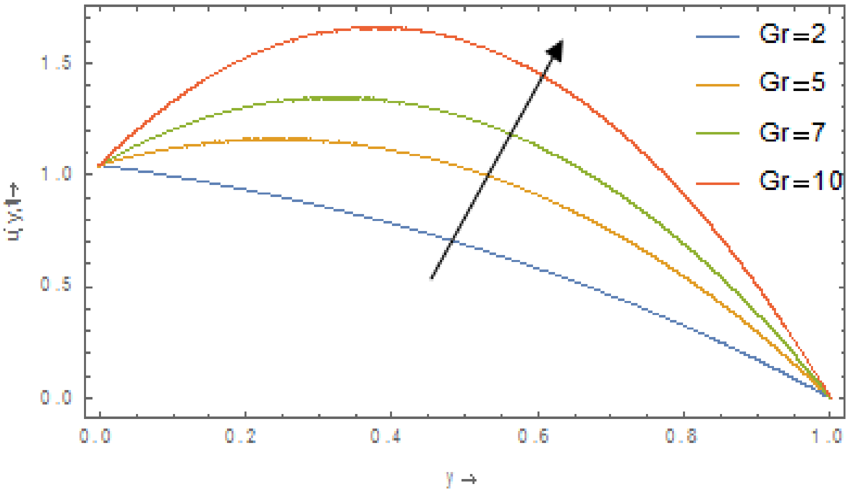

Fig 6:- Variation of Gr with M=1, R=0.5, S=0.2, Pr=0.71, K=0.3,ω=0.5, ϵ=0.05, t=1

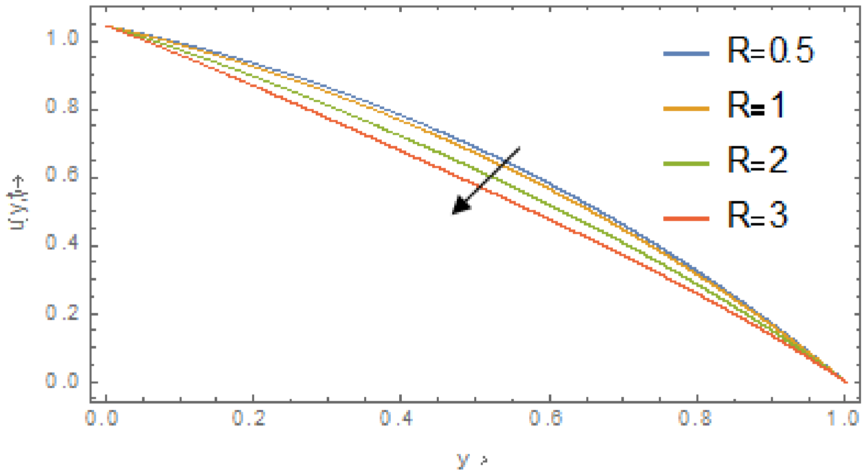

Fig 7:- Effect of R on the velocity when M=1, Pr=0.71, S=0.2, Gr=2, K=0.3,ω=0.5, ϵ=0.05, t=1

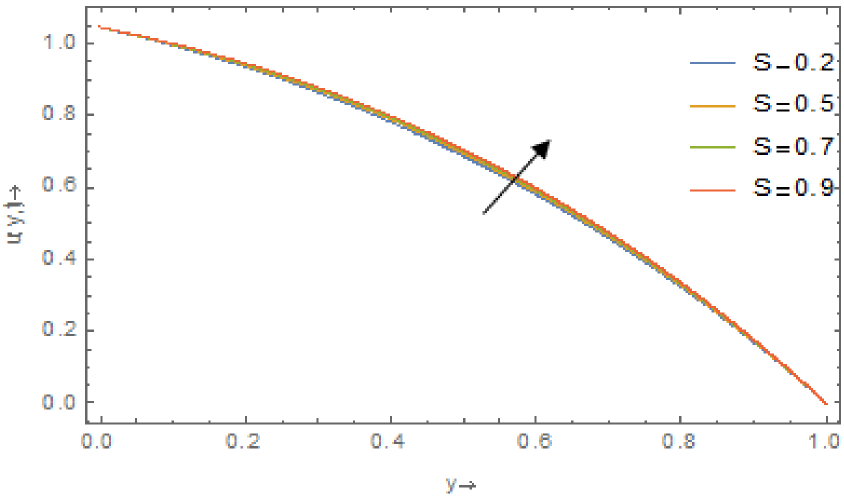

Fig 8:- Variation of S with M=1, R=0.5, S=0.2, Gr=2, K=0.3,ω=0.5, ϵ=0.05, t=1

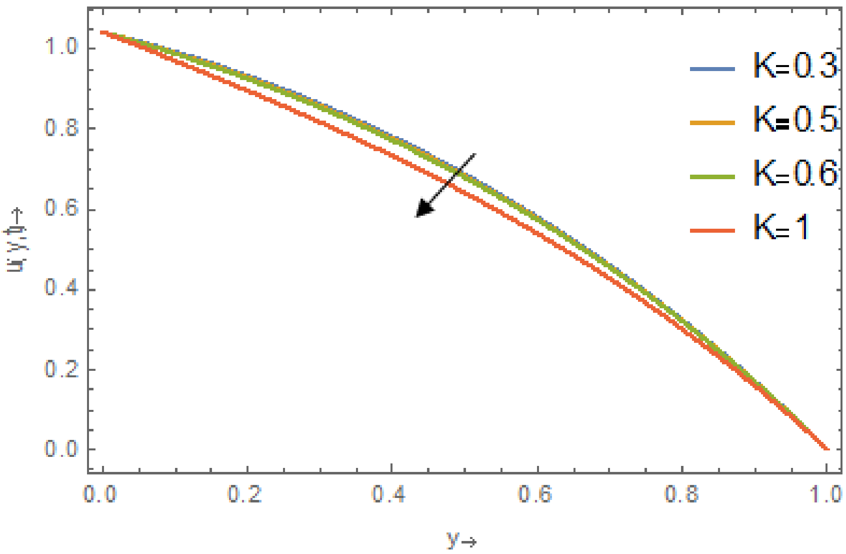

Fig 9:- Variation of K with M=1, R=0.5, S=0.2, Gr=2, Pr=0.71,ω=0.5, ϵ=0.05, t=1

V. CONCLUSION

In this paper, we have analyzed the effect of heat source on MHD free convective flow through a porous medium with periodic wall temperature. From the investigation, the observations made are summarized below:

- Varying magnetic parameter incrementally causes the velocity of the fluid to decrease.
- It can be seen clearly that the heat radiation parameter causes a decrease in the temperature of the fluid.
- Thermal Grashof number being increased caused an increase in the velocity of the fluid.
- The increase in the Prandtl number decreases the temperature of the fluid.
- Heat source parameter variation increases the temperature of the fluid.

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APPENDIX

$\alpha_1 = \sqrt{R^2 - S}$, $\alpha_2 = -\sqrt{R^2 - S}$, $D_1 = \frac{e^{\alpha_2 - 1}}{e^{\alpha_2 - e^{\alpha_1}}}$, $D_2 = \frac{1-e^{\alpha_2}}{e^{\alpha_2 - e^{\alpha_1}}}$, $D_4 = \frac{-e^{\alpha_3}}{e^{\alpha_4 - e^{\alpha_3}}}$, $D_3 = \frac{-e^{\alpha_4}}{e^{\alpha_4 - e^{\alpha_3}}}$,

$\alpha_3 = \sqrt{R^2 - S + i\omega Pr}$, $\alpha_4 = -\sqrt{R^2 - S + i\omega Pr}$, $A_1 = \frac{1}{e^{\alpha_2 - e^{\alpha_1}}}$,$A_2 = \frac{1}{e^{\alpha_6 - e^{\alpha_5}}}$,$A_3 = \frac{1}{e^{\alpha_4 - e^{\alpha_3}}}$,

$A_4 = \frac{1}{e^{\alpha_8 - e^{\alpha_7}}}$, $\alpha_5 = \sqrt{K^2 + M^2}$, $\alpha_6 = -\sqrt{K^2 + M^2}$, $D_{11} = D_7 e^{\propto_2} + D_8 + D_9 + D_{10} e^{\propto_1}$,

$D_{12} = D_7 e^{\propto_2 + \alpha_1} + D_8 e^{\alpha_1} + D_9 e^{\alpha_2} + D_{10} e^{\propto_1 + \alpha_2}$, $\alpha_7 = \sqrt{K^2 + M^2 + i\omega}$,$\alpha_8 = -\sqrt{K^2 + M^2 + i\omega}$,

$D_{17} = D_{15} e^{\propto_4} + D_{16} e^{\propto_3}$, $D_{18} e^{\propto_4 + \alpha_3} + D_{16} e^{\propto_3 + \alpha_4}$, $D_6 = \frac{1}{e^{\alpha_6 - e^{\alpha_5}}}[-e^{\alpha_5} + D_{11} e^{\alpha_5} - D_{12}]$

, $D_5 = 1 - \frac{1}{e^{\alpha_6 - e^{\alpha_5}}}[-e^{\alpha_5} + D_{11} e^{\alpha_5} - D_{12}]$ - $D_{11,} D_{14} = \frac{1}{e^{\alpha_8 - e^{\alpha_7}}}[-e^{\alpha_7} + D_{17} e^{\alpha_7} - D_{18}]$

, $D_5 = 1 - \frac{1}{e^{\alpha_8 - e^{\alpha_7}}}[-e^{\alpha_7} + D_{17} e^{\alpha_7} - D_{18}]$ - D_{17} , $D_7 = \frac{-GrA_1}{\propto_1^2 - (K^2 + M^2)}$, $D_8 = \frac{GrA_1}{\propto_1^2 - (K^2 + M^2)}$,

$D_9 = \frac{-GrA_1}{\propto_2^2 - (K^2 + M^2)}$, $D_{10} = \frac{GrA_1}{\propto_2^2 - (K^2 + M^2)}$, $D_{15} = \frac{-GrA_3}{\propto_3^2 - (K^2 + M^2 + i\omega)}$,$D_{16} = \frac{GrA_3}{\propto_4^2 - (K^2 + M^2 + i\omega)}$