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Development of Low Grade Robotic Arm for Small Industries

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Abstract:- This paper focused on the development of low grade robotic arm for small industries who cannot afford heavy duty and expensive arm made by most companies. Most small factories in regions like Africa are in desperate need of a device to handle repetitive tasks but they cannot meet up with the purchase and maintenance cost of companies like KUKA, Fanuc, etc. so they rely on human labour which is inaccurate and causes body pain for the workers. In this work, a 5 degree of freedom robotic arm was developed to solve this problem.

The link of the robotic arm was 3d printed using poly lactic acid filaments and the ATmega328P AVR microcontroller was used to control the arm, four metal gear servo was used to actuate the arm while one servo was used for the end effector. The critical load for the arm was established after running the stress analysis for each of the links. And forward and inverse kinematic equations was developed to track the position and rotation of the end effector in 3D space. The arm can work in two modes, sentry and user mode. A hard switch was provided to switch from one mode to the other. Sentry mode was designed for repetitive tasks. User mode allows a controller to operate the arm using a joystick to control the joints. Software stress analysis was done using Autodesk inventor and physical evaluations were conducted and the arm performed as expected.

Keywords:- *Robotic Arm, Industrial Robot, Robotics, Forward Kinematics, Inverse Kinematics, Automation.*

I. INTRODUCTION

A robot is an electro-mechanical machine, which is guided by a computer program or electronic circuitry to carry out a variety of physical tasks or actions. There have been many definitions of a robot. According to[1] a robot arm is a device which is to do performs automated task, either according to direct human supervision, pre-defined program, set of general guideline, using (artificial intelligence) techniques.

Robots have found their way into most aspects of life such as from manufacturing to our various homes. In the past few decades the robotic arm industry has experienced exponential growth, this has made it possible for vast research and development to occur [2].

Robotic arms have been around for a long time, they are used in industries for tasks such as welding, assembly lines, material handling etc. and they serve their purpose effectively. As pointed out in the study of [3], the use of the industrial robot, which became identifiable as a unique device in the 1960s along with computer-aided design (CAD) systems and computer-aided manufacturing (CAM) systems, characterizes the latest trends in the automation of the manufacturing process.

The idea to use multiple end effectors have been around for some years now. A specific arm can be well equipped and designed with all types of end effectors which are fitted for particular application. [4].

In this paper, a 5 degree of freedom robotic arm was developed. These type of robots are used in factories for tedious repetitive tasks that leads to body pain and deformation when performed by a human.

The design process taken include design stage, construction and testing. The mechanical and electrical design of the arm allows for different end effectors to be attached to perform various tasks. The robot was actuated using two types of servo motors, the MG996r metal gear servo and the SG90 9g servo. The ATmega328P was used as the Micro control unit(MCU) and it was powered using rechargeable batteries. The links of the robotic arm 3D printed using Poly lactic acid (PLA) and other materials such as wood and aluminium were employed to complete the build. The arm has a base which rotates 180 degrees around the z-axes and two links joined by a rotary joint. Only one end effector was used in the test but others can be attached depending on the task at hand.

II. MATERIALS AND METHODS

The procedures employed include the design stage, construction and testing. To track the position of the end effector in 3D-space, it was necessary to device kinematic equations to do this.

2.1 Forward Kinematic Analysis

To get the position of the end effector after rotating the joints, a forward kinematics equation was derived. Joint rotations were gotten by reading the initial and final position of the servo motors. This angles were then injected into a forward kinematics equation which was used to know both the position and rotation of the tool center position(TCP).

Figure 2.0 and table 2.0 shows translations revealed by Geometric analysis of the robotic arm.

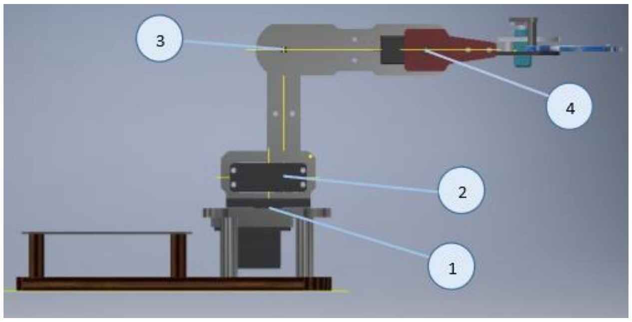

FIGURE 2.0
JOINTS IN THE ROBOTIC ARM

TABLE I

az1	ax2	ay2	az2	az3	ay3	ax4	ay4
59	11	15.5	21	90	10	100	9.5

TRANSLATIONS ON THE ROBOTIC ARM

Where az1 is the translation along the z axis between the base and joint 1
ax2 is the translation along the x axis between the joint1 and joint2
ay2 is the translation along the y axis between the joint1 and joint2
az2 is the translation along the z axis between the joint1 and joint2
ax3 is the translation along the x axis between the joint2 and joint3
az3 is the translation along the z axis between the joint2 and joint3
ay3 is the translation along the y axis between the joint2 and joint3
ax4 is the translation along the x axis between the joint3 and joint4
ay4 is the translation along the y axis between the joint3 and joint4
Combined rotation and translation was considered and the base of the arm served as the local coordinate system. Rotation matrix around each axes is given by (1), (2) and (3);

$$R_x(\theta) = \begin{matrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{matrix} \quad (1)$$

$$R_y(\theta) = \begin{matrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{matrix} \quad (2)$$

$$R_z(\theta) = \begin{matrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{matrix} \quad (3)$$

Where subscripts x, y and z are the axes around which the rotation occurs, and θ is the angle of rotation.

A homogenous transformation matrix 'T' was created by combining both rotation and translation.

$$T = \begin{bmatrix} R & \Delta \\ 0 & 1 \end{bmatrix} \quad (4)$$

Where;

$$\text{R = rotation matrix} = \begin{matrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{matrix} \qquad (5)$$

And

$$\text{Translation} = \Delta = \begin{cases} \Delta x \\ \Delta y \\ \Delta z \end{cases} \qquad (6)$$

Hence,

$$T = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ 1 \end{bmatrix} \qquad (7)$$

$$T = \begin{bmatrix} R_{11} & R_{12} & R_{13} & \Delta x \\ R_{21} & R_{22} & R_{23} & \Delta y \\ R_{31} & R_{32} & R_{33} & \Delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (8)$$

The combined rotation and translation matrix was gotten by moving from joint to joint.

From the base to joint 1(J1), translation only occurs in the z-axes and rotation is around the z-axes. The translation is denoted by az1. The matrix is shown in (9) and (10).

$$T_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & az1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (9)$$

$$T_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 59 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (10)$$

From J1 to J2, translation occurs in all 3 axes and rotation is around the y-axes. The translations are denoted by ax2, az2, ay2. The matrix is shown in (11).

$$T_2 = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 & 11 \\ 0 & 1 & 0 & 15.5 \\ -\sin\theta_2 & 0 & \cos\theta_2 & 21 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (11)$$

From J2 to J3, translation occurs in 3 axes and rotation is around the y-axes. The translations are denoted by az3, ay3. The matrix is shown in (12).

$$T_3 = \begin{bmatrix} \cos\theta_3 & 0 & \sin\theta_3 & 0 \\ 0 & 1 & 0 & 10 \\ -\sin\theta_3 & 0 & \cos\theta_3 & 90 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (12)$$

From J3 to J4, translation occurs in the x and y axes and rotation is around the y-axes. The translations are denoted by ax4, az4. The matrix is shown in (13).

$$T_4 = \begin{bmatrix} \cos\theta_4 & 0 & \sin\theta_4 & 100 \\ 0 & 1 & 0 & 9.5 \\ -\sin\theta_4 & 0 & \cos\theta_4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (13)$$

The combined rotation matrix for the arm is gotten by evaluating T_c

$$T_c = T_1 * T_2 * T_3 * T_4 \quad (14)$$

$$\text{End effector position} = T_c * \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \quad (15)$$

Where [X, Y, Z] defines the position of the base
(16) was used to predict the position of the end effector.

2.2 Inverse Kinematics Analysis

The inverse kinematic equation was used to get the angle of rotation in each joint needed to get to a desired position with the end effector. To get to a position X, Y, Z with the end effector, the required joint angles for joint 1, 2 and 3 was gotten using (16), (17) and (18).

$$J3 = 180 - arcCos\left(\frac{az3^2 + (az3 + ax4)^2 - X^2 - Z^2}{2 * (az3 + ax4) * az3}\right) \quad (16)$$

$$J2 = arcTan\left(\frac{Z}{X}\right) + arcCos\left(\frac{az3^2 - (az3 + ax4)^2 + X^2 + Z^2}{2 * az3 * \sqrt{X^2 + Y^2}}\right) \quad (17)$$

$$J1 = arcTan\left(\frac{Y}{X}\right) \quad (18)$$

Where J1 is the angle for joint 1
J2 is the angle for joint 2,
J3 is the angle for joint 3,
X is the desired position in the x axis
Y is the desired position in the y axis
Z is the desired position in the x axis
Figure 2.1 Shows the free body diagram of the arm.

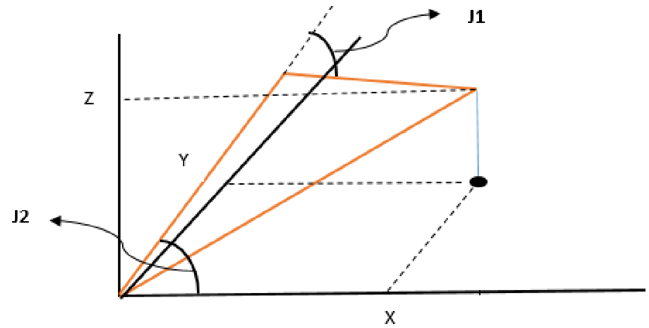

FIGURE 2.1
FREE BODY DIAGRAM OF THE ARM

2.3 Stress Analysis of Links

Deformation could only be prevented by not overloading the robotic arm, so it was necessary to determine the maximum load the arm could lift. The following design assumptions guided the design process;

- It was assumed that all bolted joints are completely rigid.
- It was assumed that the material for the linkages obey hooks law
- Maximum permissible slope throughout the arm was 0.5^0 (0.0087 radian)
- Maximum allowable deflection in the arm was 0.5mm

2.3.1 Link (3) (stress analysis)

It was necessary to obtain the centroid of link3 because that's where the self weight of the arm acts. Figure 2.2 was used to obtain the centroid.

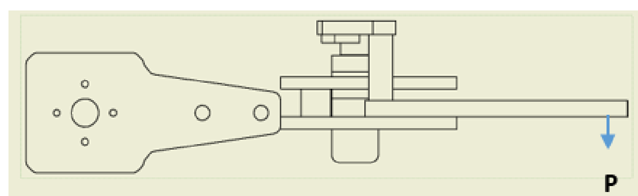

FIGURE 2.2
LINK 3 (SECTIONING)

$$A1\,X1 = 900\,X\,15 = 13500mm3$$
$$A2\,x2 = 135\,x\,87.5 = 11812.5cm3$$
$$A3\,x3 = 135\,x\,87.5 = 11812.5mm3$$
$$A4\,x\,x4 = 52.5\,x\,73.75 = 3871.875mm3$$
$$A5\,x5 = 268\,x\,119.5 = 32026mm3$$
$$A6\,x6 = 490\,x\,475 = 23275mm3$$
$$\overline{X} = \frac{A1\,X1 + A2X2 + A3\,X3 + A4\,X4 + A5\,X5 + A6\,X6}{A1 + A2 + A3 + A4 + A5 + A6}$$
$$\overline{X} = \frac{96297.875}{1980.5} = 48.62mm$$

The load (P) lifted by the arm acted at the tip of the end effector. The load diagram and free body diagram is shown in Figure 2.3 and Figure 2.4 respectively

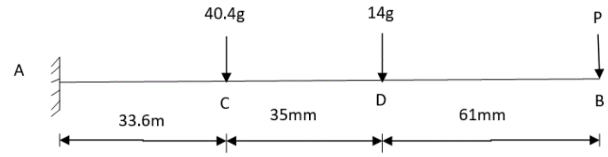

FIGURE 2.3
LOAD DIAGRAM FOR LINK 3

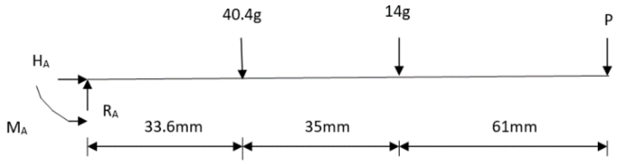

FIGURE 2.4
FREE BODY DIAGRAM FOR LINK 3

Equilibrium condition. Summation of vertical forces must be equal to zero;

$R_A = 40.4 + 14 + P$

$R_A = (54.4 + P)g \quad (\sum fv = 0) \qquad (19)$

Equilibrium condition. Summation of horizontal forces must be equal to zero;

$H_A = 0 \quad (\sum FH = 0) \qquad (20)$

Summing moments about point A yielded (21).

$M_A = (2317.84 + 129.6P)gmm \quad (\sum M = 0) \qquad (21)$

The moment M_A opposed the torque produced by the actuator. So it was necessary the actuator could produce enough torque to overcome this moment. The MG996r metal gear servo produced a maximum stall torque (T_S) of 11kgcm. [5]

Stall torque (T_S) = 11 x 1000 x 10gmm (converting to gram millimeter)

$T_S = 110000gmm$

Performance factor (F_p) was introduced to compensate for any flaws in the servo motor.

$$F_p = 0.8$$
$$T'_S = Ts\,x\,F_p$$

$T'_S = 110000\,x\,0.8$

$T'_S = 88000gmm$

Equilibrium condition is shown in (22).

$Torque\ produced\ by\ servo\ Motor = M_A \qquad (22)$

From (19) and (21);

$88000 = 2317.84 + 129.6P$

$$P = \frac{88000 - 2317.84}{129.6\text{mm}}(gmm)$$

$\boldsymbol{P = 661.13g}$

For the actuator to effectively lift the load. P must not exceed 660g.

To analyse the effect of deformations on the link, the free body diagram (FBD) in Newton is shown in Figure 2.5.

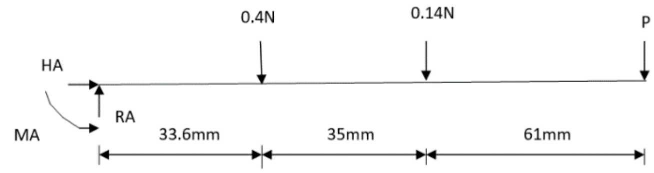

FIGURE 2.5
FREE BODY DIAGRAM IN NEWTON (LINK 3)

Bending moment equation using Macaulay principle. [6] was derived as shown in (23);

$$M_x = R_A x - M_A - 0.4\langle x - 33.6\rangle - 0.14\langle x - 68.6\rangle \qquad (23)$$

reaction at A is gotten from (19)

$R_A = 0.4 + 0.14 + P$

$R_A = (0.54 + P)\,N$

$M_A = (23.044 + 129.6P)Nmm$

From Euler's equation. [6]

$$\frac{EId^2y}{dx^2} = -M_x \qquad (24)$$

Where E is the elastic modulus of the material in use

I is the moment of inertia of the cross-section about the bending axis

y is the deflection along the beam

x is any point of interest along the beam

While M_x is the bending moment.

Double integration of (23) yields (25) and (26) which are slope and deflection equation respectively.

$EIy'' = -M_x$

$EIy'' = M_A + 0.4\langle x - 33.6\rangle + 0.14\langle x - 68.6\rangle - R_A x$

$$EIy' = M_A x + 0.2\langle x - 33.6\rangle^2 + 0.07\langle x - 68.6\rangle^2 - 0.5R_A x^2 + C1 \qquad (25)$$

$$EIy = 0.5M_A x^2 + 0.0667\langle x - 33.6\rangle^3 + 0.0233\langle x - 68.6\rangle^3 - 0.1667R_A x^3 + C1x + C2 \qquad (26)$$

Boundary Condition

$Deflection\ at\ point\ A\ (x = 0) = 0 \qquad (a)$

$Slope\ at\ point\ A\ (x = 0) = 0 \qquad (b)$

Applying condition (a) to (25);

$$E1(0) = 0.5M_A(0)^2 - 0.1667R_A(0)^3 + C1(0) + C2$$
$$C2 = 0$$

Applying condition (b) to (24);

$$E1(0) = M_A(0) - 0.5R_A(0)^2 + C1$$
$$C1 = 0$$

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Haven obtained the value of the integration constants, deflection and slope equations are written in (27) and (28) respectively;

Slope equation

$$E1\theta = M_A x + 0.2\langle x - 33.6\rangle^2 + 0.07\langle x - 68.6\rangle^2 - 0.5\, R_A x^2 \quad (27)$$

Deflection equation

$$E1y = 0.5M_A x^2 + 0.0667\langle x - 33.6\rangle^3 + 0.0233\langle x - 68.6\rangle^3 - 0.16667\, R_A x^3 \quad (28)$$

In-depth analysis of (27) and (28) revealed that slope and deflection are maximum the end of the beam. i.e. at point L.

Where L is the total length of the link

In this case $L = 129.6mm$

Elastic modules for Poly Lactic Acid is 3.5Gpa. [7].

$$E = 3.5\, x\, 10^9\, N/m2$$

$$E = 3.5\, x 10^9 \frac{N}{10^6} mm^2$$

$$E = 3.5\, x\, 10^3\ \ N/mm2$$

Moment of inertia for minimum cross section is gotten from Figure 2.6 as shown below;

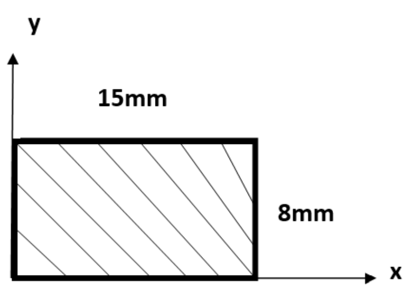

FIGURE 2.6
CROSS SECTION FOR LINK 3

$$Ixx = \frac{bd^3}{12} = \frac{15\,(8)^3}{12} = 640mm^4$$

$$EI = 3.5\, x\, 10^3\, x\, 640\, \left(\frac{N}{mm^2}\, x\, mm^4\right)$$

$$EI = 2.24\, x\, 10^6 Nmm^2$$

Substituting L into (27) gave (29)

$$EI\theta_L = M_A(129.6) + 0.2\,(129.6 - 33.6)^2 + 0.07\,(129.6 - 68.6)^2 - 0.5R_A(029.6)^2 \quad (29)$$

$$EI\theta_L = 129.6M_A + 2103.67 - 8398.1R_A (30)$$

$R_A$andM_A was substituted into (29) to give (30) as shown below;

$$EI\theta_L = 129.6\,(23.04 + 129.6P) + 2103.67 - 8398.1\,(0.54 + P) \quad (31)$$

$$EI\theta_L = 555.2 + 8393.1P \quad (32)$$

From design assumptions, $\theta_L = 0.00873\, radian$

$$2.24\, x\, 10^6\, x\, 0.00873 = 555.2 + 8393.1P$$

$$8393.1P = 19000$$

$$P = 2.264N$$

$$\boldsymbol{P = 226.38g}$$

Hence, if the maximum allowable slope in link 3 is to be limited to 0.5 degrees, the load must not exceed 225g.

Substituting L = 129.6mm into (28) gave (33);

$$E1y = 0.5M_A(129.6)^2 + 0.0667\,(129.6 - 33.6)^3 + 0.0233(129.6 - 68.6)^3 - 0.1667R_A(129.6)^3 \quad (33)$$

$$E1y = 64300.55 + 8398.1M_A - 362804.3R_A \quad (34)$$

Ra and Ma was substituted into (34) to form (35) as shown below;

$$E1y = 64300.55 + 8398.1\,(23.044 + 129.6P) - 362804.3\,(0.54 + P) \quad (35)$$

I and E are already known and y is known from the design assumptions as 0.5mm. hence the value of P was gotten as shown below;

$$P = \frac{(1.12\, x\, 10^6) - 61912.048}{725589.46}$$

$$P = 1.458N$$

$$\boldsymbol{P = 145.82g}$$

The maximum allowable load given by the actuator is 660g. And that for slope and deflection was 225g and 145g respectively. We will choose the lowest value obtained. So for link (3) not to fail under loading, the maximum value of P was taken as $145g$.

2.3.2 Link (2) (stress Analysis)

The centroid of link 2 is obtained from Figure2.7 as shown below;

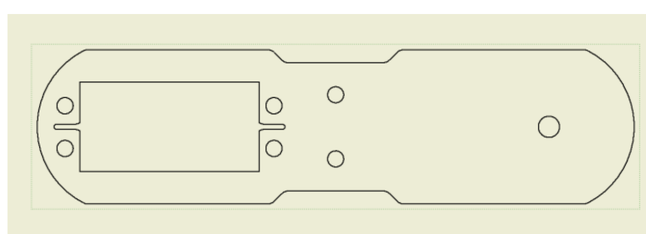

FIGURE 2.7
LINK 2 (SECTIONING)

$$A1\, X1 = 307.9 * 8 = 2463.2mm3$$

$$A2X2 = 1620 * 34.5 = 55890mm3$$

$$A3\, X3 = 660 * 68 = 44880mm3$$

$$A4\, X4 = 1620 * 101.5 = 164430mm3$$

$$A6\ X6 = 307.9 * 128 = 39411.2mm3$$

$$X = \frac{\sum AX}{\sum A} = \frac{403212.4}{5397.8} = \frac{307074.4}{4515.8}$$

$\overline{X} = 68mm$

The load diagram and free body diagram is shown in Figure 2.8 and Figure2.9 respectively;

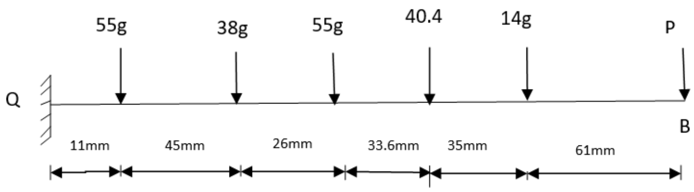

FIGURE 2.8
LOAD DIAGRAM FOR LINK 2

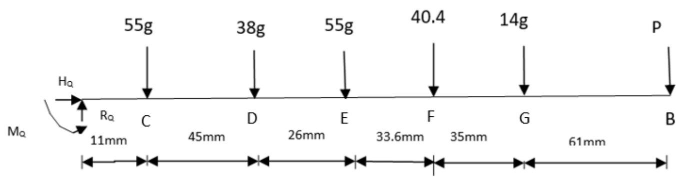

FIGURE 2.9
LOAD DIAGRAM FOR LINK 2

Equilibrium condition. Summation of vertical forces must be equal to zero;

$$\sum fv = 0$$
$$R_Q = 55 + 33 + 55 + 40.4 + 14 + P$$
$$R_Q = (202.4 + P)g \quad (36)$$
$$R_Q = (2.024 + P)N \quad (37)$$

Equilibrium condition. Summation of horizontal forces must be equal to zero;

$$\sum FH = 0$$
$$H_Q = 0 \quad (38)$$

Summing moments about point Q yields (39).

$$\sum M_Q = 0$$
$$M_Q = 55(11) + 38(56) + 55(82) + 40.4(115.6) + 14(150.6) + 211.6P \quad (39)$$
$$M_Q = (14021.64 + 211.6P)gmm \quad (40)$$
$$M_Q = (140.22 + 211.6P)Nmm \quad (41)$$

The moment M_Q opposed the torque produced by the actuator. For this joint to function, it was necessary the actuator could produce enough torque to overcome M_Q.

Actuator (MG996r servo) Stall torque (T_S) = 11 x 1000 x 10gmm (converting to gram millimeter)

$$T_S = 110000gmm$$
$$T'_S = Ts\,x\,F_p$$
$$T'_S = 110000\,x\,0.8$$
$$T'_S = 88000gmm$$

Equilibrium condition is shown in (42).

$$Torque\ produced\ by\ servo\ Motor = M_Q \quad (42)$$
$$8800 = T's = 14021.64 + 211.6P$$
$$P = \frac{88000 - 14021.64}{211.6}$$
$$\boldsymbol{P = 349.6g}$$

For the actuator to function effectively. P must not exceed 349g.

Bending moment equation using Macaulay principle.
was derived as shown in (43)

$$M_x = R_Q x - 0.55\langle x - 11\rangle - 0.38\langle x - 56\rangle - 0.55\langle x - 82\rangle - 0.404\langle x - 115.6\rangle - M_Q - 0.14\langle x - 50.6\rangle \quad (43)$$

Applying Euler's equation.

$$E1y'' = -M_x \quad (44)$$
$$E1y'' = 0.55\langle x - 11\rangle + 0.38\langle x - 56\rangle + 0.55\langle x - 82\rangle + 0.4\langle x - 115.6\rangle + 0.14\langle x - 150.6\rangle + M_Q - R_Q x \quad (45)$$

Double integration of (43) yields (46) and (47)

$$E1y' = 0.275\langle x - 11\rangle^2 + 0.19\langle x - 56\rangle^2 + 0.275\langle x - 82\rangle^2 + 0.2\langle x - 115.6\rangle^2 + 0.07\langle x - 180.6\rangle^2 + M_Q x - 0.5R_Q x^2 + C1 \quad (46)$$
$$EIy = 0.092\langle x - 11\rangle^3 + 0.0633\langle x - 56\rangle^3 + 0.092\langle x - 82\rangle^3 + 0.0667\langle x - 115.63\rangle^3 + 0.0233\langle x - 82\rangle^3 + 0.0667\langle x - 115.6\rangle^3 + 0.0233\langle x - 150.6\rangle^3 + 0.5M_Q x^2 - 0.1667R_Q x^3 + C1x + C2 \quad (47)$$

Boundary Condition

$$Deflection\ at\ point\ Q\ (x = 0) = 0 \quad (a)$$
$$Slope\ at\ point\ Q\ (x = 0) = 0 \quad (b)$$

Applying condition (a) and (b) to (46) and (47) respectively gave
$C_1 = 0$, $C_2 = 0$.

Slope and deflection equations are written in (48) and (49) respectively;
Slope equation

$$E1\theta = 0.275\langle x - 11\rangle^2 + 0.19\langle x - 56\rangle^2 + 0.275\langle x - 82\rangle^2 + 0.2\langle x - 115.6\rangle^2 + 0.07\langle x - 150.6\rangle^2 + M_Q x - 0.5R_Q x^2 \quad (48)$$

Deflection equation

$$E1y = 0.092\,\langle x - 11\rangle^3 + 0.0633\,\langle x - 56\rangle^3 + 0.092\langle x - 82\rangle^3 + 0.0667\langle x - 115.6\rangle^3 + 0.0233\,\langle x - 150.6\,\rangle^3 + 0.5M_Q x^2 - 0.1667R_Q x^3 \quad (49)$$

Moment of inertia of minimum cross section in link 2 is gotten from Figure 2.10 as shown below;

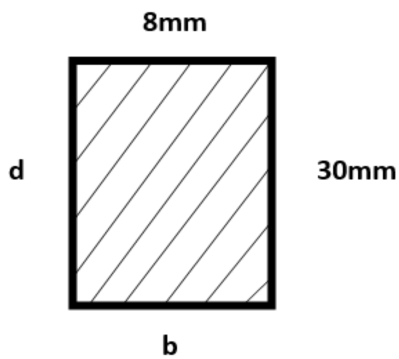

FIGURE 2.10
CROSS SECTION FOR LINK 2

$$I_{xx} = \frac{bd^3}{12} = \frac{8(30)^3}{12} = 18000mm^4$$
$$EI = 3.5\,x\,10^3\,x\,18000 = 63\,x\,10^6 N/mm^2$$

The point of minimum cross section and maximum deformation on link 2 was at;

$$x = 110mm$$

Substituting $x = 110mm$ into (48) gave (50)

$$EI\theta = 0.275\,(110 - 11)^2 + 0.19\,(110.56)^2 + 0.275\,(110 - 82)^2 + M_Q(110) - 0.5R_Q(110)^2 \quad (50)$$
$$549778.7 = 3464.915 + 110M_Q - 6050R_Q \quad (51)$$

R_Q and M_Q were substituted into (51) to give (52) as shown below;

$$46313.785 = 110\,(140.22 + 211.6P) - 6050\,(2 + P) \quad (52)$$
$$542989.585 = 17016P \quad (53)$$
$$P = 31.52N$$
$$\boldsymbol{P = 3152g}$$

Hence, if the maximum allowable slope in link 2 was to be limited to 0.5 degrees, the load must not exceed 3152g.

Substituting x = 110mm into (49) gave (53)

$$EIy = 0.092\,(110 - 11)^3 + 0.0633\,(110 - 56)^3 + 0.092\,(110 - 82)^3 + 0.5\,M_Q(110)^2 - 0.1667R_Q(110)^3 (53)$$
$$31500000 = 101254.56 + 6050R_Q - 221837.77R_Q \quad (55)$$
$$31398745.44 = 6050(140.22 + 211.6P) - 221837.77(2 + P) \quad (56)$$

$$P = 29.286N$$
$$\boldsymbol{P = 2928.6g}$$

The maximum allowable load given by the actuator is 349.6g. And that for slope and deflection was 3152g and 2928g respectively. We will choose the lowest value obtained. So for link (3) not to fail under loading, the maximum value of P was taken as349g.

2.3.3 Link 1 (Stress Analysis)

The centroid of link1 was obtained as $X = \overline{45}mm$ from Figure2.11.

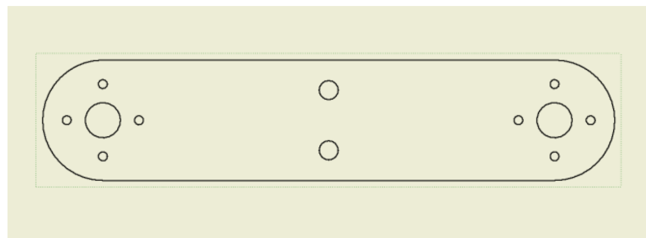

FIGURE 2.11
LINK 3 (SECTIONING)

The load diagram and free body diagram is shown in Figure 2.12 and Figure2.13 respectively

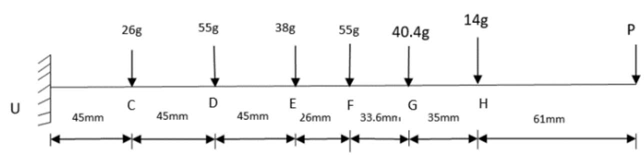

FIGURE 2.12
LOAD DIAGRAM FOR LINK 3

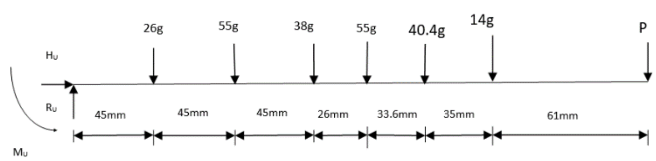

FIGURE 2.13
FREE BODY DIAGRAM FOR LINK 3

Equilibrium condition. Summation of vertical forces must be equal to zero;

$$\sum fv = 0$$
$$R_U = 26 + 55 + 35 + 55 + 40.4 + 14 + P \quad (57)$$
$$R_U = (228.4 + P)g \quad (58)$$
$$R_U = (2.284 + P)N \quad (59)$$

Equilibrium condition. Summation of horizontal forces must be equal to zero;

$$\sum fH = 0$$
$$H_U = 0$$

Summing moments about any point U yielded (60)

$$M_U = 26\,(45) + 55\,(90) + 38\,(135 + 55\,(161) + 40.4\,(194.6) + 14\,(229.6) + P\,(290.6) \quad (60)$$

$$M_U = (31181.24 + 290.6P)gmm \quad (61)$$

$$M_U = (311.8 + 290.6P)Nmm \quad (62)$$

Stall torque of MG996r servo motor (T_S) = 11 x 1000 x 10gmm (converting to gram millimeter)

$$T_S = 110000gmm$$

There are two servos in the joint, hence;

$$T_S = 2 * 110000gmm$$

$$T_S = 220000gmm$$

Performance factor (F_p) was introduced to compensate for any flaws in the servo motor.

$$F_p = 0.8$$

$$T'_S = Ts\; x\; F_p$$

$$T'_S = 220000\; x\; 0.8$$

$$T'_S = 176000gmm$$

Equilibrium condition;

$$Torque\ produced\ by\ servo\ Motor = M_U$$

$$176000 = 31181.24 + 290.6P \quad (63)$$

$$\boldsymbol{P = 498.344g}$$

For the actuator to effectively lift the arm. P must not exceed 498g.

Slope and deflection equation are derived as follows. Bending moment equation using Macaulay principle. [6] was derived as shown in (64);

$$M_U = R_U X - 0.26\,\langle x-45\rangle - 0.55\,\langle x-90\rangle - 0.38\,\langle x-135\rangle - 0.55\,\langle x - 161\rangle - 0.40\,\langle x-194.6\rangle - 0.14\langle\, x - 229.6\rangle - M_U \quad (64)$$

Introducing Euler's equation. [6]

$$EIy'' = -M_U (65)$$

$$EIy'' = 0.26\,\langle x-45\rangle + 0.55\,\langle x-90\rangle + 0.38\,\langle x-185\rangle + 0.55\,\langle x-161\rangle + 0.40.4\,\langle x-194.6\rangle + 0.14\,\langle x-229.6\rangle + M_U - R_U x \quad (66)$$

Double integration of (66) yields (67) and (68) which are our slope and deflection equations respectively.

$$EIy' = 0.13\,\langle x-45\,\rangle^2 + 0.275\,\langle x-90\,\rangle^2 + 0.19\,\langle x-135\rangle^2 + 0.275\langle x-161\,\rangle^2 + 0.20\langle\, x-194.6\,\rangle^2 + 0.7\langle x-229.6\,\rangle^2 + M_U x - 0.5R_U x^2 + C1 \quad (67)$$

$$EIy = 0.0433\,\langle x-45\rangle^3 + 0.09167\,\langle x-90\rangle^3 + 0.0633\,\langle x-135\rangle^3 + 0.09167\,\langle x-161\rangle^3 + 0.0667\,\langle x-135\rangle^3 + 0.233\langle x-194.6\rangle^3 + 0.233\,\langle x-229.6\,\rangle^3 + 0.5\,M_U x^2 - 0.1667R_U x^3 + C1x + C2 \quad (68)$$

Boundary Condition

$$Deflection\ at\ point\ U\ (x=0) = 0 \quad (a)$$

$$Slope\ at\ point\ U\ (x=0) = 0 \quad (b)$$

For link 1, maximum deformation and minimum cross section area occur at

$$x = 90mm$$

Moment of inertia for minimum cross section is gotten from Figure2.14 as shown below;

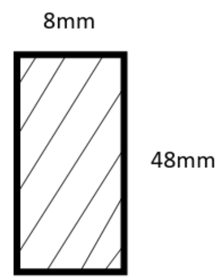

FIGURE 2.14
CROSS SECTION FOR LINK 3

$$I_{xx} = \frac{bd^3}{12} = \frac{8(48)^3}{12}$$

$$I_{xx} = 73728mm^4$$

$$EI = 3.8\,x\,10^3\,x\,73728$$

$$EI = 258\,x\,10^6 N/mm^2$$

Substituting $x = 90mm$into (67) gave (69);

$$EI\theta = 0.13\,(90-45)^2 + M_U(90) - 0.5R_U(90)^2 \quad (69)$$

$$2.25\,x\,10^6 = 263.25 + 90M_U - 4050R_U \quad (70)$$

$R_U$and M_U was substituted into (70) to give (71) as shown below;

$$2249736.75 = 90\,(311.8 + 290.6P) - 4050\,(2.284 + P) \quad (71)$$

$$P = 100.93N$$

$$\boldsymbol{P = 10092.9g}$$

Hence, if the maximum allowable slope in link 1 was to be limited to 0.5 degrees, the load must not exceed 10092 g.

Substituting $x = 90mm$ into (68) gave (72);

$$EIy = 0.0433\,(90-45)^3 + 0.5M_U(90)^2 - 0.1667R_U(90)^3 \quad (72)$$

$$129\,x\,10^6 = 3945.7 + 4050M_U - 121502.43R_U \quad (73)$$

$R_U$and M_U was substituted into (73) to give (74) as shown below;

$$128996004.3 = 4050\,(311.8 + 290.6P) - 121502.43\,(2.284 + P)(74)$$

$$P = 121.29N$$

$$\boldsymbol{P = 12128.8g}$$

Hence, if the maximum allowable deflection in link 1 was to be limited to 0.5 mm, the load must not exceed 12128 g. The maximum allowable load given by the actuator is 498g. And that for slope and deflection was 10092g and 12128g respectively. We will choose the lowest value obtained. So for link (3) not to fail under loading, the maximum value of P was taken as $498g$.

Table 2.2 was extrapolated from the stress analysis above

TABLE 2.2

Link	Critical value for P (g)
1	498.3
2	349.6
3	145.8

CRITICAL VALUES FOR P IN EACH LINK

Hence maximum load permissible for the robotic arm was taken as 146g.

2.3.4 END EFFECTOR GRIPPER ANALYSIS

It was paramount to obtain the maximum load the gripper could hold. If this load was exceeded, the gripper wont be able to handle the load. The end effector is shown in Figure2.15

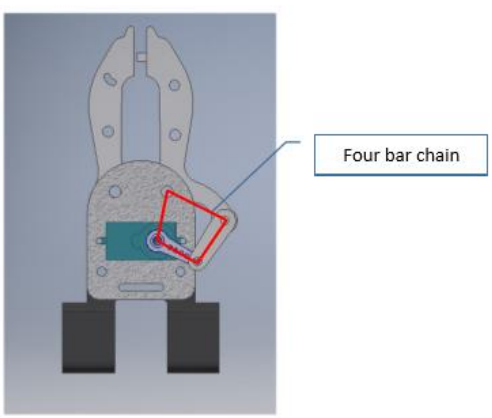

FIGURE 2.15
END EFFECTOR

The mechanism of the griper is basically a four bar chain. Free body diagram is shown below in Figure 2.16:

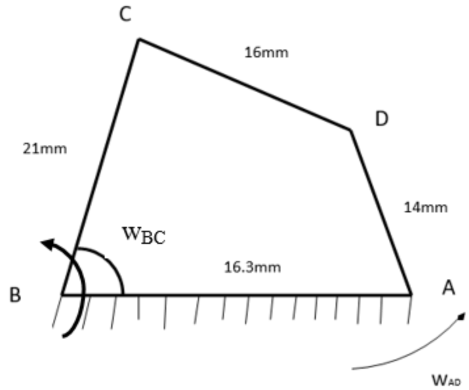

FIGURE 2.16
GRIPPER MECHANISM (FOUR BAR CHAIN)

Where w_{BC} is the angular velocity at B
w_{AD} is the angular velocity at A
F_G is the force which the gripper exerts on the load

In other to get F_G, it was necessary to get w_{BC}. w_{AD} is the angular velocity of the servo motor and was gotten from the datasheet as ($w_{AD} = 9.52\,rad/sec$)

$$Velocity\ @\ D\ (V_D) = w_{AD}\ x\ AD \quad (75)$$

$$= \frac{9.52\ x\ 14}{1000}$$

$$= 0.1333m/s$$

The velocity diagram of the mechanism is shown in Figure2.17

Scale: 10mm represent 0.05m/s

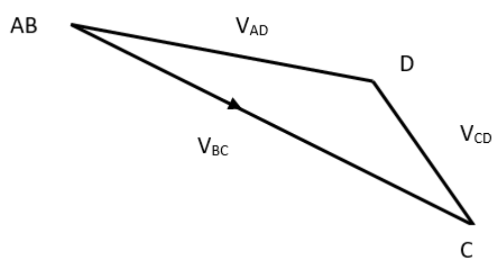

FIGURE 2.17
VELOCITY DIAGRAM OF THE FOUR BAR CHAIN

From measurement

$$V_{AD} = 26.66mm = 0.1333m/s$$

$$V_{CD} = 0.85mm = 0.00425m/s$$

$$V_{BC} = 35mm = 0.175m/s$$

$$\therefore w_{BC} = V_{BC}/BC$$

$$w_{BC} = \frac{0.175}{0.021} = 8.33rad/sec$$

The torque at B (T_B) was gotten from (76) as shown below;

$$T_A w_{AD} = T_B w_{BC} (76)$$

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The servo motor was attached at point A so the torque at A was equal to that of the servo.

$T_A = 2.5kg\ cm$

$= 2.5\ x\ 9.81\ x\ 10^{-2}$ (SG90 Servo Datasheet)

$T_A = 0.2453Nm$

From (76);

$$T_B = \frac{T_A w_{AD}}{w_{BC}} = \frac{0.2453\ x\ 9.52}{8.33}$$

$T_B = 0.28Nm$

F_G is the force of gripper. From Figure 2.18, F_G is can be gotten from (77).

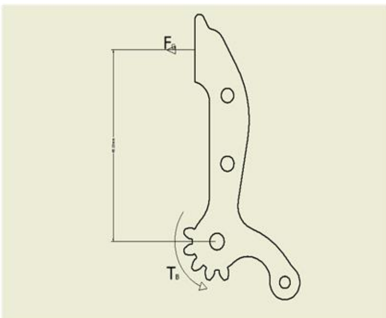

FIGURE 2.18
FORCES ACTING ON GRIPPER

$Tourque = Force * Distance (77)$

Hence;

$$F_G = \frac{T_B}{0.045226} \quad (78)$$

$F_G = 6.19N$

So the load lifted was held by a force of 6.19N. the maximum load that this force can lift was gotten by considering the effects of frictional force between the gripper material and the load material.

Figure 2.19 shows how the gripper holds the load. The higher the load magnitude, the higher the pressure needed to hold it. In this case, the higher the value of ‘P’, the higher the value of ‘FG’ required. A relationship was developed for FG and P in(80)

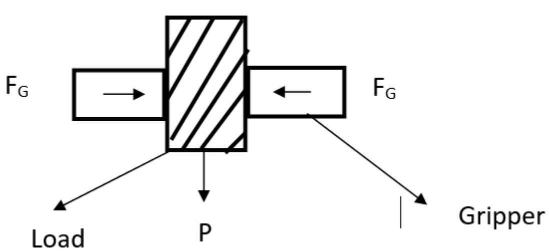

FIGURE 2.19
FRICTION ANALYSIS OF GRIPPER

$F = \mu R \quad (79)$

Where F is Frictional Force (N)
μ is the Coefficient of friction
R is the Reaction force (N)

It can hence be written that

$2F_G = \mu P \quad (80)$

$$P = \frac{2F_G}{\mu} \quad (81)$$

$\boldsymbol{P = 12.38}\mu^{-1} \quad (82)$

The value of μ varies with the material with which the load is made.

Since $\boldsymbol{P} = 146$g

$$\mu = \frac{12.38}{1.46} = 8.479$$

Maximum allowable friction coefficient for load and end effector material was 8.479

2.3.5 OVERTURNING AND SLIDING MOMENT

To prevent the arm from overturning when stretched to its full length, counter weight had to be introduced to the system. Figure2.20 shows that overturning will occur at point O.

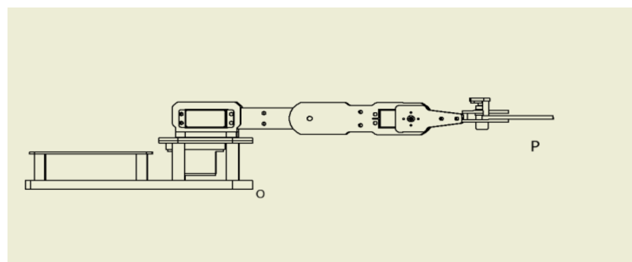

FIGURE 2.20
OVERTURNING MOMENT

The free body diagram of the arm including all the loads is shown in Figure 2.21;

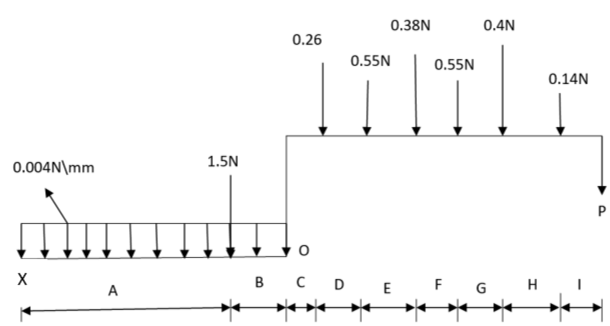

FIGURE 2.21
FREE BODY DIAGRAM

Where $A = 177mm$ $B = 45mm$ $C = 45mm$ $D = 45mm$

$E = 45 \quad F = 26mm \quad G = 33.6mm \quad H = 35mm$
$I = 61mm$

Overturning moment (M_T) was gotten by summing all the clockwise moment about O as shown in (83);

$$Overturning\ moment\ (M_o) = P\,(290.6) +).14\,(229.6) + 0.4\,(194.6) + 0.38\,(135) + 0.55\,(161) + 0.55(90) + 0.26\,(45) \quad (83)$$

$$M_o = 290.6P + 311.034 \quad (84)$$

$$M_o = 290.6\,(1.46) + 311.034 \quad (85)$$

$$M_o = 735.31Nmm$$

Restoring moment was gotten by summing counter clockwise moment about point O as shown below;

$$Restoring\ Moments\ (M_R) = 1.5\,(45) + 0.004\,(222)\,(111)$$

$$M_R = 166.07Nmm$$

Overturning moment is greater than the restoring moments (M_0>M_R). To prevent the structure from overturning, a counter weight was introduced to nullify the resultant moment (ΣM).

$$\Sigma M = M_O - M_R$$

$$\Sigma M = 735.31 - 166.07$$

$$\Sigma M = 569.24Nmm$$

The counter weight wasassumed to be placed 30mm away from X. To prevent overturning from ever occurring, a factor of safety against overturning (F_O = 1.5) was introduced;

$$F_O = \frac{Restoring\ Moment\ (M_R)}{Overturning\ Moment\ (M_O)} \quad (86)$$

$$1.5 = \frac{M_R}{M_O}$$

Considering counter weight (W_C)

$$1.5 = \frac{M_R + W_C(192)}{M_O}$$

$$1.5 = \frac{166.07 + 192W_C}{659.8}$$

$$W_C = \frac{1.5\,(659.8) - 166.07}{192}$$

$$W_C = 4.3N$$

$$W_C = 428.97g$$

Where W_C is the magnitude of the required counter weight and was placed 30mm from point X. With respect to sliding, it was ensured that no horizontal force was acting on the system. Hence (87) was obeyed.

$$\Sigma F_H = 0 \quad (87)$$

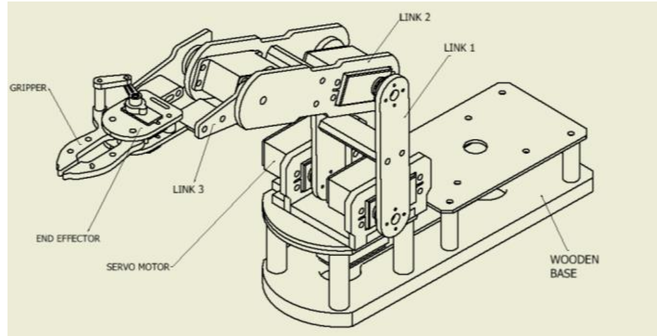

FIGURE 2.22
COMPLETE DIAGRAM OF ROBOTIC ARM

III. PERFORMANCE EVALUATION

In a case of overloading, maximum deformation would occur in link 3, so this link would fail first if the value is exceeded followed by the next link with lowest factor of safety(FOS). The factor of safety of each link is computed in table 3.1. the actuator based FOS tells how safely the servos at that joint will lift the load while the material based FOS shows how safe the material is at that value of load. The higher the FOS, the better.

TABLE 3.1

Link		**P-value (g)**	**Factor of safety**
1	Actuator based	498.3	3.40
	Material based	10092.9g	69.13
2	Actuator based	349.6	2.39
	Material based	2928.6	20.06
3	Actuator based	661	4.53
	Material based	145.8	0.99

FACTOR OF SAFETY IN EACH LINK

At a load of 146g, link (3) has a low material based factor of safety. This means the material will fail first. To avoid this, we will choose the value of P to be 120g. Table 3.2 shows the increased value for factor of safety for each link.

TABLE 3.2

Link		P-value (g)	Factor of safety
1	Actuator based	498.3	4.2
	Material based	10092.9g	84.1
2	Actuator based	349.6	2.9
	Material based	2928.6	24.4
3	Actuator based	661	5.5
	Material based	145.8	1.2

ACTUAL FACTOR OF SAFETY IN EACH LINK

The deformation in the arm at 146g load proved minimal and safe from our calculation and is also verified by Autodesk inventor Stress analysis software as shown in Figure 3.1. when the load safe load is exceeded, say a 500N load is applied, severe deformation is observed as shown in Figure 3.2.

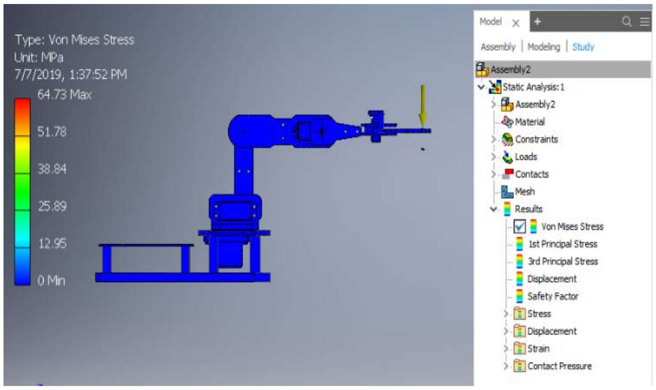

FIGURE 3.1
DEFORMATION AT 146G LOADING

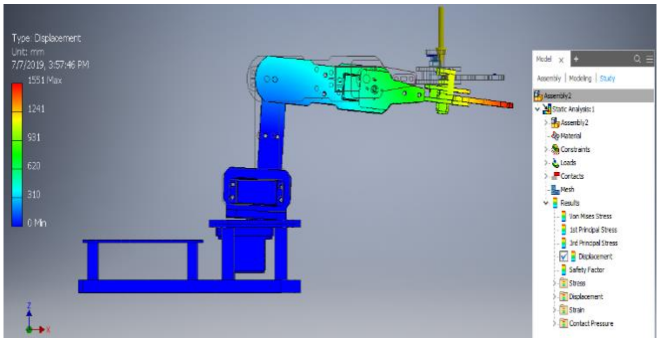

FIGURE 3.2
DEFORMATION AT 50000G LOADING

The transformation matrix derived for combined rotation and translation was very effective in tracking the position of the end effector. Although it didn't perform as expected due to lack of rigidity in the joints of the robot. With better prototyping tools and resources, an approximately perfect performance can be achieved.

IV. RESULTS AND DISCUSSION

The safe maximum load for the arm to lift was obtained as 120g. The weakest link in the arm is link 3 with a safety factor of 1.2. There are still some unanswered questions with maximum load with respect to end effector/load friction coefficient. But this can only be obtained if the coefficient of friction between the load material and the end effector material is known. Safe operating temperature range is 0 ℃ $\leq$T $\leq$60 ℃. So all parts and components will maintain their efficiency within this temperature range.

V. CONCLUSION

The aim was to develop a low grade robotic arm with basic abilities so factories with low budget can easily purchase and maintain the arm. The aim was achieved. To increase the load, the arm can lift, the links can be made with stronger material.

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